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The Application of Wavelet Threshold on Compressive Sensing in Wireless Sensor Networks

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Abstract

Compressive sensing (CS) is a novel framework which exploits both the sparsity and the intra-correlation of the signal in structural health monitoring (SHM) based on wireless sensor networks (WSNs). It contains sparse signal representation, the measurement matrix selection and the reconstruction algorithm. The SHM signal is recovered by M measurements following the restricted isometry constant (RIC). However, the signal should be denoised before reconstruction. This paper discusses two wavelet noise reduction methods, soft threshold and hard threshold method, and verifies the performance of different methods for SHM signal reconstruction. Experimental results show that wavelet hard threshold method has much better effect on SHM sparse signal reconstruction than soft threshold method. Meanwhile, we can get a more accurate corresponding relation of RIC that is $M \geq CK * \log(N/K) - 33$.

Keywords: *compressive sensing; wireless sensor networks; structural health monitoring; noise reduction; reconstruction error*

1. Introduction

Compressive sensing (CS) provides an alternative to Shannon/Nyquist sampling when signal under acquisition is known to be sparse or compressible [1, 2]. It combines sampling and compression parts together to realize sampling below Nyquist rate. Using this technology, we can reconstruct a sparse signal with much fewer measurements than traditional means. With the development of CS theory, it has been applied to wireless sensor networks (WSNs), medical imaging, remote sensing [3], structural health monitoring (SHM) [4, 5] and so on.

However, in SHM based on WSN, the signal data with noise is not suitable for CS process. Before the CS compression, the original data should be de-noising. The wavelet threshold method is simple but effective for one-dimensional signal de-noising process. The threshold quantization process of wavelet threshold method has soft threshold and hard threshold which have different effect on CS compression and reconstruction. In this paper, we first introduced the CS theory, and proposed the wavelet noise reduction method in the SHM sparse signal based on WSN. At the same time, we analyzed the signal reconstruction effects between soft threshold and hard threshold. Experimental results verify the restricted isometry constant (RIC) accuracy and give a more accurate corresponding relation of RIC. And these results also indicate that the SHM sparse signal reconstruction effect on wavelet hard threshold is better than that on wavelet soft threshold.

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The rest of the paper is structured as follows. In Section 2 we provide a description of CS theory. In Section 3 the process of noise reduction in CS is introduced. In Section 4 experimental verification steps are described in detail which also substantiates our scheme. Section 5 concludes the paper.

2. Compressive Sensing Theory

The essence of CS [6] is using an irrelevant measurement ($M \times N, M << N$) with a matrix transform base (dimension) to put the original high-dimensional sparse signal or approximate sparse signal sequence $N \times 1$ project in a low dimensional space (M dimension) to achieve signal compression. Mainly, CS theory includes three parts:

- the sparse representation of the signal;
- the measurement matrix ensuring the data minimal information loss which should be satisfied the Restricted Isometry Property (RIP) ;
- The reconstruction algorithm using the no-distortion observed value to reconstruct signals.

Assuming that $x(n)$ ($x(n) \in R^N$) is an N-dimension signal which could be also regarded as a R^N column vector and there are K-nonzero elements among it that is $\|x\|_0 \le K$, then this kind of signal is K sparse. Another situation is that $x(n)$ is an approximate sparse signal or it is K sparse in the transform domain Ψ if there is an orthogonal sparse basis Ψ which can denote $x = \Psi\alpha$ and $\|\alpha\|_0 \le K$ .And for any N-dimension discrete-time signal $x \in R^N$, introducing the $N \times N$ orthonormal basis matrix *Ψ,* the signal $x \in R^N$ can be expressed as: $x = \sum_{i=1}^{N} \Psi_i \alpha_i$ *or* $x = \Psi\alpha$. Suppose that a measurement matrix $\Phi \in R^{M \times N} (M << N)$ is introduced to produce compressed sensing coefficients y, then for a sparse signal $x \in R^N$, the linear measuring values in the measurement matrix $y \in R^M$ could be defined as $y{=}\Phi x{=}\Phi\Psi\alpha{=}\Theta\alpha$ where $\Theta = \Phi\Psi$ is a $M \times N$ sensing matrix, Φ represents a $M \times N$ measurement matrix and Ψ is a $N \times N$ transformation matrix. If y and $\Theta = \Phi\Psi$ meet with the RIP [7], K-sparse decomposition coefficients α can be reconstructed by solving the l_0 norm [8] from y, $\hat{\alpha}{=}\arg\min\|\alpha\|_0$ $s.t.$ $\Theta\alpha{=}y$ where $\hat{\alpha}$ is the only exact solution of decomposition coefficients α . Finally, the exact solution $\hat{x}$ can be obtained by reconstructing $\hat{\alpha}$ under the orthogonal transform basis Ψ shown as $\hat{x} = \Psi\hat{\alpha}$.

