Min-max Vehicle Routing Problem Based on Ant Colony Algorithm

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Abstract

To minimize the length of travelling distance of the longest sub-route in vehicle routing problem, the max-min ant system with parameter adaptation is adopted, which can be applied to different datasets in practice. Routes are constructed by sequential and parallel methods for the customers with clustering and random distribution respectively. Since the behavior of ant colony algorithm depends strongly on the given parameter values, these parameters include expectation heuristic factor, choice probability, level of pheromone persistence, and the number of ants, are self-adaptive at different stages in the course of algorithm execution, which help to accelerate convergence and enhance the searching around optimal solution, as well as to guarantee the diversity of solution to avoid falling into local optimization. Seven classical instances are tested for min-max vehicle routing problem; the results demonstrate that max-min ant system with parameter adaptation has high effectiveness, fast convergence speed, and robustness in solving these problems.

Keywords: *logistics engineering, parameter self-adaptation, max-min ant system, min-max vehicle routing problem, ant colony algorithm*

1. Introduction

Min-max vehicle routing problem (MMVRP) refers to a series of delivering points (or receiving points) in an appropriate traveling route, the vehicles pass through them in an sequential way, and can satisfy certain constraints to make the longest sub-route distance or the time shortest in the whole traveling line, such as the air-dropped material collection in an emergency, the formulation of school bus route with consideration of social factors, and so on [1].

MMAS (max-min ant system) is put forward by German scholar Stützle, *etc.* in 1997, which is the improved algorithm based on ant system, has been successfully applied to various combinatorial optimization problems, and is one of the best ant colony optimization algorithm in performance currently [2-4]. In the application and study of MMAS, the parameter configuration directly influences the performance of algorithm searching for the optimal [5-7]. Due to the parameters have their respective function and interaction, the relationship of each other is relatively complex, thus the parameter configuration becomes the important work in algorithm design and debug. Most of the existing parameter adjustments adopt the test method, through single parameter changing one by one, to analyze and compare results to obtain the optimal value of each parameter; finally, which are applied to solve the problem. This method is time-consuming and strenuous, especially when the problem changes, the parameters also need to be adjusted, so the research of MMAS algorithm which can adaptively adjust parameters has important theoretical and practical significance [8-10]. Aiming to min-max vehicle routing problem, this paper puts forward the max-min ant system with the adaptive parameters, adopts examples to test, and has obtained the good effect.

2. The Min-max Vehicle Routing Problem

According to the literature [11], the mathematical model of min-max vehicle routing problem can be expressed as follows. The objective function is:

$$F = D + mL_0 = Min\{\max(\sum_{i=0}^{n}\sum_{j=0}^{n} d_{ij}x_{ij}^{k})\} + mL_0, \quad k = 1,...,m \tag{1}$$

Constraints:

$$\sum_{i=0}^{m}\sum_{k=1}^{m} x_{ij}^{k} = 1, \quad j = 0,...,n; \tag{2}$$

$$\sum_{j=0}^{m}\sum_{k=1}^{m} x_{ij}^{k} = 1, \quad i = 0,...,n; \tag{3}$$

$$\sum_{i=0}^{n} x_{ip}^{k} - \sum_{j=1}^{n} x_{pj}^{k} = 0, \quad p = 1,...,n; \tag{4}$$

$$\sum_{i=0}^{n}\sum_{j=1}^{n} q_i x_{ij}^{k} \le Q \tag{5}$$

In the expressions, D represents to start from center parking yard (which is expressed in 0), it is the longest sub-line length in the whole traveling route; L_0 represents the total distance of entire route via the nearest neighbor method to construct the route; m represents the number of used vehicle; n represents the customer number, d_{ij} represents the distance from customer i to customer j , if the coordinate of client nodes is known, often the Euclidean method is used to calculate the distance; q_i represents the freight volume of customer i ; Q represents the struck capacity of each truck, if the vehicle k arrives customer j from the customer i , x_{ij}^{k} is 1, otherwise 0. Constraints (2) and (3) ensure that each customer point is passed once by only one truck; Constraint (4) ensures the continuity of the sub-routes, namely the vehicle reaches a dispatch point for goods, which must leave from that point; Constraints (5) is a capacity constraint, the goods load of each sub-route can not exceeds the capacity of the vehicle.

