International Journal of Hybrid Information Technology
Vol.6, No.5 (2013), pp.277-286
http://dx.doi.org/10.14257/ijhit.2013.6.5.26

Computational Complexity of Adaptive Synthesis Filter Banks for Image Compression

Abdul Nadeem and Yangyu Fan

School of Electronics and Information,
Northwestren Polytechnical University, Xi'an, China

labraznad@yahoo.com, fan_yangyu@sina.com

Abstract

This paper explores the contributions of all reconstruction combinations of the synthesis delay filters in adaptive synthesis filter banks and their effects on objective performance in image compression. It presents a novel comprehensive approach to manage all reconstruction combinations and their placing configuration in the adaptive synthesis filter banks. Based on the contributions of reconstruction combinations, it suggests the most suitable reconstruction combinations in order to reduce the computation complexity of adaptive synthesis filter banks. The simulations are carried out by using popular bi-orthogonal 9/7 filters on a number of standard test images with different design examples, which show that suggested reconstruction combinations of the synthesis filters play a very vital role on objective performance.

Keywords: *Computational complexity, adaptive synthesis filter banks, reconstruction combinations, delay filters*

1. Introduction

Adaptive filter banks have gained popularity over conventional filter banks especially in signal processing community for more than two decades. Motivated largely by the application of signal compression, these systems have been developed significantly over the years for audio, image, and video coding [1-4]. Presently, wavelet /subband image coding systems represent the best approach known for image compression, which allow signals to be decomposed into subbands and facilitate for efficient and effective processing. In these systems, the subbands are quantized, which in turn introduces noise into the systems. The traditional approach is to use high quality exact or near exact reconstruction of analysis and synthesis filters, in conjunction with good quantization and entropy coding schemes. This works well relatively speaking, but naturally reconstruction quality degrades when bit rates are lowered. However, this longstanding challenge has been resolved by designing adaptive filter banks (in form time varying filter banks) for subband image compression systems that have shown improved reconstruction quality at low bit rates.

Adaptive filter banks have been explored by a number of authors, by changing the analysis and synthesis filters coefficients in response to the input [5-7]. This can be done by dynamically switching back and forth among analysis/synthesis filters with different spectral and temporal (step response) characteristics, such that in regions where no major transitions occur, the filter set with good magnitude response characteristics is used. When transition regions or object edges are encountered, the

ISSN: 1738-9968 IJHIT

system switches to the filter set with good step response properties. In this approach, the analysis filter banks are selected dynamically from among a set of candidate filters according to the properties of the input. Such type of switching between different types of filters with asymmetric impulse responses and varying group delays has shown favorable reconstruction properties at edges. A degree of performance improvement has been reported, but a major issue of this approach, however, is that the synthesis filters must be changed in lock step with the analysis filters in order to perform reconstruction, which requires dynamic synchronization. This dynamic synchronization in many situations is unattractive as the interval between switching decreases and the number of switches increases.

Fixed analysis and adaptive synthesis filters have eliminated the issue of maintaining synchrony and simplified the operation of the filter bank [8-9]. The synthesis filters were used to exploit the phase diversity in the synthesis section, by means of a combination of only two delay and linear phase filters. These filters were used only in first level of subband structure by using all the possible reconstruction combinations. Although, it was shown that exact reconstruction could be possible and at the same time reconstruction quality could be enhanced, but this approach neither accommodates a rich set of synthesis filters nor reduces the computational complexity of synthesis filter bank.

In this paper, we discuss computational complexity of adaptive synthesis filter banks in a comprehensive manner. Adaptive synthesis filter banks are used to exploit the phase diversity of the synthesis section, by using a large number of delay filters in synthesis filter bank. One of the most critical issues of adaptive synthesis filter banks is that they must be less computational complex. In this paper, we highlight the contributions of all the reconstruction combinations of synthesis delay filters on objective performance and then suggest the most suitable ones in order to reduce the computational complexity of adaptive synthesis filter banks. We describe a new method to manage all the reconstruction combinations of the delay filters by grouping them into three categories. The category I and category II each have two sub groups, whereas category III has different number of sub groups, depending upon number of delay filters that takes part in image reconstruction. We also present a novel approach of placing configuration of various sub groups of different categories in the adaptive synthesis filter banks and the same configuration is used in first level of subband tree.