Candes and Tao [7] have also given Restricted isometry constant (RIC) concept denoted as δ_m , which is the infimum of RIP established for all parameters. The relationship between the M-dimensional measurement matrix Φ and K can be obtained based on RIC conditions. Studies show that if Φ adopts a random matrix, we can ensure that Φ can be able to meet with the incoherence and constraint conditions by the great probability where M is fulfill with the equalization (1).

$$M \ge CK * \log(N / K) \tag{1}$$

Where 'C' is a constant related with the recovery accuracy. In this experiment，we intends to adopt $M \ge K * \log(N / K)$ as the measuring number condition to get the signal reconstruction and verify the accuracy of the formula (1).

3. Noise Reduction in CS

In CS application, there are a variety of choices for the orthonormal basis Ψ such as the Fourier transform and the wavelet basis of the Wavelet transform. Haar wavelet basis is the first choice as the preferred orthogonal transform base in our experiments with higher sparsity. But the practical analysis found that both sparse effects didn't reach our paper goal. Most the minimum value of the sampling points is very close to zero. However, due to the sparseness of statistical is determined by the number of non-zero values in the sparse signals, it is necessary to make appropriate process on the signal after thinning. In addition, the noise signal has a great impact on the signal sparse representation, so that the denoising process must be executed. Study found that the wavelet threshold process just handles the above two problems simultaneously to denoise signal, retain the larger value and remove the minimum value. Moreover, the threshold algorithm has low computational complexity, so that it is an easy but very effective method. The following is a brief introduction for wavelet threshold process.

3.1 Wavelet threshold noise reduction

After the signal sparse decomposition in CS, it is necessary to eliminate noise and process threshold value to get better sparsity, and to improve the data compression ratio and obtain precise reconstruction. The wavelet threshold method is simple but effective for one-dimensional signal denoising process.

A mathematical model of one-dimensional signal contained noise is usually defined as follows (2):

$$s(n) = x(n) + \sigma * e(n) \qquad (n = 0,1,2,...N-1) \tag{2}$$

Where $x(n)$ donates the original signal, $s(n)$ represents a signal with noise, $e(n)$ is a noise signal, and σ means noise intensity. In the simplest case, amusing $e(n)$ is Gaussian white noise and $\sigma = 1$. The purpose of the wavelet threshold noise reduction is to try to suppress the noise signal $e(n)$ in order to reconstruct the original signal $x(n)$.

(1) Wavelet threshold denoising steps

As we all know that the signal denoising process essentially inhibits the unwanted part of the signal and restore the useful part. The main steps of wavelet threshold denoising for one-dimensional signal are consists of noise signal decomposition, threshold quantization and signal reconstruction from it. Since there have been some wavelet decomposition and signal reconstruction mature algorithms, then how to select the threshold and threshold quantization approach became the core of the wavelet threshold method which directly determine the quality of the noise signal reduction in a considerable degree.

(2) The threshold Value Selection

There are basically two types of obtained thresholds for the wavelet threshold method as shown on the Table 1.

Table 1. Two kinds of thresholds selection methods

<table>
<tr><th>Threshold selection method</th><th>Classification</th></tr>
<tr><td rowspan="2">Based on the original signal</td><td>Donoho-Johnstone threshold</td></tr>
<tr><td>Birge-Masart penalty function and Penaltythreshold</td></tr>
<tr><td rowspan="4">Nonlinear wavelet transform threshold selection</td><td>Minimaxi variance threshold</td></tr>
<tr><td>sqtwolog</td></tr>
<tr><td>unbiased risk estimation threshold (rigrsure)</td></tr>
<tr><td>heursure</td></tr>
</table>

(3) The threshold quantization process method

After getting the threshold, the important step is to quantify it which has two kinds' process methods as shown in the Table 2.