3. The Max-min Ant System with Adaptive Parameter

Aiming at the min-max vehicle routing problem, this paper puts forward the max-min ant system with adaptive parameters, its main steps are: (1) the parameters initialization, pheromone initialization; (2) for each loop do; (3) for each ant colony do; (4) for each ant do; (5) route building; (6) local search; (7) evaluation route (8)end for; (9) pheromone update; (10) end for; (11) parameters update; (12) end for; (13) output.

3.1. Line building

In actual vehicle routing problem, the geographical position of nodes may randomly generate, and also present the clustering distribution. According to the characteristics of different data in the application examples, this article uses two kinds of route generating methods which are sequential method and parallel method. The first sub-route randomly starts from a client node, according to the selection rule to determine the next node, when the vehicle does not meet the capacity constraint, it will end the current sub-line, starts a new sub-line from center parking yard, and repeats this process until all the clients are visited.

Parallel generating route is according to the nearest domain method to determine the required vehicles number, all sub-lines are parallelly built, each sub-line starts from center parking yard, according to the selection rules to determine the next node. When the vehicle does not meet the capacity constraint, the current sub-line will be terminated; if all sub-routes are completed and there are also some unvisited clients, the current set is set as unfeasible solution, the target value is set as a larger value. When the node's geographical location randomly generates, the generation approach of parallel route can build each sub-line at the same time, it is advantageous to the optimization of whole route; when node's location presents clustering distribution, the client nodes concentrated in the same area are often put into the same sub-line in the optimal route. Because using parallel method can scatter concentrated client node locations in different sub-lines, adopting sequential route generating method is more suitable for node selection of customers with clustering distribution.

The rule of each ant selecting next client node is very important to the quality of generating solutions, this article adopts the pseudo-random proportion rule, that is, for ant l located at node i to select next node s according to expression (6).

In the expression,

$$s = \begin{cases} \arg\max\ j \in Allowed_l [\tau(i,j)]^{\alpha}[\eta(i,j)]^{\beta}, q \le q_0 \\ expression(7), \qquad q > q_0 \end{cases}$$

(6)

$$p_i(i,s) = \begin{cases} \dfrac{[\tau(i,s)/\tau_0]^{\alpha}[\eta(i,s)]^{\beta}}{\sum_{j \in Allowed_l} [\tau(i,j)/\tau_0]^{\alpha}[\eta(i,j)]^{\beta}}, s \in Allowed_l \\ 0, \qquad s \notin Allowed_l \end{cases}$$

(7)

In the expression, $Allowed_l$ represents the customer who is allowed to visit by ant l located at node i, in order to improve search efficiency, the maximum number can be set; $\tau(i,j)$ is the pheromone on edge (i,j); τ_0 is the pheromone initial value on each edge; $\eta(i,j)$ is the heuristic function, $\eta(i,j) = 1/d_{ij}$, α is the heuristic factor of information, which represents the relative importance of track, and reflects the role of accumulated information in the movement process while ants choosing a path, its value is larger, then the ant tend to choose the route that other ants pass through, interoperability among ants is more stronger; β is the expectation heuristic factor, which represents the importance of visibility, reflects the importance of heuristic information in motion in the route selection of ants, the greater is the value, the more it is close to the greedy rule; q is the uniformly distributed random number in [o, 1] interval, $q_0 (0 \le q_0 \le 1)$ is the given probability of path choice, when $q \le q_0$, according to a priori knowledge to choose the best edge, which is shown in the expression (6), otherwise to select another edge according expression (7).

3.2. Pheromone update

In order to make full use of the optimal solution at present, after each cycle, only the ants with global optimal are allowed to release pheromone, pheromone update rule is shown in expression(8). In the initial stage of a searching, the number of nodes for selection is numerous; the probability of getting an optimal line is high, with the increase of node number, the nodes for selection will gradually decrease, especially in the late stage of each sub-line, due to the constraints of capacity, there are often passive choice. Therefore this paper sets the amount of pheromone update $\mathrm{V}\tau(i, j)$ as expression (9).

$$\tau(i, j) = \rho\tau(i, j) + \mathrm{V}\tau(i, j) \tag{8}$$

$$\mathrm{V}\tau(i, j) = \begin{cases} 1/D^*(1-v/n); & \textit{if side } (i, j) \textit{ is the global optimal edge} \\ 0; & \textit{otherwise} \end{cases} \tag{9}$$

In the expression, ρ is the pheromone continuous parameter; D^* is the longest sub-line length corresponding to the optimal target value. v represents the number of visited nodes. In order to avoid the stop of searching, the pheromone of each edge is limited as $[\tau_{\min}, \tau_{\max}]$, in order to make ants search more new solutions in the initialization stage, the pheromone initialization value is set as $\tau_{\max}$.