In the sections to come, we first present adaptive synthesis filter banks and then propose a general design approach to handle all the reconstruction combinations of adaptive synthesis filter banks. Afterwards, simulations are performed on various bench mark images; by using popular bi-orthogonal 9/7 filters for different design examples and finally, results are discussed and concluded.

2. Adaptive Synthesis Filter Banks

In adaptive synthesis filter banks, the analysis section is conventional, while the synthesis section is adaptive. The motivation in such type of filter banks comes from the recognition that the phase diversity (or equivalently diversity in group delay) in the synthesis section, which is comprehensively exploited. Consider the fact that quantization error in a coded image is directly related to the signal amplitude and that the characteristics of the error are influenced by the group delay characteristics of the synthesis filters. The adaptive synthesis filter banks implicitly reconstruct the input image with a multiplicity of reconstruction filters, each with different group delay characteristics but same magnitude response. Each of these filter pairs (low pass and high pass) generates a unique reconstruction. As the reconstructions are performed on the same quantized signal, the resulting reconstructions will each contain the signal plus the associated noise spatially displaced. Since spatial regions with high amplitude changes generate proportionately higher quantization noise and this noise is spatially shifted across the diverse reconstructions and suppressed as part of the process of merging the images together. The final image is constructed by choosing the most accurate pixels (on a pixel by pixel basis) from each of the reconstructed images to enhance the reconstruction quality, thereby exploiting the phase diversity of the system.

The block diagram of adaptive synthesis filter bank is shown in Figure 1. The synthesis section consists of "n" filters (where n is an odd integer) and it is composed of "n-1" delay filters along with linear phase filters. Odd length filters are preferred because of their superior performance in compression.

If "K" is the length of the longer analysis filter, then "2K−4" synthesis filters can be designed by using the following time domain equation

$$\mathbf{AS = B}$$

where "A" is a block Toeplitz matrix of analysis filter coefficients, "S" is a matrix of synthesis filter coefficients, and "B" is the reconstruction matrix.

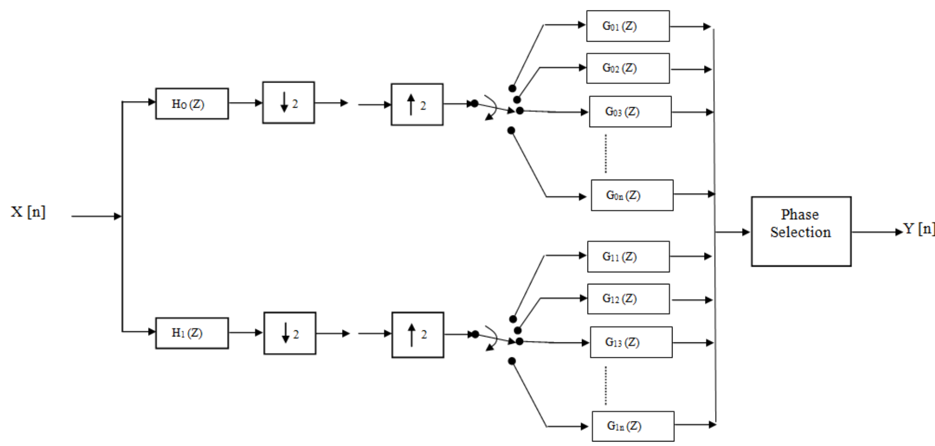

Figure 1. Block diagram conventional analysis and adaptive synthesis filter bank

In a more expanded form, above equation can be expressed as

$$\underbrace{\begin{bmatrix} \mathbf{P}_0^T & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{P}_1^T & \mathbf{P}_0^T & \vdots & \mathbf{0} & \mathbf{0} \\ \mathbf{P}_2^T & \mathbf{P}_1^T & \vdots & \vdots & \\ \mathbf{P}_3^T & \mathbf{P}_2^T & \ddots & \ddots & \\ \vdots & \vdots & \ddots & \ddots & \\ \mathbf{P}_{K-1}^T & \mathbf{P}_{K-2}^T & \cdots & \mathbf{P}_0^T & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{K-1}^T & \ddots & \mathbf{P}_1^T & h_0(0) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_{K-1}^T & h_0(L-1) \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \mathbf{Q}_0 \\ \mathbf{Q}_1 \\ \vdots \\ \mathbf{Q}_{K-2} \\ \mathbf{Q}_{K-1} \end{bmatrix}}_{\mathbf{S}} = \underbrace{\begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{J}_R \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}}_{\mathbf{B}}.$$

