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Low Bit Rate Analysis of Adaptive Synthesis Filter Banks for Image Compression

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Abstract

The aim of this paper is to analyze the performance of adaptive synthesis filter banks for image compression at low bit rates. In adaptive synthesis filter banks, analysis section comprises of linear phase filters whereas, synthesis section manages a combination of linear phase and non linear filters (in form of delay filters). The adaptive synthesis filter banks have advantages over conventional adaptive filter banks because they have no synchronization issues between analysis and synthesis filters and are compatible with the existing subband coding systems. Their performance is evaluated against various compression ratios by using biorthogonal filters '4.4' and '5.5' which suggest adaptive synthesis filter banks achieve significant gain in PSNR at compression ratio 0.3 ~ 0.6 bit per pixel.

Keywords: *Compression ratio, adaptive synthesis filter banks, peak signal to noise ratio, biorthogonal filters, bit rate*

1. Introduction

Adaptive analysis and adaptive synthesis filter banks have been widely used over two decades in many application areas for time-frequency decomposition, processing, and reconstruction. In many of these applications, including speech, audio, image, and video subband coding systems, the signal properties and statistics are varied by changing the analysis and synthesis filters coefficients in response to the input [1-4]. This can be done by dynamically switching back and forth among analysis/synthesis filters with different spectral and temporal (step response) characteristics.

For image compression applications, the subbands are quantized, which in turn introduces noise into the system. The traditional approach is to use high quality exact or near exact reconstruction analysis-synthesis filters, in conjunction with good quantization and entropy coding schemes. This works well relatively speaking, but naturally reconstruction quality degrades when bit rates are lowered. However, this longstanding challenge has been resolved by designing adaptive filter banks (in form time varying filter banks) for subband image compression systems that have shown improved reconstruction quality at low bit rates. A degree of performance improvement has been reported, but a major issue of these filter banks however, is that the synthesis

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filters must be changed in lock step with the analysis filters in order to perform reconstruction, which requires dynamic synchronization. Furthermore, they become more complex as the interval between switching decreases and the number of switches increases. This issue was explored by Arrowood [5-6] that synchronization could be done in either a backward adaptive or forward adaptive mode. But in either way, it adds a layer of computational overhead to the encoder, which in many situations is unattractive.

Adaptive synthesis filter banks [7] have no issue of maintaining dynamic synchronization between the analysis and synthesis filters as analysis section is non adaptive, which also simply the operation of filter banks. They have the ability to improve the reconstruction quality in particular at lower bit rates, by exploiting the phase diversity of the synthesis section in an adaptive mechanism. Such systems are also compatible with the existing sub band coding systems.

In the sections to come, we first present adaptive synthesis filter banks in details and then quality measures of image compression system. Afterwards, simulations are performed on number of bench mark images by using bi-orthogonal filters ‘4.4’ and ‘5.5’ to evaluate the performance of adaptive synthesis filter banks at low bit rates and finally results are discussed and concluded.

2. Adaptive Synthesis Filter Banks

In adaptive synthesis filter banks, the analysis section is conventional, while the synthesis section is adaptive. The motivation in such type of filter banks comes from the recognition that the phase diversity (or equivalently diversity in group delay) in the synthesis section, which is comprehensively exploited. Consider the fact that quantization error in a coded image is directly related to the signal amplitude and that the characteristics of the error are influenced by the group delay characteristics of the synthesis filters. The adaptive synthesis filter banks implicitly reconstruct the input image with a multiplicity of reconstruction filters, each with different group delay characteristics but same magnitude response. Each of these filter pairs (low pass and high pass) generates a unique reconstruction. As the reconstructions are performed on the same quantized signal, the resulting reconstructions will each contain the signal plus the associated noise spatially displaced. Since spatial regions with high amplitude changes generate proportionately higher quantization noise and this noise is spatially shifted across the diverse reconstructions and suppressed as part of the process of merging the images together. The final image is constructed by choosing the most accurate pixels (on a pixel by pixel basis) from each of the reconstructed images to enhance the reconstruction quality, thereby exploiting the phase diversity of the system.

The general block diagram of adaptive synthesis filter bank is shown in Figure 1. The synthesis section consists of “n” filters (where n is an odd integer) and it is composed of “n-1” delay filters along with linear phase filters. Odd length filters are preferred because of their superior performance in compression.