Analyzed the Table 2, we found that Compulsory denoising process is simple and the signal denoising is smooth but it is easy to lose a useful component in the signal, however the threshold value process has taken the low-frequency and high-frequency part of the signal into account, especially the soft threshold is the smoother way as well as the Hard threshold could retain more of the characteristics of real signal spikes.

Table 2. The threshold quantization process method

<table>
<tr><th colspan="2">Threshold process classification</th><th>Approach</th></tr>
<tr><td colspan="2">Compulsory denoising process</td><td>All high frequency coefficients of the wavelet decomposition structure is set to 0， and filter out all the high frequency part</td></tr>
<tr><td rowspan="2">The threshold value process</td><td>Soft threshold</td><td>Compared the signal absolute value with the threshold value, set the point value to 0 which is not greater than the threshold, and make the point value which is larger than the threshold with the difference between them.</td></tr>
<tr><td>Hard threshold</td><td>Compared the signal absolute value with the threshold value, set the point value to 0 which is not greater than the threshold, and keep the original value when the point value is larger than the threshold.</td></tr>
</table>

Soft threshold and hard threshold mathematical models are described as follows (3) and (4). In order to facilitate the description, amusing that SORH='s' denotes the soft threshold and the SORH='h' represents the hardware one.

$$SORH = 's' \ : \ \eta^S(w,T) = \begin{cases} sign(w)(|w|-T) & |w| \geq T \\ 0 & |w| < T \end{cases} \tag{3}$$

$$SORH = 'h' \ : \ \eta^H(w,T) = \begin{cases} w & |w| \geq T \\ 0 & |w| < T \end{cases} \tag{4}$$

4. Simulation and Performance Evaluation

In order to get the effective and real data of the experiments, we designed a data acquisition experimental system which sensor node is a common node without compression function. The whole size of the original data about 208M is used to simulate the CS processing. In the simulation, we take Gaussian random matrix as the measurement matrix; use OMP as the reconstruction algorithm. During the process of noise reduction, the threshold selection criteria (TPTR) choose unbiased risk estimation threshold (rigrsure), the data length N is 1024. In this paper, we Use the matlab to achieve signal compression and reconstruction. Such a process is repetitive, so as to observe convergence of CS.

4.1 The evaluation standards for CS applications in SHM

(1) Compression ratio (CR)

The compression ratio is one of the indicators to measure the degree of data compression whose definition is the compression ratio between the original signal data quantity and the compressed data amount written as the follows (5), where N_o, N_{co} denote the signal data quantity and compressed data amount. The larger the CR is, the better the performance of the compression is, and the smaller the traffic load on the network is.

$$CR = N_o / N_{co} \tag{5}$$

(2) Reconstruction error ξ

Reconstruction error is on behalf of the similarity degree of the reconstructed signal and the original one. It is an import indicator to measure the effects of data decompression after refactoring which formula is (6), where $\hat{x}$, x separately indicated the reconstructed signal and the original one. The smaller the reconstruction error is, the higher the data recovery accuracy of the compressed sensing reconstruction algorithm is.

$$\xi = \frac{\|\hat{x} - x\|_2}{\|x\|_2} \tag{6}$$

4.2 The soft and hard threshold selection in CS

Make M_stand=K*log(N/K), and control the number of measurements in M_stand-98<=M<=M_stand+98. From Figure 1 and Figure 2, we can see that under the situation TPTR 'rigrsure', taking the soft and hardware threshold methods on the signal which length is N=1024, the sparisity generated by both is K=99 and CR is 90.33%.

As we all know that the theoretical number of measurements is M_stand=231, actually, the number of measurements can be less than its theoretical times under the situation of that the signal length is larger. Figure 1 and Figure 2 have indicated that the minimum number of measurements for both hardware and soft thresholds could not be less than M_stand-33, otherwise, reconstructing the signal will fail. In order to further analyze the comparison of reconstruction error for the soft and hardware threshold methods, we control the measurement times in [M_stand-33, M_stand+98] showing on the Figure 3. On the Figure 3, within the valid reconstruction measuring number ranged [M_stand-33, M_stand+98], the reconstruction error for the hardware threshold method is less than the soft threshold one.