…………..……………(1)

The length of shorter filter is adjusted to “K” by zero padding at the back end. The submatrices of “P” are defined as

$$\mathbf{P}_i^T = [h_0(i) h_1(i)]$$

where “$h_0(i)$” and “$h_1(i)$” represent the lowpass and highpass analysis filters and the “Q” matrices contain the lowpass and highpass synthesis filter coefficients. The “Q” matrices are given by

$$\mathbf{Q}_0 = [g_0(0) g_0(1)],$$

$$\mathbf{Q}_1 = [g_1(0) g_1(1)],$$

$$\mathbf{Q}_2 = [g_0(2) g_0(3)],$$

$$\mathbf{Q}_3 = [g_1(2) g_1(3)],$$

and so on until all the synthesis filter coefficients are included. Finally, “J_R” in reconstruction matrix “B” defined as

$$\mathbf{J}_R = \begin{bmatrix} 0 & 0 & \dots & 1 \\ 0 & & \cdot & 0 \\ \vdots & \cdot & & 0 \\ 1 & 0 & \dots & 0 \end{bmatrix}.$$

The position of “J_R” in the reconstruction matrix “B” controls the phase characteristics of the synthesis filters. Given a desired sample delay, “J_R” is positioned in the “d−1” location of matrix “B” where “d” is the desired system delay. Thus we can easily design optimal filters with group delays ranging from minimum to maximum phase.

The synthesis delay filters are computed from common analysis filters by using equation (1) as

$$\mathbf{S_1 = (A^T A)^{-1} A^T B_1}$$

$$\mathbf{S_2 = (A^T A)^{-1} A^T B_2}$$

$$\mathbf{S_3 = (A^T A)^{-1} A^T B_3}$$

$$\vdots$$

$$\mathbf{S_m = (A^T A)^{-1} A^T B_m} \qquad \text{........................ (2)}$$

The reconstruction errors of synthesis delay filters are computed by using the equation (2) as

$$\varepsilon_{r1} = \| AS_1 - B_1 \|_F^{\,2}$$

$$\varepsilon_{r2} = \| AS_2 - B_2 \|_F^{\,2}$$

$$\varepsilon_{r3} = \| AS_3 - B_3 \|_F^{\,2}$$

$$\vdots \qquad \vdots$$

$$\varepsilon_{rm} = \| AS_m - B_m \|_F^{\,2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

and reconstruction error of these filters is minimized by optimizing their coefficients. For low pass filters, the sum of the odd coefficients and the sum of the even coefficients both are made approximately equal to 0.7071. Similarly, for high pass filters, the sum of the odd coefficients and the sum of the even coefficients are made approximately equal to 0.7071 and -0.7071 respectively.

The synthesis delay filters are then divided into “m/2” reconstruction groups (where “m” is the number of delay filters and it is even) by exploiting the optimal phase diversity of the synthesis section. Each group comprises of two delay filters along with linear phase filters. If “S_d” and “S_p” are delay and linear phase synthesis filters respectively and “d” represents the delay, where

$$d = 1, 2, 3, \ldots\ldots\ldots\ldots m$$

then synthesis filters are grouped as

$$G_1 = S_1, S_m, S_p$$

$$G_2 = S_2, S_{m-1}, S_p$$

$$G_3 = S_3, S_{m-2}, S_p$$

$$\vdots \qquad \vdots$$

$$G_{m/2-1} = S_{m/2-1}, S_{m/2+2}, S_p$$

$$G_{m/2} = S_{m/2}, S_{(m/2)+1}, S_p \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

3. Computational Complexity of Adaptive Synthesis Filter Banks

In an adaptive synthesis filter bank comprising of "n" filters, there will be "n^2" possible reconstruction combinations as

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \ldots & a_{1m} & a_{1p} \\ a_{21} & a_{22} & a_{23} & a_{24} & \ldots & a_{2m} & a_{2p} \\ a_{31} & a_{32} & a_{33} & a_{34} & \ldots & a_{3m} & a_{3p} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & a_{m4} & \ldots & a_{mm} & a_{mp} \\ a_{p1} & a_{p2} & a_{p3} & a_{p4} & \ldots & a_{pm} & a_{pp} \end{pmatrix}_{n \times n}$$