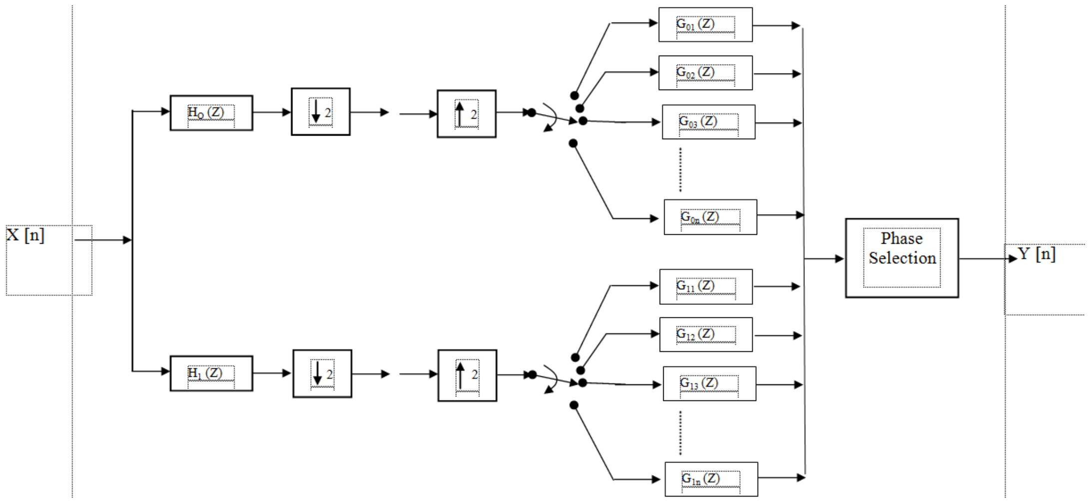

Figure 1. General block diagram conventional analysis and adaptive synthesis filter bank

If “K” is the length of the longer analysis filter, then “2K−6” synthesis delay filters along with linear phase filters can be designed by using the following time domain equation

$$\mathbf{AS} = \mathbf{B}$$

where “A” is a block Toeplitz matrix of analysis filter coefficients, “S” is a matrix of synthesis filter coefficients, and “B” is the reconstruction matrix.

In a more expanded form, above equation can be expressed as

$$\underbrace{\begin{bmatrix} \mathbf{P}_0^T & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{P}_1^T & \mathbf{P}_0^T & \vdots & \mathbf{0} & \mathbf{0} \\ \mathbf{P}_2^T & \mathbf{P}_1^T & \vdots & \vdots & \\ \mathbf{P}_3^T & \mathbf{P}_2^T & \ddots & \ddots & \\ \vdots & \vdots & \ddots & \ddots & \\ \mathbf{P}_{K-1}^T & \mathbf{P}_{K-2}^T & \cdots & \mathbf{P}_0^T & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{K-1}^T & \ddots & \mathbf{P}_1^T & h_0(0) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_{K-1}^T & h_0(L-1) \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \mathbf{Q}_0 \\ \mathbf{Q}_1 \\ \vdots \\ \mathbf{Q}_{K-2} \\ \mathbf{Q}_{K-1} \end{bmatrix}}_{\mathbf{S}} = \underbrace{\begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{J}_R \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}}_{\mathbf{B}} .$$

………..……………(1)

The length of shorter filter is adjusted to “K” by zero padding at the back end. The submatrices of “P” are defined as

$$\mathbf{P}_i^T = [h_0(i)h_1(i)]$$

where “$h_0(i)$” and “$h_1(i)$” represent the lowpass and highpass analysis filters and the “Q” matrices contain the lowpass and highpass synthesis filter coefficients. The “Q” matrices are given by

$$\mathbf{Q}_0 = [g_0(0) g_0(1)],$$

$$\mathbf{Q}_1 = [g_1(0) g_1(1)],$$

$$\mathbf{Q}_2 = [g_0(2) g_0(3)],$$

$$\mathbf{Q}_3 = [g_1(2) g_1(3)],$$

and so on until all the synthesis filter coefficients are included. Finally, “J_R” in reconstruction matrix “B” defined as

$$\mathbf{J}_R = \begin{bmatrix} 0 & 0 & \ldots & 1 \\ 0 & & \cdot & 0 \\ \vdots & \cdot & & 0 \\ 1 & 0 & \ldots & 0 \end{bmatrix}.$$

The position of “J_R” in the reconstruction matrix “B” controls the phase characteristics of the synthesis filters. Given a desired sample delay, “J_R” is positioned in the “d−1” location of matrix “B” where “d” is the desired system delay. Thus we can easily design optimal filters with group delays ranging from minimum to maximum phase.

