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Backlash analysis of RV reducer based on Error Factor Sensitivity and Monte-Carlo Simulation

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Abstract

Error factors of RV reducer are not completely considered, leads to backlash precision is limited, so an improved backlash estimation model is proposed. RV reducer structure and working principle is deeply analyzed, a variety of error factors are considered, and the backlash estimation model is improved according to error propagation. Furthermore, sensitivity of all the error factors are analyzed, RV reducer backlash is obtained through Monte-Carlo simulation, and the simulation results are compared with the traditional computation results. It is conductive to taking fix action in the design stage, reducing backlash, improving the transmission accuracy.

Keywords: *RV reducer; backlash; sensitivity analysis; Monte-Carlo simulation*

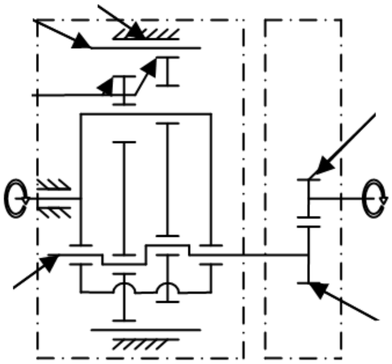
1. Introduction

RV reducer is widely used in national defense, aerospace and industrial production. It has been a main topic of much discussion as its transfer accuracy is seriously affected by backlash. Blanche studied rotating accuracy of cycloid and lantern gear reducer using pure geometry method [1]. Wu Yongkuan established mathematical model for the geometric backlash of high accurate RV reducer used in robot [2]. Han Linshan discussed dynamic transmission accuracy of the RV reducer [3-4]. However, all the error factors of RV reducer were not completely considered, as a result, the actual situation of RV reducer was not truly reflected. Therefore, all the error factors from multi sources were comprehensively analyzed, and exact results of RV reducer backlash were given. What’ s more, the sensitiveness of the error factors was discussed, and the RV reducer backlash was simulated through Monte-Carlo method. By this way, it could make a great contribution to take corrective measures to reduce backlash to improve the transmission accuracy at the design stage.

2. Structure of RV Reducer

The rated input speed of RV reducer is 15 r/min, and rated output torque is 784 Nm. Mechanism block diagram of the reducer is shown in Figure 1, which is mainly composed of input gear, three planet gears, three crankshafts, two cycloidal and pin gear pairs, hold flange and output.

The RV reducer is a 2-stage reduction gear. The first stage is spur gear reduction. An input gear engages with and rotates spur gears that are coupled to crankshafts. The second stage is epicyclical gear reduction. Crankshafts driven by the spur gears cause an eccentric motion of two epicyclical gears called RV gears that are offset 180 degrees from one another to provide a balanced load. The eccentric motion of the RV gears causes engagement of the cycloidal gear teeth with cylindrically pins located around the inside edge of the case. In the course of one revolution of the crankshafts the teeth of the RV gear move the distance of one pin in the opposite directions of the rotating cranks. The motion of the RV gear is such that the teeth remain in close contact with the pins and many teeth share the load simultaneously. The case is fixed, so the shaft is output.



1-involute planet gear 2-center gear 3-case 4-pin 5-cycloidal gear 6-crankshaft

Figure 1. RV reducer transmission diagram

3. RV Reducer Backlash Analysis

3.1. Influencing factors of backlash

The influencing factors of the involute planetary transmission and the cycloid planetary transmission are as follows:

(1) The involute transmission’s error factors

1) Deviation of base tangent length

2) Deviation of central circular

3) Radial run-out

4) Parallelism of axis

5) Radial composite error (including tooth profile error, cumulative pitch error, tooth alignment error)

