

Comparative Study of Optical Unipolar Codes for Incoherent DS-OCDMA system

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Abstract

In incoherent Direct Sequence Optical Code Division Multiple Access system (DS-OCDMA), the Multiple Access Interference (MAI) is one of the main limitations. To mitigate the MAI, many types of codes can be used to remove the contributions from users. In this paper, we study two types of unipolar codes used in DS-OCDMA system incoherent which are optical orthogonal codes (OOC) and the prime code (PC). We developed the characteristics of these codes i.e factors correlations, and the theoretical upper bound of the probability of error. The simulation results showed that PC codes have better performance than OOC codes.

Keywords: *Direct-sequence optical code-division multiple-access (DS-OCDMA), multiple access interference (MAI), optical orthogonal codes (OOC), prime code (PC), conventional correlation receiver (CCR)*

1. Introduction

The Code Division Multiple Access (CDMA), a multiplexing technique is newer than the TDMA and FDMA. In this multiple access technique, users share the same frequency space and transmitted over the same time intervals [1]. This is, in this case, assign a code to each transmitter, also called signatures or code sequence, which allows it to transmit information to avoid interfering with messages from other users. MAI reduction is obtained only in the case of the use of strictly orthogonal code sequences [7].

The choice of a family of codes with better correlation properties minimizes the multiple access interference (MAI), and therefore ensures better performance in OCDMA systems.

Unlike radio frequency CDMA systems where the use of bipolar codes presents no major difficulties, the implementation of codes in optical systems is faced with the problem of non-conservation of the phase of optical signals. For this, unipolar optical codes have been developed. They are implemented at the cost of stress relief on the correlation properties of sequences used. The codes are used in optical CDMA optical orthogonal codes (OOC) and the prime code (PC) [2, 6].

2. Architecture of System DS-OCDMA

In DS-OCDMA system, a transmission, the spectral spreading is carried out directly by multiplying the codes sequences of users with their data to be transmitted. The encoded data of each user are coupled and sent simultaneously via the same fiber. A reception, the receiver will be multiplexed with the received signal sequence each address to extract the data sent as illustrated in Figure 1 [3]:

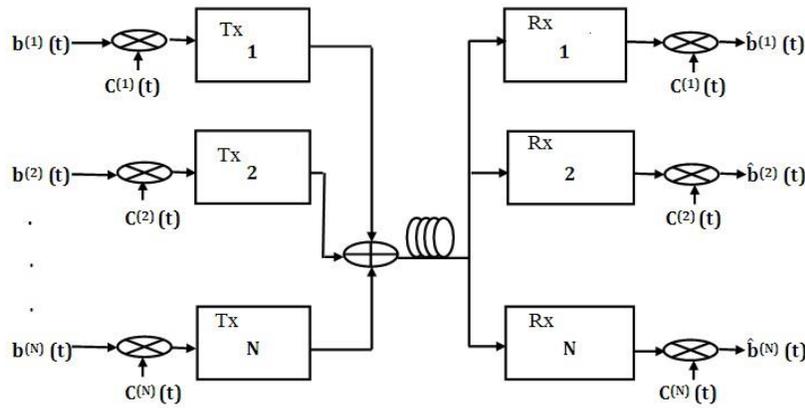


Figure 1. A transmission system DS-OCDMA

3. Optical Codes

To overcome the problem of phase control, unipolar sequences can be used. These are sequences that take values from $\{0, 1\}$, but we can't have a strict orthogonality with these codes (factors of auto and cross-correlation non-zero). These codes are used for optical CDMA time. In this paper, we consider the Optical Orthogonal Codes (OOC) and Prime Codes (PC) [4].

3.1. Optical Orthogonal Codes (OOC)

A family of codes unipolar is defined by the length of the sequence denoted by F representing the number of chips, by W the weight of the code indicating the number of chips of level "1" in a code sequence and the maximum number of users N that can be multiplexed using this family of sequences. Also two parameters such as, the auto-correlation λ_a , and cross-correlation λ_c constraints are equal to one.