The synthesis delay filters are computed from common analysis filters by using equation (1) as

$$\mathbf{S_1 = (A^T A)^{-1} A^T B_1}$$

$$\mathbf{S_2 = (A^T A)^{-1} A^T B_2}$$

$$\mathbf{S_3 = (A^T A)^{-1} A^T B_3}$$

$$\vdots \qquad \vdots$$

$$\mathbf{S_m = (A^T A)^{-1} A^T B_m} \qquad \text{......................... (2)}$$

The reconstruction errors of synthesis delay filters are computed by using the equation (2) as

$$\begin{aligned}
\boldsymbol{\varepsilon_{r1}} &= \| \mathbf{AS_1} - \mathbf{B_1} \|_F^{\,2} \\
\boldsymbol{\varepsilon_{r2}} &= \| \mathbf{AS_2} - \mathbf{B_2} \|_F^{\,2} \\
\boldsymbol{\varepsilon_{r3}} &= \| \mathbf{AS_3} - \mathbf{B_3} \|_F^{\,2} \\
&\ddots \qquad \ddots \\
\boldsymbol{\varepsilon_{rm}} &= \| \mathbf{AS_m} - \mathbf{B_m} \|_F^{\,2} \qquad \ldots\ldots\ldots\ldots\ldots \quad (3)
\end{aligned}$$

and reconstruction error of these filters is minimized by optimizing their coefficients. For low pass filters, the sum of the odd coefficients and the sum of the even coefficients both are made approximately equal to 0.7071. Similarly, for high pass filters, the sum of the odd coefficients and the sum of the even coefficients are made equal to 0.7071 and -0.7071 respectively.

The synthesis delay filters are then divided into “m/2” reconstruction groups (where “m” is the number of delay filters and it is even) by exploiting the optimal phase diversity of the synthesis section. Each group comprises of two delay filters along with linear phase filters. If “S_d” and “S_p” are delay and linear phase synthesis filters respectively and “d” represents the delay, where

$$\mathbf{d = 1, 2, 3, \ldots\ldots\ldots\ldots\ m}$$

then synthesis filters are grouped as

$$\begin{aligned}
\mathbf{G_1} &= \mathbf{S_1, S_m, S_p} \\
\mathbf{G_2} &= \mathbf{S_2, S_{m-1}, S_p} \\
\mathbf{G_3} &= \mathbf{S_3, S_{m-2}, S_p} \\
&\ddots \qquad \ddots \\
\mathbf{G_{m/2\,-1}} &= \mathbf{S_{m/2\,-1}\,, S_{m/2\,+2}, S_p} \\
\mathbf{G_{m/2}} &= \mathbf{S_{m/2}\,, S_{(m/2)\,+1}, S_p} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)
\end{aligned}$$

To illustrate the phase diversity of the synthesis filters, consider the step response of the lowest delay (minimum phase) and the highest delay (maximum phase) filters shown in

Figure 2 [8]. The top plot shows the output of a minimum phase filter after filtering an image step edge. The bottom plot shows the same for a maximum phase filter. The phase dispersion results in massive oscillation on the trailing part of the edge for the maximum phase case but no such oscillation on the leading part of the edge. The reverse characteristics are evident for the minimum phase filter case. Furthermore, each of these outputs is shifted spatially by different amounts.

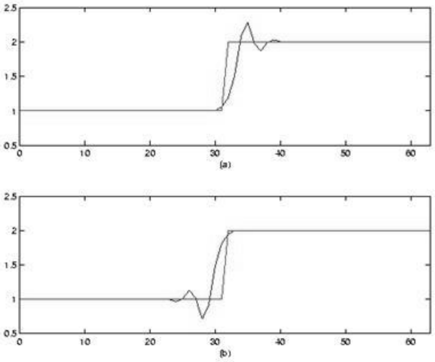

Figure 2. (a) Lowest delay (minimum phase) lowpass filter step response. (b) Highest delay (maximum phase) lowpass filter step response.