6) Eccentricity of rolling bearing

(2) The cycloidal transmission’s error factors

1) Mending tooth shape of the cycloidal gear

2) Total cumulative pitch error of the cycloidal gear

3) Radial run-out of the cycloidal gear

4) Radius of the circle error of the pin gear

5) Radius of the pin gear error

6) Fit of clearance of the gear pin and hole

7) Total cumulative pitch error of the pins

8) Mending tooth shape error of the cycloidal gear

9) Clearance of crankshaft

10) Eccentricity of crankshaft

3.2. Mathematical model of backlash

The backlash of RV reducer is composed of the involute planetary transmission error and the cycloid planetary transmission error. According to the relationship of error transmission, the mathematical model of backlash is shown in (1),

$$\Delta\varphi = \frac{180\times 60\Delta j_1}{i_{16}^{5}\pi r_1} + \frac{180\times 60}{\pi a z_c}\Delta j_2 + \frac{180\times 60}{\pi a_0}\Delta j_3 \tag{1}$$

Where Δ ф is total backlash of transmission system to the output shaft, Δ j1 is the involute gear transmission part of the backlash, Δ j2 is the cycloidal gear transmission part of the backlash, Δ j3 is rolling bearing part of the backlash, is reduction ratio of RV reducer.

Obviously, the errors of all the influencing factors should be calculated firstly. The backlash expressions are listed in Table 1.

Table 1. Backlash caused by the error factors

No	Error factors	Backlash	No	Error factors	Backlash
1	Deviation of base tangent length	$-\frac{E_{ss}+E_{si}}{2\cos\alpha}$	12	Moving distance modification	$-2\Delta r_p\sqrt{1-K_1^2}$
2	Deviation of central circular	$\Delta f_a \tan\alpha$	13	Radial run-out of the cycloidal gear	$\frac{F_r}{4}$
3	Radial run-out	$F_t \tan\alpha$	14	Radius of the circle error of the pin gear	$\delta r_p\sqrt{1-K_1^2}$
4	Parallelism of x axis	$\frac{1}{2} f_x \tan\alpha$	15	Radius of the pin gear error	δr_{rp}
5	Parallelism of y axis	$\frac{1}{2} f_y$	16	Fit of clearance of the gear pin and hole	$\frac{d_{j\max} - d_{j\min}}{2}$
6	Tooth profile error	$\frac{f_f}{\cos\alpha}$	17	Pitch error of the pins	$\frac{F_p' K_1}{2}$
7	Cumulative pitch error	f_{Pt}	18	Modification error	$\delta\Delta r_{rp}$
8	Tooth alignment error	f_β	19	Moving distance modification error	$\delta\,\Delta r_p\sqrt{1-K_1^2}$
9	Eccentricity of rolling bearing	$2e_g \tan\alpha$	20	Clearance of crankshaft	$\frac{\Delta u_{\max}-\Delta u_{\min}}{2}$
10	Pitch error of the cycloidal gear	$F_p K_1$	21	Eccentric error of crankshaft	$2k_n\delta a$
11	Modification of equidistance	$2\Delta r_{rp}$			

Thus the Eq. (1) can be rewritten,

$$\Delta\varphi = \frac{180\times 60}{i_{16}^{5}\pi r_1}\sum_{i=1}^{8}\Delta j_i + \frac{180\times 60}{\pi a z_c}\sum_{i=9}^{19}\Delta j_i + \frac{180\times 60}{\pi a_0}\sum_{i=20}^{21}\Delta j_i \tag{2}$$

3.3. Calculation of backlash

The circumference backlash ΔE_s caused by deviation of base tangent length can be written in (3)

$$j_{t\Delta E_s} = -\frac{\Delta E_s}{\cos\alpha} \tag{3}$$

Where Ess and Esi are upper deviation and lower deviation. Δ Es is noted as a random variable, which obeys normal distribution according to engineering experience. Shown in Fig.2.The digital features ofΔ Es are shown in (4) and (5),

$$\mu(\Delta E_s) = -\frac{E_{ss}+E_{si}}{2\cos\alpha} \tag{4}$$

$$\sigma(\Delta E_s) = -\frac{E_{ss}+E_{si}}{6\cos\alpha} \tag{5}$$

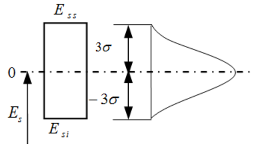


Figure 2. Statistical distribution of the deviation

To suppose all the error factors are random variables which obey Normal distribution. To refer GB2363-90 and JB 10419-2005, tolerance of all the factors are substituted into equations above. The results are shown in Table 2.