It was shown by Salehi as the cardinality of a code OOC ($F, W, \lambda_a=1, \lambda_c=1$) is given by:

$$N = \left\lfloor \frac{F-1}{W(W-1)} \right\rfloor \quad (1)$$

Where $\lfloor \cdot \rfloor$ is the operator which takes the integer portion of a lower value by number. It should be noted that this theoretical expression gives an upper bound on the number of possible users in a family code.

The autocorrelation λ_a values and cross-correlation λ_c of codes are key parameters for evaluating performance system in the presence of multiple users. λ_a and λ_c to set, a family of OOC codes should check:

$$|AC_{x,x}(l)| = \left| \sum_{n=1}^F x_n x_{n+l} \right| = \begin{cases} \omega & \text{if } l = 0 \\ \leq \lambda_a & \text{if } 1 \leq l \leq F \end{cases} \quad (2)$$

$$|CC_{x,y}(l)| = \left| \sum_{n=1}^F x_n y_{n+l} \right| = \leq \lambda_c \quad \text{if } 1 \leq l \leq F \quad (3)$$

The equation (2) amounts to determining, for all values of circular shift, the number of chips to '1' in common between the code and its shifted version.

The equation (3) amounts to determining the number of chips in common between two unit family codes, for all values of circular shift between two codes.

3.2. Prime codes (PC)

The specificity of these sequences is that their construction algorithm is based on the choice of a prime number p .

i and j as consider two numbers between 0 and $p-1$. A series of sequences is calculated by the following equation [7]:

$$\begin{cases} S_{i,j} = \{s_{i,0}, s_{i,1}, \dots, s_{i,j}, \dots, s_{i,p-1}\} \\ s_{i,j} = i \cdot j \text{ mod } (p) \end{cases} \quad (4)$$

The prime code sequences

$C_i = \{c_{i,0}, c_{i,1}, \dots, c_{i,k}, \dots, c_{i,F-1}\}$ that will be used to encode and multiplex the signals associated with different users, are generated by:

$$c_{i,k} = \begin{cases} 1 & \text{pour } k = s_{i,j} + j \cdot p \\ 0 & \text{ailleurs} \end{cases} \quad (5)$$

We can define:

$N=P$ Multiplexing Capability of a family of sequence codes.
 $F=P^2$ size or length of a first code sequence.
 $W=P$ weight of a first code sequence.

As for OOC codes, we can calculate the Auto-correlation and cross-correlation of family PS codes (P^2, P).

The properties of self and cross-correlation of PS codes are described by the equations:

$$Z_{ii}(l) = \sum_{n=0}^{p^2-1} c_i^p(n) \cdot c_i^p(n-l) \quad (6)$$

$$Z_{i_1 i_2}(l) = \sum_{n=0}^{p^2-1} c_{i_1}^p(n) \cdot c_{i_2}^p(n-l) \quad (7)$$

With $i_1 \neq i_2$

And:

$$Z_{ii}(l) = \begin{cases} p & \text{if } l = 0 \\ \leq \lambda_a = p - 1 & \text{otherwise} \end{cases} \quad (8)$$

With $0 \leq i < p$

$$Z_{i_1 i_2}(l) \leq \lambda_c = 2 \quad (9)$$

With $0 \leq i_1, i_2 < p$ and $i_1 \neq i_2$

4. Reception System of DS-OCDMA

The reception structure used at the end of the transmission chain is a very important element, whose function is to receive the signal transmitted in the optical fiber and then, from this signal, estimate the data transmitted by the desired user. Different structures of receipt may be used for DS-OCDMA system.

In this work, we develop the structure of a conventional correlation receiver (CCR: Conventional Correlation Receiver) [5].