Each group of synthesis filters in (4) consists of eight reconstruction combinations of delay filters along with linear phase filters "a_{pp}", which are further divided into four sub groups as,

$$\mathbf{G_{ia} = a_{ii}, a_{jj}, a_{pp}} \quad \ldots\ldots\ldots\ldots \quad (5)$$
$$\mathbf{G_{ib} = a_{ij}, a_{ji}, a_{pp}} \quad \ldots\ldots\ldots\ldots \quad (6)$$
$$\mathbf{G_{ic} = a_{ip}, a_{jp}, a_{pp}} \quad \ldots\ldots\ldots\ldots \quad (7)$$
$$\mathbf{G_{id} = a_{pi}, a_{pj}, a_{pp}} \quad \ldots\ldots\ldots\ldots \quad (8)$$

where "a", "b", "c" and "d" are constant, $j = m - (i-1)$ and $i = 1, 2, 3, \ldots\ldots\ldots m/2$. The reconstruction combinations of sub groups "G_{ia}" and "G_{ib}" are of "category I", whereas the reconstruction combinations of sub groups "G_{ic}" and "G_{id}" are of "category II".

When two groups, *e.g.*, "G_1" and "G_2" participate in image reconstruction, then sub groups formed due to union of these two groups will be

$$\mathbf{G_{12} = a_{12},\ a_{21},\ a_{pp}}$$
$$\mathbf{G_{1(m-1)} = a_{1(m-1)},\ a_{(m-1)1},\ a_{pp}}$$
$$\mathbf{G_{m2} = a_{m2},\ a_{2m},\ a_{pp}}$$
$$\mathbf{G_{m(m-1)} = a_{m(m-1)},\ a_{(m-1)m},\ a_{pp}}$$

and if reconstruction is accomplished by means of all the groups i.e. "m/2" then sub groups formed due to union of the "m/2" groups will be

$$\mathbf{G_{12} = a_{12},\ a_{21},\ a_{pp}}$$
$$\mathbf{G_{1(m-1)} = a_{1(m-1)},\ a_{(m-1)1},\ a_{pp}}$$
$$\mathbf{G_{m2} = a_{m2},\ a_{2m},\ a_{pp}}$$

$$G_{m(m-1)} = a_{m(m-1)}\,,\ a_{(m-1)m}\,,\ a_{pp}$$
$$G_{13} = a_{13}\,,\ a_{31}\,,\ a_{pp}$$
$$\vdots$$
$$G_{(m/2-1)(m/2)} = a_{(m/2-1)(m/2)}\,,\ a_{(m/2)(m/2-1)}\,,\ a_{pp}$$
$$G_{(m/2-1)(m/2+1)} = a_{(m/2-1)(m/2+1)}\,,\ a_{(m/2+1)(m/2-1)}\,,\ a_{pp}$$
$$G_{(m/2+2)(m/2)} = a_{(m/2+2)(m/2)}\,,\ a_{(m/2)(m/2+2)}\,,\ a_{pp}$$
$$G_{(m/2+2)(m/2+1)} = a_{(m/2+2)(m/2+1)}\,,\ a_{(m/2+1)(m/2+2)}\,,\ a_{pp} \qquad \text{(9)}$$

The reconstruction combinations of sub groups formed due to union of "m/2" groups as in (9) are of "category III".

If "k" groups are used in image reconstruction, where

$$k = 1, 2, 3, 4, \ldots\ldots\ldots\ldots\ldots\ldots\ m/2$$

then total number of reconstruction combinations will be $(2k+1)^2$, which are further divided into $((2k+1)^2 - 1)/2$ sub groups. Each of the group will have four individual sub groups of categories I and II as in (5) ~ (8) and number of sub groups formed due to the union of the "k" groups will be

$$\begin{array}{ll} 4(k-1) & \text{for } k = 2 \\ 4(k-1) + 4(k-2) & \text{for } k = 3 \\ 4(k-1) + 4(k-2) + 4(k-3) & \text{for } k = 4 \end{array}$$

and so on.