Digital feature in table 2 were introduced into Eq. (2), and then the total system backlash is obtained, which is φ∑=μ∑±3σ∑=6.12±1.77 (arc min). The expectation is 6.12(arc min), and the fluctuation range from 4.35(arc min) to 7.89(arc min).

3.4. Sensitivity analysis

Each error caused by the influencing factor is considered to be a random variable, and is recorded as $x=[x_1,x_2,\ldots,x_k]$, then $\Delta x=[\Delta x_1,\Delta x_2,\ldots,\Delta x_k]$. Backlash of RV reducer can be denoted as $\varphi(x)=\varphi(x_1,x_2,\ldots,\ x_k)$. From above, the sensitivity of each error factor can be written as follows:

$$S_i = \nabla\varphi(x) = \left[\frac{\partial\varphi}{\partial x_1},\frac{\partial\varphi}{\partial x_2},\ldots,\frac{\partial\varphi}{\partial x_k}\right] = [S_1,S_2,\ldots,S_k] \tag{6}$$

Table 2. Digital feature and results of the error factors

No.	Error factors	Digital feature	
		μ	σ
1	Deviation of base tangent length	$-\frac{E_{ss}+E_{si}}{2\cos\alpha}$=-15.963	$-\frac{E_{ss}+E_{si}}{6\cos\alpha}$=-5.321
2	Deviation of central circular	0	$\frac{1}{3}\Delta f_a\tan\alpha$=1.577
3	Radial run-out	0	$\frac{F_t\tan\alpha}{3}$=2.184
4	Parallelism of x axis	$\frac{1}{2}f_x\tan\alpha$=1.902	$\frac{1}{6}f_x\tan\alpha$=0.364
5	Parallelism of y axis	$\frac{1}{2}f_y$=1.5	$\frac{1}{6}f_y$=0.5
6	tooth profile error	$\frac{f_f}{\cos\alpha}$=9.578	$\frac{f_f}{3\cos\alpha}$=3.193
7	cumulative pitch error	0	$\frac{1}{3}f_{pt}$=2.167
8	Tooth alignment error	0	$\frac{1}{3}f_{\beta}$=2
9	Eccentricity of rolling bearing	0	$\frac{2e_g\tan\alpha}{3}=3.6\times10^{-4}$
10	Total cumulative pitch error	0	$\frac{1}{3}F_pK_1$=7.68
11	Modification of equidistance	0	$\frac{2\Delta r_{rp}}{3}$=3.73
12	Moving distance modification	0	$\frac{-2\Delta r_p\sqrt{1-K_1^2}}{3}$=5.04
13	Radial run-out of the cycloidal gear	0	$\frac{F_r}{12}$=1.917
14	Radius of the circle error of the pin gear	0	$\frac{1}{3}\delta r_p\sqrt{1-K_1^2}$=1.241
15	Radius of the pin gear error	0	$\frac{1}{3}\delta r_{rp}$=1.333
16	Fit of clearance of the gear pin and hole	$\frac{\delta_{j\max}-\delta_{j\min}}{2}$=8.75	$\frac{\delta_{j\max}-\delta_{j\min}}{6}$=2.92
17	Total cumulative pitch error of the pins	$\frac{F_p^{'}K_1}{2}$=18	$\frac{F_p^{'}K_1}{6}$=6
18	Modification of equidistance error	0	$\frac{\delta\Delta r_{rp}}{3}$=0.667
19	Moving distance modification error	0	$\frac{\delta\Delta r_p}{3}\sqrt{1-K_1^2}$=4.963
20	Clearance of crankshaft	$\frac{\Delta u_{\max}+\Delta u_{\min}}{2}$=2.5	$\frac{\Delta u_{\max}-\Delta u_{\min}}{6}$=0.5
21	Eccentric error of crankshaft	0	$\frac{2k_n\delta a}{3}=3.3\times10^{-5}$

Normalized the sensitivities by (7), and their weight are obtained.

$$g_i=\frac{S_i}{\sum_{i=1}^{k}S_i} \tag{7}$$

The results are listed in Table 3.