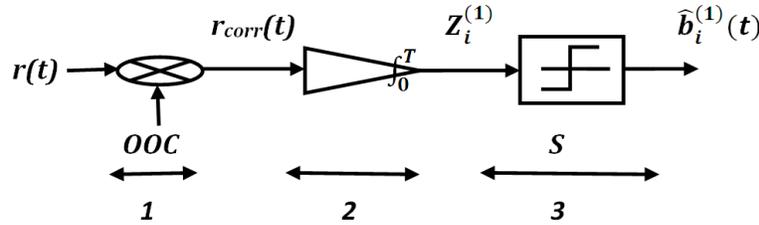


Figure 2. Conventional receiver of user 1

Figure 2 shows the principle of detection and estimation of the transmitted data, and which comprises 3 functions:

1. Multiplying the received signal by the code of the desired user. This step is equivalent to the fabrication of a mask between the received signal and the code sequence. It allows keeping only the power present in the chip unit code.
2. The integration of the signal obtained on the bit time: this step assesses the total power present in the signal previously obtained during the interval of time a bit. This step provides the value of the decision variable.
3. Decision making compared to a threshold: comparing the decision variable with the decision threshold provides the estimated data.

From a mathematical point of view, the successive operations result in the following expressions:

- Multiplying the signal received by the code of the desired user and which can be expressed as follows:

$$\begin{aligned}
 r_{corr}(t) &= r(t) \cdot c_1(t) \\
 &= \left(\sum_{k=1}^N b_i^{(k)} \cdot c_k(t) \right) \cdot c_1(t) \\
 &= b_1(t) c_1^2(t) + \sum_{k=2}^N b_i^{(k)} \cdot c_k(t) \cdot c_1(t)
 \end{aligned} \tag{10}$$

- The integration of the signal obtained provides the decision variable $Z_i^{(1)}$ the i bit of data the user # 1 and has the following form:

$$\begin{aligned}
 Z_i^{(1)} &= \int_0^{T_b} b_i^{(1)} \cdot c_1(t) dt + \sum_{k=2}^N b_i^{(k)} \int_0^{T_b} c_k(t) \cdot c_1(t) dt \\
 &= W \cdot b_i^{(1)} + \sum_{k=2}^N b_i^{(k)} \int_0^{T_b} c_k(t) \cdot c_1(t) dt
 \end{aligned} \tag{11}$$

- Decision making by comparison with a threshold S follows the decoding rule is written as follows:

$$\begin{cases}
 si & Z_i^{(1)} \geq S \Rightarrow \hat{b}_i^{(1)} = 1 \\
 si & Z_i^{(1)} < S \Rightarrow \hat{b}_i^{(1)} = 0
 \end{cases} \tag{12}$$

To determine the error probability of CCR, we must analyze the expression of the decision variable. For this, consider the decision variable $Z_i^{(1)}$ of the i th bit of data $b_i^{(1)}$ user # 1:

$$Z_i^{(1)} = W \cdot b_i^{(1)} + \sum_{k=2}^N b_i^{(k)} \cdot \int_0^{T_b} c_k(t) \cdot c_1(t) dt \tag{13}$$

- * The first term $W \cdot b_i^{(1)}$ corresponds to the contribution of the desired user.

$$\begin{cases}
 si & b_i^{(1)} = 0 \Rightarrow W \cdot b_i^{(1)} = 0 \\
 si & b_i^{(1)} = 1 \Rightarrow W \cdot b_i^{(1)} = W
 \end{cases} \tag{14}$$

- * The second term I_I corresponds to the interference from other users: it is the multiple access interference (MAI).

$$\begin{aligned}
 I_I &= \sum_{k=2}^N b_i^{(k)} \cdot \int_0^{T_b} c_k(t) \cdot c_1(t) dt \\
 &= \sum_{k=2}^N I_k^{(1)}
 \end{aligned} \tag{15}$$

The interference term $I_k^{(1)}$ of the user non-desired # k on the desired user # 1, depends on:

