# The Study of a Digital Equivalent Capacitor Circuit 

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#### Abstract

Resistors, inductors and capacitor are indispensable components in modern electronics field. While digital potentiometer is capable of conducting digital adjustments on resistances, the flexibility of adjusting the capacitance of most capacitors is still very limited even until now. This paper introduced a digital capacitor circuit based on impedance transformations. The impedance transformation circuit was set up by assembling an integrated operational amplifier and small amounts of reststors and capacitors. The capacitance of this circuit can be adjusted by controlling the resistance of a certain resistor using a digital potentiometer. Simulation results showed this circuit has many advantages including great flexibility if contolted with a single chip, wide capacitance adjustment range, insensitivity to temperature and pressure, and the size of the circuit remaining the same while the capacitance increases.


Keywords: Programmable; Digital capactor, Impedance transformation

## 1. Introduction



As the electronic technology deyelops, there are significantly increasing needs of digital programmable devices in this field Nov days, there are few digital adjustable capacitor products in the market. Most of a vailable capacitors use simple analog switches to control the capacitance This kind of capacitor has many disadvantages including size limitation and narrow adjustment range (only several to less than 20 pF ). Also, most equivalent capacitor circuits consist of capacitors and resistors connecting in series or parallel instead of just capacitors alone. This narrows the freedom of the resistor in the circuit and subsequently limits the range of the equivalent capacitance. The digital capacitor introcuced in this paper, using impedance changing circuit, yields pure capacitance equivalem outputs. In order to make the integration of the circuit easy, the traditional method of directly using analog switches to control the capacitance was abandoned. Insead, the capacitance was controlled by adjusting the resistance in the circuit. Using this method, the equivalent capacitance has large adjustable range and high accuracy, and the size of the equivalent circuit remains the same regardless the value of the capacitance output. This circuit, if adding a single chip to control the output capacitance, has the potential to be widely used in many different kinds of electronic devices.

## 2. Impedance Generator Principle

### 2.1. Equivalent Capacitor Circuit

The equivalent capacitor circuit based on an integrator is shown in Figure 1.


Figure 1. Equivalent Capacitor and Equivalent Circuit

It can be obtained from the circuit above that

$$
\begin{gather*}
V_{o} \cdot j \omega C_{1}\left(R_{3}+\frac{1}{j \omega C_{1}}\right)=V_{1}  \tag{1}\\
\frac{V_{1}-V_{o}}{R_{1}}=I_{1} \tag{2}
\end{gather*}
$$

Where $V_{1}$ and $V_{0}$ are the input and output voltages of the equivalent apacitor; $1 / j \omega C_{1}$ is the capacitance of the capacitor $C_{1} ; R_{1}$ and $R_{3}$ are the resistances of resistors $R_{1}$ and $R_{3}$ respectively; $I_{1}$ is the input current of the equivalent capacitor.

Combine Equation (1) and Equation (2)

$$
\begin{equation*}
R_{1}+\frac{1}{1 S C_{1} R}=\frac{V_{1}}{1} \tag{3}
\end{equation*}
$$

It can be seen from Equation 3 that the equivalent impedance

$$
\begin{equation*}
Z_{\text {in }}=\frac{V_{i}}{I_{i}}=R_{1}+\frac{C_{j} C_{1} \frac{R_{3}}{R_{1}}}{j} \tag{4}
\end{equation*}
$$

Therefore the valuê of the equivalent capacitance $C_{\mathrm{E}}=C_{1} R_{3} / R_{1}$.
$I_{\mathrm{L}}$ is the equiyalent leakage current passing through the equivalent capacitor and its value can becalculated using the following equation

$$
\begin{equation*}
I_{L}=\frac{V_{i e}+I_{i e} R_{3}}{R_{1}} \tag{5}
\end{equation*}
$$

where $V_{\mathrm{ie}}$ is the bias voltage; $I_{\mathrm{ie}}$ is the bias current.
An assurnption that $R_{1}=R_{2}$ was made while deriving equation (5) to minimize $I_{\mathrm{L}}$. Leakage current could potentially impact the stability of the capacitor, result electromagnetic problems and/or affect the frequency characteristic of the circuit. A high-quality operational amplifier is usually desirable in order to control the leakage current. Also, the equivalent capacitor would be non-polarized only if the capacitor $C_{1}$ is non-polarized. As it can be seen from the equations above, resistor $R_{1}$ is important in determining the value of the equivalent impedance $Z_{\text {in }}$; resistor $R_{2}$ together with the operational amplifier form an in-phase gain amplifier circuit; $R_{1}$ and $R_{3}$ together determine the value of the equivalent capacitance $\mathrm{C}_{\mathrm{E}}$.