Table 3. Sensitivities and weights of error factors

No.	Error factors	Sensitivity	Weight
1	Deviation of base tangent length	$\frac{180\times60}{i_{16}^{5}\pi r_1\cos\alpha}$	1.799×10^{-3}
2	Deviation of central circular	$\frac{2\times180\times60\times\tan\alpha}{i_{16}^{5}\pi r_1}$	1.157×10^{-3}
3	Radial run-out	$\frac{2\times180\times60\times\tan\alpha}{i_{16}^{5}\pi r_1}$	1.157×10^{-3}
4	Parallelism of x axis	$\frac{180\times60\times\tan\alpha}{i_{16}^{5}\pi r_1}$	0.643×10^{-3}
5	Parallelism of y axis	$\frac{180\times60}{i_{16}^{5}\pi r_1}$	1.671×10^{-3}
6	Tooth profile error	$\frac{2\times180\times60}{i_{16}^{5}\pi r_1\cos\alpha}$	3.470×10^{-3}
7	Cumulative pitch error	$\frac{2\times180\times60\times\tan\alpha}{i_{16}^{5}\pi r_1}$	1.286×10^{-3}
8	Tooth alignment error	$\frac{2\times180\times60}{i_{16}^{5}\pi r_1}$	3.341×10^{-3}
9	Eccentricity of rolling bearing	$\frac{2\times180\times60}{i_{16}^{5}\pi r_1}$	3.341×10^{-3}
10	pitch error of the cycloidal gear	$\frac{2\times180\times60}{\pi \mathrm{r}_c^{'}}$	0.129
11	Modification of equidistance	$\frac{2\times180\times60}{\pi \mathrm{r}_c^{'}}\sqrt{1-K_1^2}$	0.080
12	Moving distance modification	$\frac{2\times180\times60\times K_1}{\pi \mathrm{r}_c^{'}}$	0.101
13	Radial run-out of the cycloidal gear	$\frac{180\times60}{2\pi \mathrm{r}_c^{'}}$	0.032
14	Radius of the circle error of the pin gear	$\frac{2\times180\times60}{\pi \mathrm{r}_c^{'}}\sqrt{1-K_1^2}$	0.080
15	Radius of the pin gear error	$\frac{2\times180\times60}{\pi \mathrm{r}_c^{'}}$	0.129
16	Fit of clearance of the gear pin and hole	$\frac{180\times60}{\pi \mathrm{r}_c^{'}}$	0.064
17	Total cumulative pitch error of the pins	$\frac{180\times60\times K_1}{2\pi \mathrm{r}_c^{'}}$	0.025
18	Modification of equidistance error	$\frac{2\times180\times60}{\pi \mathrm{r}_c^{'}}$	0.129
19	Moving distance modification error	$\frac{2\times180\times60}{\pi \mathrm{r}_c^{'}}\sqrt{1-K_1^2}$	0.080
20	Clearance of crankshaft	$\frac{180\times60}{\pi a_0}$	0.064
21	Eccentric error of crankshaft	$\frac{180\times60\times k_n}{\pi a_0}$	1.799×10^{-5}

Conclusions are obtained from Table 3,

(1) The error factors in 1st stage have less influence on system backlash. Their sensitivity values are less than 0.1.

(2) The error factors in 2nd stage have a major influence on system backlash, especially radius of the pin gear error, total cumulative pitch error of the cycloidal gear and modification of equidistance error. These factors with the largest sensitivity are the greatest-affected factors. However, the sensitivity of eccentric of crankshaft is very small. So the system backlash is lighted influenced by them, which can be ignored.