- Of the data transmitted $b_i^{(k)}$
- Of cross-correlation between the code of the desired user $c_1(t)$ and that of the user non-desired $c_k(t)$.

$$\begin{cases}
 if & b_i^{(k)} = 0 \Rightarrow I_k^{(1)} = 0 \\
 if & b_i^{(1)} = 1 \Rightarrow I_k^{(1)} = \int_0^{T_b} c_k(t) \cdot c_1(t) dt
 \end{cases} \tag{16}$$

Accordingly:

- * If $b_i^{(1)} = 1$, then the decision variable is written $Z_i^{(1)} = W + I_1$ with $I_1 \geq 0$. So: $Z_i^{(1)} \geq W$. If $S \leq W$, according to the decoding rule, we cannot go wrong decision on $\hat{b}_i^{(1)}$ where $b_i^{(1)} = 1$.
- * If $b_i^{(1)} = 0$, then the decision variable is written $Z_i^{(1)} = I_1$ with $I_1 \geq 0$. Thus: $Z_i^{(1)} \geq 0$. If $Z_i^{(1)} = I_1 \geq S$, we can make an error $\hat{b}_i^{(1)}$.

On the other hand, the receiver must perfectly decode the data in a single user in the chain of transmission. So, in absence of noise, it is necessary that:

$S \leq W$ to detect correctly a '1'

$S > 0$ for correctly detecting a '0'

In this case, it is necessary that $0 < S \leq W$. Based on the foregoing analysis, this means that, whatever the number of active users, given a '1' will always be correctly detected. To reduce the maximum number of errors on a '0', and thus obtain the lowest probability of error, the optimal threshold is as high as possible and this when: $S_{opt\ CCR} = W$

We obtain the error probability for the CCR which is a maximum cross-correlation is '1':

$$P_{eCCR} = \frac{1}{2} \sum_{i=S}^{N-1} C_{N-1}^i \left(\frac{R}{2}\right)^i \left(1 - \frac{R}{2}\right)^{N-1-i} \quad (21)$$

With R the probability of having a chip '1' in common between two codes.

For codes OOC (L, W, 1.1), we have $R = W^2/F$ and the probability of having a chip in common between two codes is given by

$$P_{eCCR} = \frac{1}{2} \sum_{i=S}^{N-1} C_{N-1}^i \left(\frac{W^2}{2F}\right)^i \left(1 - \frac{W^2}{2F}\right)^{N-1-i} \quad (22)$$

The error probability of CCR code for a maximum cross-correlation which is 2:

$$P_{eCCR} = \frac{1}{2} \left(1 - \sum_{i_2=0}^{\lfloor (S-1-i_1)/2 \rfloor} C_{N-1}^{i_1} C_{N-1-i_1}^{i_2} \left(\frac{R_1}{2}\right)^{i_1} \left(\frac{R_2}{2}\right)^{i_2} \left(1 - \frac{R_1}{2} - \frac{R_2}{2}\right)^{N-1-i_1-i_2} \right) \quad (23)$$

Where R_1 is the probability of having a chip '1' in common between two codes, and R_2 the probability of having two chips '1' in common between two codes.

For PS (P^2 , P) has been

$$\begin{cases} R_1 = \frac{2P^2+P+2}{3P^2} \\ R_2 = \frac{(P-2)(P+1)}{6P^2} \end{cases} \quad (24)$$

Consequently, we obtain the probability of error CCR for a code of PS (P^2, P)

$$P_{\alpha CCR} = \frac{1}{2} \left(1 - \sum_{i_1=0}^{s-1} \sum_{i_2=0}^{(s-1-i_1)} C_{N-1}^{i_1} C_{N-1-i_1}^{i_2} \left(\frac{2P^2+P+2}{6P^2} \right)^{i_1} \left(\frac{(P-2)(P+1)}{12P^2} \right)^{i_2} \left(1 - \frac{2P^2+P+2}{2} \cdot \frac{(P-2)(P+1)}{2} \right)^{N-1-i_1-i_2} \right) \quad (25)$$

5. Performance Analysis

5.1. Validation of Theoretical Analysis

The Table 1 below shows the results obtained for OOC codes (61, 5, 1,1) and $N = 3$ with the method of BIBD. These results are compared with the results of the work [9].