In the reconstruction process synthesis filters are selected in an adaptive mechanism, *i.e.*, at edges delay filters are used and in the smooth regions, reconstruction is accomplished by linear phase filters. The selection is based on comparison of three outputs of each group on pixel by pixel basis. Let $y_o(i)$, $y_n(i)$ and $y_h(i)$ are the outputs of the lowest delay, linear phase and highest delay filters respectively, then if

$$\mathbf{y_o(i) \approx y_n(i)}$$

and

$$\mathbf{y_h(i) \neq y_n(i)}$$

reconstruction is accomplished by the highest delay filters. Similarly, if

$$\mathbf{y_h(i) \approx y_n(i)}$$

and

$$\mathbf{y_o(i) \neq y_n(i)}$$

reconstruction is done by the lowest delay filters. In case, if none of the delay filter of each group meet the above criteria, then reconstruction is accomplished by linear phase filters.

Since for “n” synthesis filters, there will be “n^2” possible reconstruction combinations. However, this computational complexity can be reduced to “2n-1” by combining each set of delay filters with corresponding linear phase filters only because other combinations do not significantly contribute in optimal image reconstruction [9].

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \ldots\ldots & a_{1m} & a_{1p} \\ a_{21} & a_{22} & a_{23} & a_{24} & \ldots\ldots & a_{2m} & a_{2p} \\ a_{31} & a_{32} & a_{33} & a_{34} & \ldots\ldots & a_{3m} & a_{3p} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & a_{m4} & \ldots\ldots & a_{mm} & a_{mp} \\ a_{p1} & a_{p2} & a_{p3} & a_{p4} & \ldots\ldots & a_{pm} & a_{pp} \end{pmatrix}_{n \times n}$$

Placing configuration of various reconstruction groups in the synthesis filter bank is very important, as it places the groups at different locations in order to give them different priorities. Because at a given pixel valve, different reconstruction groups have different contributions. Placing of the delay filters is made before the linear phase filters by keeping the delay filters of the groups

$$\mathbf{G_{m/2},\ G_{(m/2)-1},\ G_{(m/2)-2},\ \ldots\ldots\ldots\ldots\ldots\ G_1}$$

in the order of

$$\mathbf{1^{st},\ 2^{nd},\ 3^{rd},\ \ldots\ldots\ldots\ldots\ldots\ m/2}$$

positions respectively in the synthesis filter bank and the same placing configuration is implemented up to “level 3” of the subband tree and in all the remaining levels, linear phase filters are used.

3. Quality Measures of Image Compression System

The performances of image compression techniques are mainly analyzed on the basis of two measures: Compression Ratio (CR) and Peak Signal to Noise Ratio (PSNR). The compression ratio is defined as ratio of the size of original data set to the size of the compressed data set [10].

$$Comprassion\_ratio = \frac{A}{B} \times 100$$

where A is number of bytes in the original data set and B is number of bytes in the compressed data set. The PSNR provides a measurement of the amount of distortion in a signal, with a higher value indicating less distortion. For n-bits per pixel image, PSNR is defined as

$$PSNR = 20\log_{10}\frac{2^{R}-1}{RMSE}db$$

where, RMSE is the root mean square difference between two images. The Mean Square Error (MSE) is defined as

$$MSE = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \left| y(m,n) - x(m,n) \right|^2$$

where x(m,n), y(m,n) are respectively the original and recovered pixel values at the mth row and nth column for MxN size image. The PSNR is given in decibel units (Db), which measures the ratio of the peak signal and the error signal (difference between two images).

4. Design Example and Simulation Results

In this section, we implement adaptive synthesis filter banks for n = 5 by means of biorthogonal filters “bior 4.4” and “bior 5.5”, whose longer analysis filter lengths are 9 and 11 respectively. So, we can design 12 and 16 different synthesis delay filters from the respective biorthogonal filters. In this design example, the adaptive synthesis filter banks are comprised of a set of four delay synthesis filters along with linear phase filters and same placing configuration is implemented upto level 3 of subband tree, while linear phase filters are used in all the remaining levels. We have incorporated adaptive synthesis filter banks in popular SPHIT (Set Partitioning in Hierarchical Trees) coder and modify synthesis section so that all the delay and linear phase filters take part in optimal image reconstruction.