To improve the reconstruction quality, phase diversity of synthesis delay filters is exploited in an adaptive mechanism. Let $y_o(i)$, $y_n(i)$ and $y_h(i)$ are the outputs of reconstruction combinations "a_{ii}", "a_{pp}" and "a_{jj}" of the sub group "G_{ia}" of (5) respectively, then if

$$y_o(i) \approx y_n(i)$$

and

$$y_h(i) \neq y_n(i)$$

then reconstruction is made by "a_{jj}". Similarly, if

$$y_h(i) \approx y_n(i)$$

and

$$y_o(i) \neq y_n(i)$$

then reconstruction is accomplished by "a_{ii}". In case, if none of the reconstruction combinations of each sub group meet this criteria, then reconstruction is carried out by linear phase filters "a_{pp}".

Placing configuration of sub groups of different categories in the synthesis filter bank is very important, as it places the categories at different locations in order to give them different priorities. Because at a given pixel valve, a number of reconstruction combinations have different contributions. In placing configuration, the sub groups of categories II, I and III in each group are given 1st, 2nd and 3rd priorities respectively in the synthesis filter bank before the linear phase filters "S_p". So, the overall placing configuration of adaptive synthesis filter bank is

$\mathbf{G_{m/2}}$ .... (Category II)
$\mathbf{G_{m/2}}$ .... (Category I)
$\mathbf{G_{m/2} \cup G_{m/2-1} \cup \ldots\ldots\ldots \cup G_2 \cup G_1}$ .... (Category III)
$\mathbf{G_{m/2-1}}$ .... (Category II)
$\mathbf{G_{m/2-1}}$ .... (Category I)
$\mathbf{G_{m/2-1} \cup G_{m/2-2} \cup \ldots\ldots\ldots \cup G_2 \cup G_1}$ .... (Category III)
⋱
$\mathbf{G_2}$ .... (Category II)
$\mathbf{G_2}$ .... (Category I)
$\mathbf{G_2 \cup G_1}$ .... (Category III)
$\mathbf{G_1}$ .... (Category II)
$\mathbf{G_1}$ .... (Category I)
$\mathbf{S_p}$

and this placing configuration is implemented in level 1 of the subband tree, whereas in all the remaining levels, linear phase filters are used.

After, analyzing the role of various reconstruction combinations of the three categories on objective performance, the most contributive category is identified and chosen for image reconstruction.

4. Design Examples and Simulation Results

We have chosen three design examples of adaptive synthesis filter banks for image reconstruction by using bi-orthogonal "9/7" filters, which are comprised of one, two and three groups of synthesis filters represented by "G_1", "G_2", and "G_3" respectively. Since, each group of synthesis filters is composed of two delay filters along with linear phase filters, so there will be six delay filters of the three groups along with linear phase filters. For these design examples; four bench mark images "House", "Cameraman", "Peppers" and "Chemical Plant" are selected and simulations are performed for compression ratio 0.5 bpp. We have incorporated the adaptive synthesis filter bank in conventional popular SPHIT (Set Partitioning in Hierarchical Trees) coder and modified the synthesis section so that all the delay and linear phase filters take part in optimal image reconstruction.

When reconstruction is accomplished by one group "G_1", then reconstruction combinations of the delay filters are only of category I and category II, and each having two sub groups. In this case, reconstruction is first tried with sub groups of category II, then with that of category I and finally with the linear phase filters. Similarly, if image is reconstructed by means of two groups "G_1", "G_2" and three groups "G_1", "G_2", "G_3", then there will be four and twelve sub groups of category III respectively, in additional to that of categories I and II. In the placing configuration, the sub groups of category II have top priority, whereas sub groups of category III have the lowest priority in each group. When image is reconstructed with two groups; "G_2" and "G_1" are placed at 1st and 2nd positions, whereas with three groups; "G_3", "G_2" and

"G_1" are kept at 1st, 2nd and 3rd positions respectively, in the synthesis filter banks before the linear phase filters.

The computational complexity of the adaptive synthesis filter banks is minimized by selecting only those reconstruction combinations of a category in all the three categories, whose contribution is quite significant and plays a very vital role on objective performance. Since, reconstruction combinations of category II comprising of linear phase filters are the most contributive ones, so these are chosen for image reconstruction in proposed adaptive synthesis filter banks. After reducing the computational complexity, the results of proposed adaptive synthesis filter banks for compression ratio 0.5 bpp along with results of adaptive synthesis filter banks by using all possible reconstruction combinations are shown in Table 1. Although, there are slight objective improvements of adaptive synthesis filter banks by using all possible reconstruction combinations over proposed adaptive synthesis filter banks. However, for n = 3, n = 5 and n = 7, in all design examples the computational complexities of the proposed adaptive synthesis filter banks are significantly less as compare to that of adaptive synthesis filter banks using all possible reconstruction combinations.