The value of the input impedance of the equivalent capacitor is computed using the equation below

$$
\begin{equation*}
Z_{i n}=R_{1}+\frac{1}{1+j \omega C_{E}}=R_{1}+\frac{R_{1}}{1+j \omega C_{1} R_{3}} \tag{6}
\end{equation*}
$$

It can be seen from Equation (6), this equivalent circuit, consisting of a capacitor and a resistor connecting in series, is not a pure-capacitance equivalent capacitor.

### 2.2. Impedance Generator Principle

The impedance transformation circuit which consists of operational amplifiers and resistors is shown in Figure 2.


Figure 2. Impedance Converting Circuit
The following equations can be obtained aecording to Kirhhoff's law (assuming the operational amplifier $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ in Figure 2 are deal)

where $e_{1}, e_{2}, e_{3}, e_{4}$, and $e_{5}$ are node voltages of the points shown in Figure $2 ; Z_{1}, Z_{2}, Z_{3}, Z_{4}$ and $Z_{5}$ are the resistance values of resistors $Z_{1}, Z_{2}, Z_{3}, Z_{4}$ and $Z_{5}$, respectively. $i_{1}$ is the value of che current passing through resistor $Z_{1}$.

Afeording to Equation (7) ~ (10), the equivalent impedance $Z_{\text {in }}$ in Figure 2 can be expressed as

$$
\begin{equation*}
Z_{i n}=\frac{\mathrm{e}_{1}}{i_{1}}=\frac{Z_{1} Z_{3} Z_{5}}{Z_{2} Z_{4}} \tag{11}
\end{equation*}
$$

Using Equation (7) to (10), we can also get

$$
\begin{equation*}
e_{4}=e_{1}\left(1+\frac{Z_{4}}{Z_{5}}\right) \tag{12}
\end{equation*}
$$

From Equation (7), (9) and (12), the following equation is obtained

$$
\begin{equation*}
e_{2}=e_{1}\left(1-\frac{\mathrm{Z}_{2} \mathrm{Z}_{4}}{Z_{3} Z_{5}}\right) \tag{13}
\end{equation*}
$$

According to Equation (11) ~ (13), it can be known that $Z_{\text {in }}$ is proportional to $Z_{1}, Z_{3}$ and $Z_{5}$, and inversely proportional to $Z_{2}$ and $Z_{4}$. Phase and amplitude changes of $e_{1}$ are limited by $e_{2}, e_{4}, Z_{2}, Z_{3}, Z_{4}$ and $Z_{5}$. In order to let operational amplifier work within its dynamic range, $e_{4}$ and $e_{2}$ should work within the maximum voltage output ranges of operational amplifier $A_{1}$ and $A_{2}$, respectively. Therefore $Z_{2}, Z_{3}, Z_{4}$ and $Z_{5}$ together determine the dynamic range of the operational amplifier's output voltages and $Z_{1}$ determines the dynamic range of the input currents.

The resistors in this circuit must meet the requirements that the operational amplifier must form a DC loop and the phase change of the negative feedback loop cannot cause oscillation. For example, if $Z_{1}, Z_{2}, Z_{3}, Z_{4}$ and $Z_{5}$ are all capacitors, the circuit cannot work due to lack of negative feedback loop.

From Equation (11) we can see when $Z_{1}=R_{1}, Z_{2}=R_{2}, Z_{3}=R_{3}, Z_{4}=R_{4}$ and $Z_{5}=1 / j C_{6} C_{5}, Z_{\text {in }}$ equals to the impedance of a capacitor with an equivalent capacitance $C_{\mathrm{E}}$ expressed as

$$
\begin{equation*}
C_{E}=C_{5} \frac{R_{2} R_{4}}{R_{1} R_{3}} \tag{14}
\end{equation*}
$$

## 3. Digital Equivalent Capacitor Circuit

The digital equivalent capacitor circuit is shown below in Figure 3.


Figure 3. Digital Equivalent Capacitor Circuit
Operational amplifier $\mathrm{A}_{2}$ and $\mathrm{A}_{4}$ in Figure 3 serve as buffers. When the equivalent capacitor works within its high frequency range, they increase the output currents of operational amplifier $\mathrm{A}_{1}$ and $\mathrm{A}_{3}$. The range of output equivalent capacitance is controlled by the analog switch SGM3005. An X9110 digital potentiometer with an adjustable range of $0-100 k \Omega$ is used serving as resistor $R_{4}$. The X9110's SPI serial port SI is used to adjust the value of the equivalent capacitance. The resistances of resistor $R_{1}$ and $R_{3}$ are $100 \Omega$ and $1 k \Omega$ respectively. The unit of the equivalent capacitance is converted by changing the resistance of resistor $R_{2}$ and the capacitance of capacitor $C_{5}$ with $S_{0}$ and $S_{1}$ respectively. The output gears of the equivalent capacitor are shown in Table 1.