Table 1. Positions of the chips unit from a family of OOC code (61, 5, 1, 1)

<i>Utilisateurs</i>	P_1	P_2	P_3	P_4	P_5
C_1	1	10	21	35	59
C_2	5	15	20	37	50
C_3	14	16	17	23	57

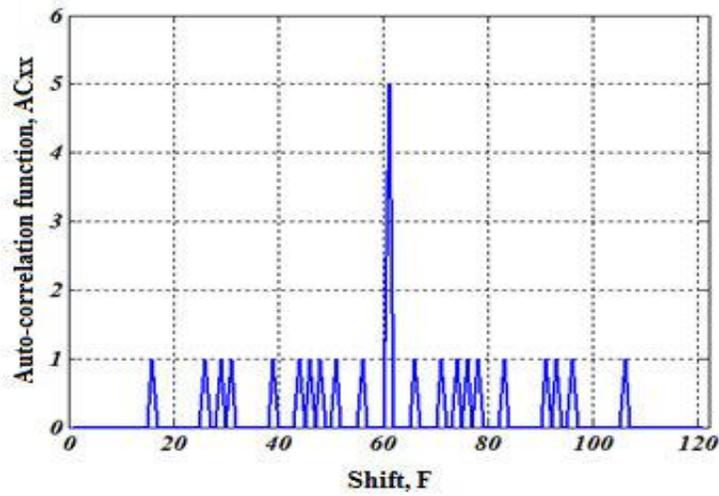


Figure 3. Auto-correlation values for the user C_2 of OOC (61, 5, 1, 1)

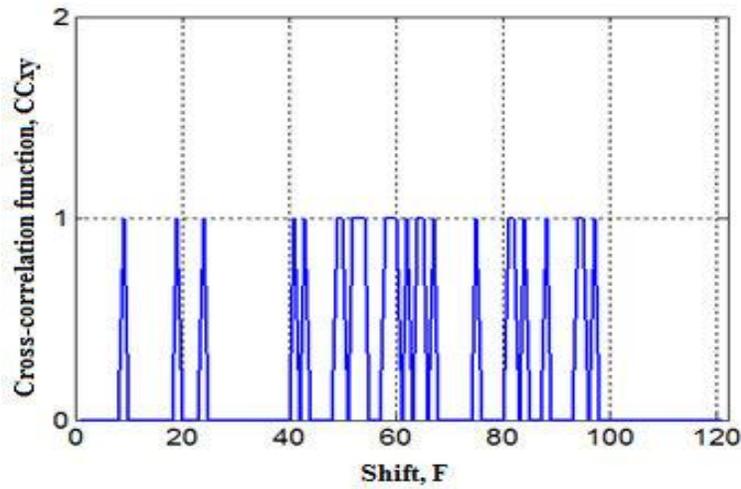


Figure 4. Cross-correlation values for the couple of users (2, 3) of OOC (61, 5, 1, 1)

Figure 3 shows the autocorrelation values obtained by the code sequence C_2 family of OOC sequence given in table 1. It shows that the maximum peaks of this function, the offset values F non-zero, $\lambda_a = 1$.

Figure 4 shows that the cross-correlation is always positive or zero, which the cross-correlation is not zero, due to the unipolar codes. As the maximum value of the cross-correlation $\lambda_c = 1$.

Therefore, the strict orthogonality can't be obtained for their OOC because unipolarity does not have a zero auto-correlation and cross-correlation whatever the shift. Therefore, the best possible for orthogonality OOC is obtained for $\lambda_a = 1$ and $\lambda_c = 1$

Table 2 presents the results obtained codes for PC (25.5), check that the results of the work [9].