Table 1: Comparison of results of proposed adaptive synthesis filter banks in term of computational complexity with adaptive synthesis filter banks using all possible reconstruction combinations for compression ratio = 0.5 bits per pixel

Image	Adaptive Synthesis Filter Banks *(using all possible reconstruction combinations)*						Proposed Adaptive Synthesis Filter Banks					
	No. of filters (n=3) C Complexity = 9		*No. of filters (n=5) C Complexity = 25*		*No. of filters (n=7) C Complexity = 49*		*No. of filters (n=3) C Complexity = 5*		*No. of filters (n=5) C Complexity = 9*		*No. of filters (n=7) C Complexity = 13*	
	MSE	*PSNR*	*MSE*	*PSNR*	*MSE*	*PSNR*	*MSE*	*PSNR*	*MSE*	*PSNR*	*MSE*	*PSNR*
House	25.5142	34.06	25.3012	34.10	25.2376	34.11	25.5373	34.06	25.3449	34.09	25.2882	34.10
Cameraman	57.3491	30.54	57.2175	30.56	57.1786	30.56	57.3744	30.54	57.2414	30.55	57.2046	30.56
Peppers	43.3106	31.76	43.1749	31.78	43.1082	31.79	43.3231	31.76	43.2019	31.78	43.1366	31.78
Chemical Plant	87.2221	28.72	86.8753	28.74	86.8177	28.74	87.2411	28.72	86.9119	28.74	86.8535	28.74

5. Conclusion

The contributions of all reconstruction combinations of synthesis delay filters in the enhancement of image reconstruction have been discussed by using a new approach, which efficiently handles all the reconstruction combinations along with their placing configuration in the adaptive synthesis filter banks. Based on the contributions of reconstruction combinations of various categories, only the most contributive ones are selected for image reconstruction. Hence, by using this method the computational complexity of adaptive synthesis filter banks can be significantly reduced without compromising their efficiency.

References

[1] J. Arrowood, T. Randolph and M. J. T. Smith, "Filter Bank Design", Digital Signal Processing Handbook, Ed. Vijay K. Madisetti and Douglas B. Williams, Boca Raton: CRC Press LLC, **(1999)**.

[2] R. L. de Queiroz and D. R. Rao, "Time-varing lapped transforms and wavelet packets", IEEE Trans. on Signal Processing, **(1993)** December, pp. 3293–3305.

[3] S.-M. Phoong and P. P. Vaidyanathan, "Time-Varying Filters and Filter Banks: Some Basic Principles", IEEE Transactions on Signal Processing, vol. 44, no. 12, **(1996)** December.

[4] J. Arrowwood, "Theory and Application of Adaptive Filter Banks", PhD thesis, Georgia Institute of Technology, **(1999)**.

[5] K. Nayebi, T. P. Barnwell III and M. J. T. Smith, "Analysis-synthesis systems with time varying filter bank structures", proceedings of IEEE Int. Conf. Acoustics, Speech, and Signal Processing, **(1992)** March, pp. 617–620.

[6] I. Sodagar, K. Nayebi, T. P. Barnwell III and M. J. T. Smith, "A new approach to time-varying FIR filter banks", proceedings of IEEE Int. Conf. Signals, Systems and Computers, **(1993)** November, pp. 1271–1275.

[7] J. L. Arrowood Jr. and M. J. T. Smith, "Exact reconstruction analysis/synthesis filter banks with time-varying filters", proceedings of IEEE Int. Conf. Acoust., Speech and Signal Processing, **(1993)** April, pp. 3:233– 236.

[8] C. A. Lettsome, "Fixed-Analysis Adaptive-Synthesis Filter Banks", PhD thesis, Georgia Institute of Technology, **(2009)** May.

[9] C. A. Lettsome and M. J. T. Smith, "Image Interpolation Exploiting Phase Diversity", proceedings of IEEE DSP Workshop, Marco Island, Florida, **(2009)** January.