Simulations are carried out on bench mark images “House”, “Camerman”, Peppers”, “Lena” and “Chemcial Plant” at low bit rates for compression ratio ranging from 0.05 bpps to 0.9 bpps. The results of adaptive synthesis filter banks and conventional filter bank by using ‘bior 4.4’ and ‘bior 5.5’are shown in Table 1 and Table 2 respectively.

Table 1, Comparison of results of adaptive synthesis filter banks and conventional filters by using biorthogonal filters “4.4” for compression ratios 0.05 ~ 0.9 bits per pixel

CR (bpp)	Conventional Filter Bank (PSNR in dB)					Adaptive Synthesis Filter Bank (PSNR in dB)				
	House	Cameraman	Peppers	Lena	Chemical Plant	House	Cameraman	Peppers	Lena	Chemical Plant
0.05	23.42	21.69	22.36	22.58	21.45	23.44	21.70	22.36	22.58	21.46
0.1	25.56	23.75	24.58	24.62	23.11	25.59	23.76	24.59	24.62	23.12
0.2	29.17	26.26	27.12	27.50	25.16	29.21	26.27	27.13	27.51	25.18
0.3	31.30	27.85	28.75	29.36	26.51	**31.39**	27.87	28.77	29.37	26.54
0.4	32.80	29.27	30.41	31.23	27.70	32.87	29.30	**30.45**	31.24	27.73
0.5	34.04	30.52	31.75	32.55	28.70	34.10	**30.56**	31.78	**32.57**	**28.74**
0.6	35.08	31.52	32.74	33.79	29.57	35.13	31.55	32.76	33.80	29.60
0.7	35.93	32.49	33.80	35.04	30.29	35.95	32.52	33.82	35.05	30.31
0.8	36.56	33.61	34.97	36.07	31.02	36.56	33.63	34.98	36.08	31.03
0.9	37.30	34.73	35.93	36.90	31.75	37.27	34.75	35.93	36.90	31.75

Table 2. Comparison of results of adaptive synthesis filter banks and conventional filters by using biorthogonal filters "5.5" for compression ratios 0.05 ~ 0.9 bits per pixel

CR (bpp)	Conventional Filter Bank (PSNR in dB)					Adaptive Synthesis Filter Bank (PSNR in dB)				
	House	Cameraman	Peppers	Lena	Chemical Plant	House	Cameraman	Peppers	Lena	Chemical Plant
0.05	22.40	21.14	22.18	22.40	21.25	22.48	21.20	22.20	22.41	21.26
0.1	24.95	23.24	24.08	24.18	22.59	25.06	23.31	24.11	24.20	22.61
0.2	28.28	25.71	26.68	27.01	24.76	28.42	25.82	26.73	27.04	24.78
0.3	30.58	27.24	28.14	28.62	26.03	**30.76**	27.36	**28.21**	28.67	26.06
0.4	32.11	28.24	29.91	30.82	26.98	32.28	**28.41**	29.97	30.87	27.01
0.5	33.01	29.85	31.10	32.07	28.40	33.17	29.98	31.16	32.12	**28.45**
0.6	34.35	30.88	31.94	33.02	29.22	34.44	30.98	32.00	**33.08**	29.26
0.7	35.31	31.68	32.65	34.72	29.86	35.32	31.77	32.70	34.70	29.88
0.8	35.92	32.48	34.19	35.78	30.41	35.90	32.56	34.19	35.73	30.42
0.9	36.50	33.67	35.31	36.47	30.97	36.46	33.69	35.26	36.40	30.97

5. Conclusions

Adaptive synthesis filter banks have shown improvements in PSNR at low bit rates and their performance degrades when bit rate is increased. When compression ratio is increased from 0.05 bpp to 0.2 bpp, there are slight improvements in PSNR and the improvements in general become quite significant for compression ratios 0.3bpp ~ 0.6bpp. Peak improvements in PSNR vary from image to image and are highlighted in bold. From compression ratios 0.7 bpp to 0.9 bpp, the improvements in PSNR are declined in most of images. So in image compression applications, adaptive synthesis filter banks should be used for compression ratio 0.3bpp ~ 0.6bpp in order to obtain maximum gain in PSNR.

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