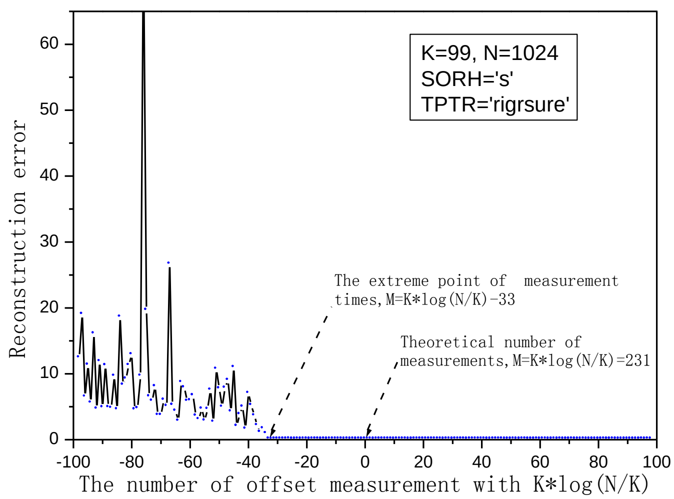

Figure 1. The soft threshold reconstruction error

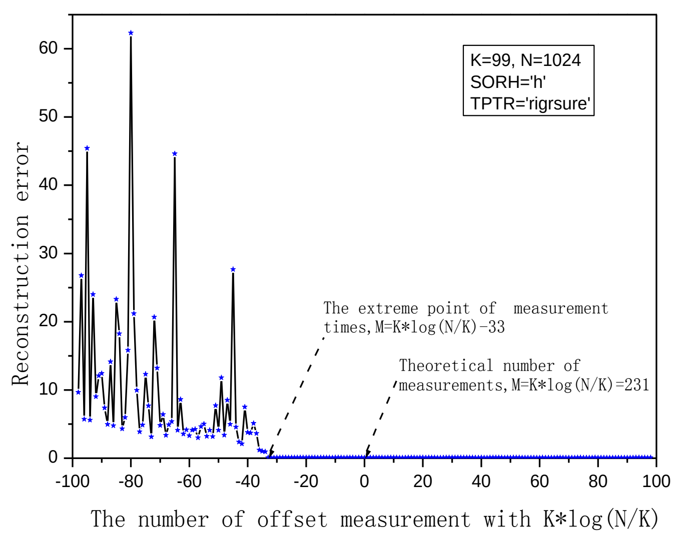

Figure 2. The hard threshold reconstruction error.

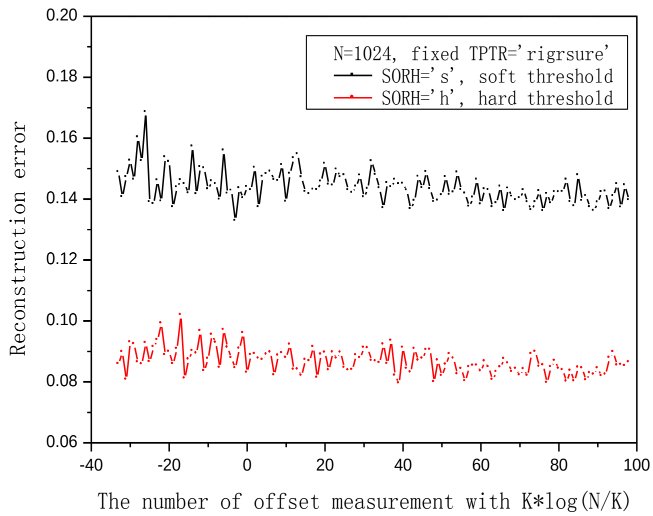

Figure 3. reconstruction error comparison within the effective measurement between soft and hard threshold

5. Conclusions and the Future Works

In this paper, two kind of method of wavelet threshold quantization were discussed. One is soft threshold method, another is hard threshold method. The threshold value process has taken the low-frequency and high-frequency part of the signal into account, especially the soft threshold is the smoother way as well as the hard threshold could retain more of the characteristics of real signal spikes. Experiments show that the RIC theory is also suitable for SHM based on WSN, which is proposed by Candes and Tao [7]. In SHM, we can draw more accurate conclusion $M \geq CK * \log(N/K) - 33$. Meanwhile, we can find that the effect of hard threshold is to surpass soft threshold. In the future, we will extend our scheme to improve a new threshold to combine the two different methods' advantages to improve the approach.

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