Table 1. The Output Gears of the Equivalent Capacitor

| $\mathrm{S}_{1} \mathrm{~S}_{0}$ | $\mathrm{R}_{2}(\Omega)$ | $\mathrm{C}_{5}$ | Unit of $\mathrm{C}_{\mathrm{E}}$ |
| :---: | :---: | :---: | :---: |
| 00 | 100 | 10 pF | pF |
| 01 | 100 | 10 uF | nF |
| 10 | 100 K | 10 pF | uF |
| 11 | 100 K | 10 uF | mF |

According to Figure 3 and Table $1, S=2^{1} \times S_{1}+2^{0} \times S_{0}$. The equivalent capacitance can be expressed using the following equation

$$
\begin{equation*}
C_{E}=C_{5} \frac{R_{2} R_{4}}{R_{1} R_{3}}=D 10^{3 S-12} \tag{15}
\end{equation*}
$$

where $C_{\mathrm{E}}$ is the equivalent capacitance; $C_{5}$ is the capacitance of the capacitor $C_{5} ; \mathrm{D}$ is an integer number between 0 and 999.

## 4. Integrated Chip of the Digital Equivalent Capacitor

An integrated digital adjustable capacitor chip can be achieved by integrating the digital equivalent capacitor in a single chip as shown in Figure 4.


Figure 4. Pins oythe Digital Adjustable Equivalent Capacitor
Table 2. Pin Descriptions

| PIN | SYMBOI | FUNCTION | PIN | SYMBOL | FUNCTION |
| :---: | :---: | :--- | :---: | :---: | :--- |
| 1 |  | Analog Supply Voltage | 8 | V- | Analog Supply Voltage |
| 2 | SI | Serial data input | 9 | A0 | Device Address |
| 3 | SCK | Serial Clock | 10 | $\overline{\mathrm{CS}}$ | Chip select pin |
| 4 | $\mathrm{~S}_{1}$ | Unit converting controllers | 11 | Cin | Input of the equivalent capacitor |
| 5 | $\mathrm{~S}_{0}$ | Unit converting controllers | 12 | $\overline{\mathrm{~W} ~ P}$ | Hardware Write Protect |
| 6 | DGND | Digital ground | 13 | $\overline{\mathrm{HOLD}}$ | Device Select |
| 7 | AGND | Analog ground | 14 | Vcc | Power-supply |

## 5. Comparison of Experimental and Simulation Results

The low pass filtering circuits of a real capacity and resistances Figure 5 and the equivalent capacitance and resistances Figure 6 are shown below. The cut-off frequency is
318.42 kHZ . Figure 7 shows the frequency amplitude characteristics curve of the real capacitor. Figure 8 shows the frequency amplitude characteristics curve of the equivalent capacitance. The simulation results showed that the amplitude frequency characteristic curves of the equivalent capacitor and the real capacitor fit very well.


Figure 5. Real Capacity Simulation Circuit


Figure 6. Equivalent Capacitance Simulation Circuit


Figure 7.Cuve of Rearcapacity

Figure 8. Curve of Equivalent Capacitance

## 6. Application of Digital Equivalent Capacitor

The circuit of multivibrator consisting of an NE555 timer and an equivalent capacitor is shown in Fidure 9. The charge-discharge capacitor in the multivibrator was replaced by an equivale it orpacitor. The oscillating period of the multivibrator is controlled by adjusting the capacitance of the equivalent capacitor CE using a single chip. The single chip used in this paper was ATmega8_06, the serial communication interface (SCI) of which PB3 was connected to the serial interface (SI) of the equivalent capacitor to control the capacitance input. The single chip's pin PB5 was connected to the SCK of the equivalent capacitor to provide clock signal. PC1 and PC2 were connected to $\mathrm{S}_{1}$ and $\mathrm{S}_{0}$ respectively to adjust the unit of the capacitance. CS and $\mathrm{A}_{0}$ were control by $\mathrm{PC}_{4}$ and $\mathrm{PC}_{3}$ respectively.


Figure 9. The Multivibrator Consisting of NE555 Timer and Equivalent Capacitor

## 7. Conclusion

The simulation and experimental results showed that the amplitude frequency characteristics of the equivalent capacitor and real capacitor yere basially the same. The differences between the simulation and experimental results can be explained by the non-ideality of the experimental devices such as the operational amplifer. Those differences could be minimized if considering this factor while setting up the circuit. With different amplifiers or circuit settings, different types e.g. with high power, high frequency, high voltage or high precision etc.) of equivalent capacitance circuits can be obtained. Using single chips to control the value of the equivalent capaciance is able to make the equivalent capacitor programmable. This design has the potential to bring great convenience to electronic product designs and scientific researches integrating the programmable equivalent capacitor circuit into a single chpa shown in-Figure 3, a new programmable capacitor chip will be obtained, which will greatly benefit the tield of digital electronic circuit.

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