Table 2. Code PC (25.5)

I	S_i	C_i
0	00000	1000010000100001000010000
1	01234	100000100000010000001000001
2	02413	1000000010000000101000000010
3	03142	100000000100100000000100100
4	04321	10000000001000100010001000

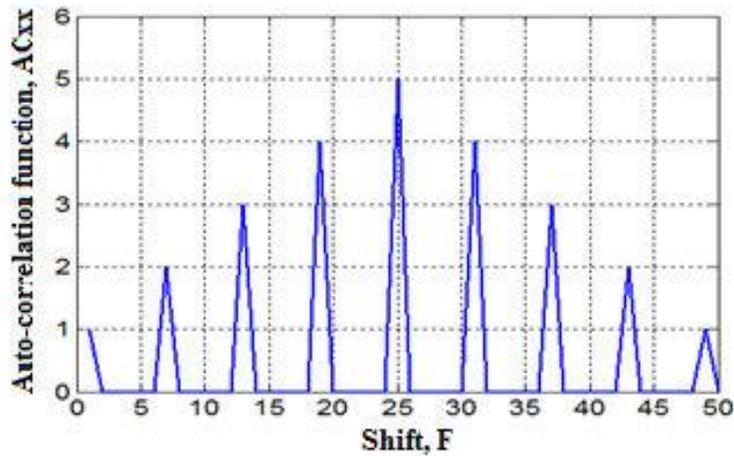


Figure 5. Auto-correlation values for the user C_2 code PC (25, 5)

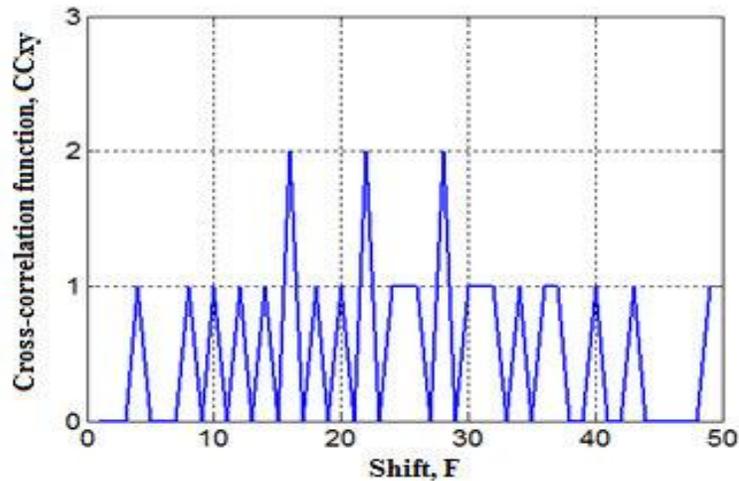


Figure 6. Cross-correlation values for the pair of users (1.4) code PC (25.5)

Figure 5 shows that the auto-correlation values obtained for a sequence length of PC to a family of code to the multiplexing of five users. It shows that the maximum level of auto-correlation peaks for non-zero values of F, corresponding to $\lambda_a = p - 1 = 4$.

Figure 6 shows the cross-correlation values obtained for a PC sequence length to a family of codes allowing multiplexing of five users. It shows that the maximum cross-correlation peaks for non-zero values F of, corresponding to $\lambda_c = 2$

5.2 Performances Analysis

Before studying the theoretical variations of the probability of error according to N, we can show of all, which the best alternative of the threshold of detection S is equal to W.

We have traced two following figures the evolution of the simulated performances of the CCR for a code OOC (361, 4, 1, 1) with N=8 users and code PC (841, 29) with N=25 users, as a function of the threshold S.