(3) To find the important influencing error factors of backlash is in favor of seizing the main contradiction during the design. What’ s more, we can target to take corrective action and reduce system backlash quickly efficiently.

4. RV Reducer Backlash Monte-Carlo Simulation

Random number is generated by Monte Carlo simulation according to certain probability distribution [5]. In fact, the RV reducer backlash is combined of several kinds of random variables which obey normal distribution. It’ s difficult to determine the system backlash distribution and parameters composed of different probability distribution by traditional method. As a result, influences cased by multiple error factors are considered comprehensively, Monte Carlo simulation is utilized to simulate all kinds of situation may appear. By this way, the RV system backlash can be computed quickly and easily. It can provide decisions basis for the backlash control.

4.1. Monte Carlo simulation procedure

(1) To determine the probability distribution and digital feature, and calculate their inverse transformation.

(2) To program by MATLAB, using uniform distributed random number generator to generate 1000 random numbers. Substitute the digital feature of the error favors into inverse transformation, and then the values of simulation are taken.

(3) The simulation results are obtained by 1000 sampling values of RV transmission backlash.

4.2. Inverse transformation of error factors

If R1 and R2 are both random variable of uniform distribution in [0, 1], the inverse transformation generated normal distribution variable X can be written by (8)

$$X = \mu + \sigma\sqrt{-2\ln R_1}\sin(2\pi R_2) \tag{8}$$

Where μ and σ are digital feature of error factors[6].

5. Results Analysis

Monte-Carlo simulation is carried out in MATLAB according to the methods above, as shown in Figure 4.

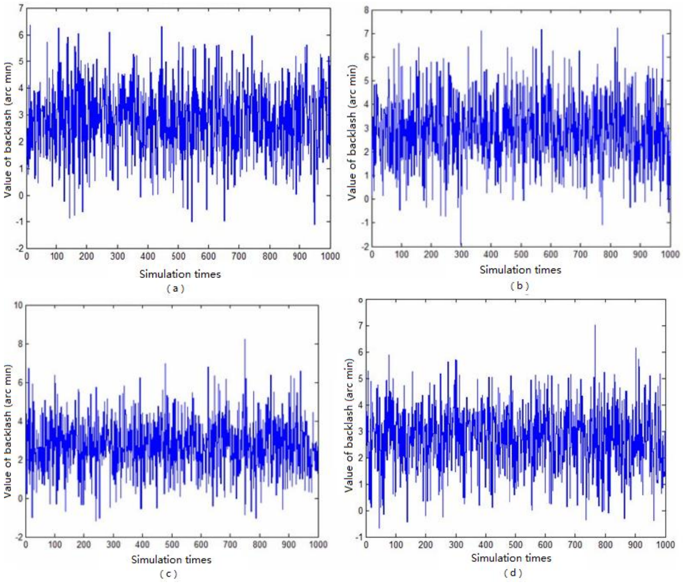


Figure 4. Simulation results

(a) In the 1st Figure, when the precision in the two stages is 6-grade, the system backlash is less than 8(*arc* min) in all probability which is consistent with the result of direct calculation. From the actually simulation, system backlash range is from 2(*arc* min) to 6(*arc* min).

(b) In the 2nd Figure, if the precision is 6-grade in the second stage and the precision in the first stage is 5-grade. Total backlash is improved little.

(c) In the 3rd Figure, if the precision is 5-grade in the second stage and the precision in the first stage is 6-grade. The backlash change heavily. Obviously, the accuracy of the second stage plays an important role in system backlash.

(d) In the 4th Figure, the precision of error factors which has larger sensitivity are changed to be 5-grade according to Table 2, the system backlash reduced greatly.

6. Conclusion

In this paper, structure of the RV reducer and its backlash has been researched. Furthermore, sensitivity of error factors that influence RV transmission backlash has been analyzed. With the help of statistical knowledge, each factor's digital feature has been deduced. The backlash of RV reducer is simulated by using Monte-Carlo. Although the simulation has error, system backlash can be predicted in the design stage, especially for mass production, it has very practical significance.

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