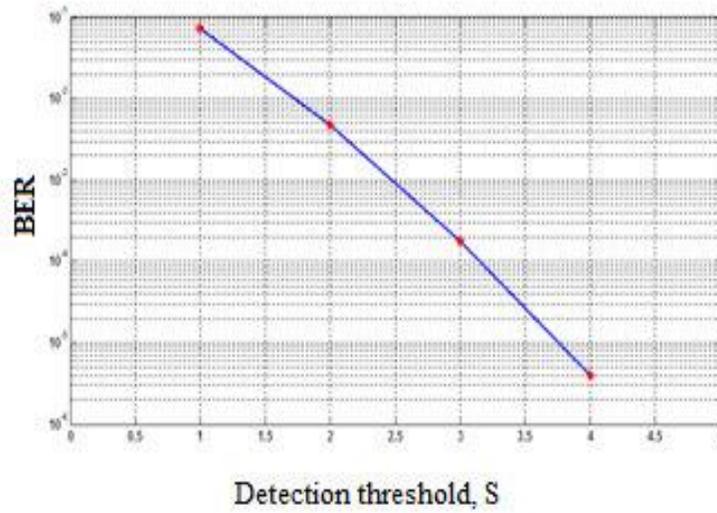


Figure 7. BER of a code OOC (361, 4, 1, 1) with N=8 users as a function of the threshold of decision

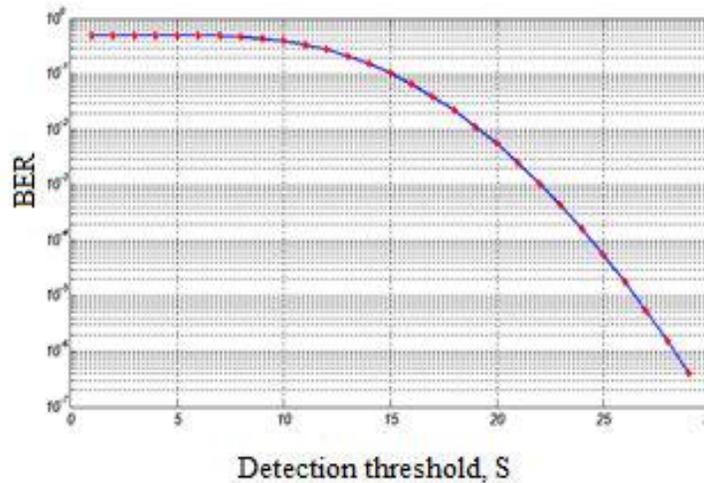


Figure 8. BER of code PC (25.5) with N=25 users as a function of the threshold of decision

We can be seen in the Figure 7 that the minimum value $BER_{OOC}(\approx 4.10^{-6})$ is obtained for $S=W=4$. So $S=W=4$ is the integer value of the optimal threshold of CCR.

We can notice that the lowest value $BER_{PC}(\approx 4.10^{-7})$ is obtained for $W=S=P=29$ (Figure. 8). So $S=W=P=29$ is the integer optimal threshold of CCR. So we will consider $S=W=P$ for CCR when PC codes is used.

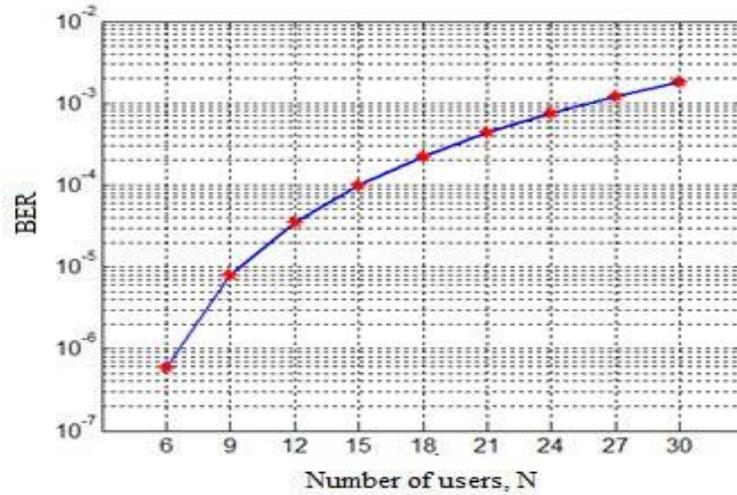


Figure 9. Error probability for code OOC (361, 4, 1.1) with N users

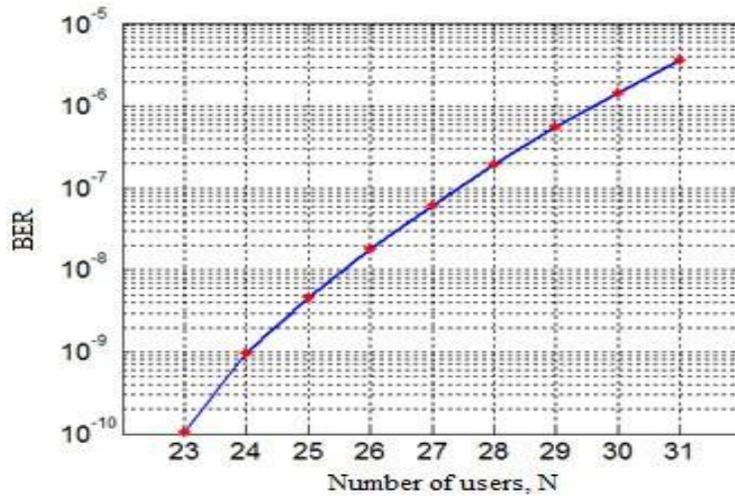


Figure 10. Error probability for code PC (961, 31) with N users.

In Figure 9 and Figure 10, we have traced the evolution of the probability of error (BER) as a function of the number of users N, for a threshold fixed at the weight (optimal threshold),

We can notice in these figures that as the BER increases when the number of users increases (performance degrade) with each types of codes. Indeed, we have saw in the case of an incoherent optical system, families of code used can't be orthogonal. This limitation is that each user communicates simultaneously with the desired user can be interfere. Therefore, the number of users increases, MAI increases and thus more performance degrades. We can also observe that the use of PC codes can achieve better performance with a large number of users that OOC codes. For example: ($BER = 10^{-10}$, $N=23$) for PC Code, and ($BER=7 \times 10^{-4}$, $N=23$) for OOC codes.

Finally, the comparison between the two types of codes can be done by calculating the minimal code to obtain a performance inferior to 10^{-9} with a number of users N determined. The results are reported in Table 3 below:

Table 3. Code parameters (F, W) with the minimal length L for a BER<10⁻⁹

<i>N</i>	<i>OOO (F, W)</i>	<i>PC (F, W)</i>
<i>10</i>	<i>(561,8)</i>	<i>(289, 17)</i>
<i>15</i>	<i>(1081,9)</i>	<i>(529, 23)</i>
<i>20</i>	<i>(1441,9)</i>	<i>(841, 29)</i>
<i>25</i>	<i>(1801,9)</i>	<i>(1369, 37)</i>
<i>30</i>	<i>(2161,9)</i>	<i>(1681, 41)</i>

We can observe that the use of PC codes can reduce the code length. In return, the weight of the code is slightly increased, and this when several wavelengths are used. Accordingly, if the only critter is to minimize the code length, the codes PC are more efficient.

Thus, we consider that bandwidth is 10 GHz, while the data rate for performance 10⁻⁹ and N=30 is D=B/F=4.7Mbit/ s for OOC, D=6Mbit/s for PC. Thus, the decrease of the code length permits to increase the data rate of each user.

6. Conclusion and Future Work

In this work, we have characterized a DS-OCDMA system incoherent working in single-user detection with two families of unipolar code, OOC codes and PC. The comparative study has showed that the correlation factors of OOC codes are better than PC codes. Against by the use of codes PC offers better performance than the OOC codes in terms of BER

Now, the next works will be based to study and improve of new types of detection for better extract the bits information (minimize BER).

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