3.3. Parameter selection

In order to analyze the influence of max-min ant system parameters for the convergence performance and target result of min-max vehicle routing problem, based on the CMT-3 as the research object, through a lot of simulation experiments, this paper discusses the effect on the algorithm about parameters $\alpha, \beta, \rho, q_0, N_A$ (ant amount), and brings forward parameter adjustment method of max-min ant system applying to the different problems. For analysis and comparison, the base values $\alpha, \beta, \rho, q_0, N_A$ are set as 1, 3, 0.9, 0, and 25.

3.3.1. Information heuristic factor α : Information heuristic factor α reflects the relative important degree of accumulated information amount in guiding ant colony to search path in the moving process. The greater is the value, the bigger is the possibility that ants choose former path, the searching randomicity is decreased; when the heuristic factor's value α is too small to make the ant colony search early fall into local optimum. In the experiment, the α is set as 20, 10, 5, 3, 1, 0.5, and the rest parameters use basic values. The experimental results are shown in Figure 1.

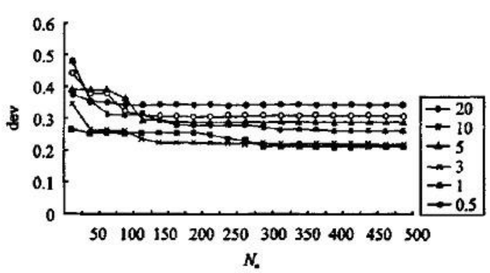

Figure 1. MMAS with various values of α

Among them, the horizontal axis is the iteration number N_c, which represents the execution cycles of max-min ant system; Vertical axis is the deviation value dev, $dev = (D^* - D_{low}) / D_{low}$, D_{low} represents the low bound of longest sub-line length, in the max-min vehicle route problem, the value can be expressed in Low-bound, that is, a truck starts from a parking yard, arrives at the farthest node, and return back the parking yard, the value of whole traveling distance is taken as the low bound of the farthest sub-line, when the length of longest sub-line is equal to Low-bound, which means this is the optimal value. dev is used to represent the deviation degree between the longest sub-line length and its lower bound, the value is smaller, which shows the longest sub-line length is smaller.

From Figure 1, when other initialization parameters are same, heuristic factor α has great impact on the performance of algorithm. When α is too small, not only the convergence speed is slow, but also the algorithm is easily trapped in local optimal solution, when α is less than 1, the effect of algorithm is poor; when α is too large, which is equivalent to give the importance of pheromone in ant search process with full attention, this will lead to positive feedback effect very strong in local optimum path, the algorithm will also appear premature convergence phenomenon, when α exceeds 10 and reaches to 20 in particular, it is difficult to get the optimal solution.

3.3.2. Expectation heuristic factor β: Expectation heuristic factor β reflects the relative important degree of heuristic information in the process of guiding ant colony to search; its size reflects the strength of apriority and uncertainty factors in the process of ant colony optimization. Its value is bigger, the possibility is greater that ants at some local point to select local shortest path, although at this time the convergence speed of algorithm is accelerated, the randomness of ant colony to search the optimal path is abated, it is easy to trap into local optimum. In the experiment, β is set as 20, 10, 5, 3, 1, 0.5, the rest of the parameters use basic values. The experimental results are shown in Figure 2.

The Figure 2 shows that in ant colony algorithm the expected heuristic factor β has a great influence on the algorithm performance. When β is within a certain range, the greater is the initial target value, the faster is the convergence speed; but when β is too large, the convergence speed of algorithm increases, the convergence performance is trending to change bad; when β is too small, it will lead ants to trap into pure random searching, in this case, generally it is difficult to find the optimal solution. In the case of $\alpha = 1$, it is better to keep the value of β in the range [3, 10].

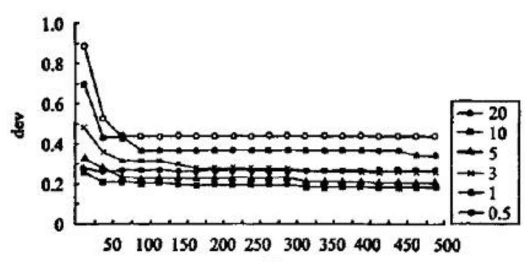

Figure 2. MMAS with various values of β

3.3.3. The pheromone lasting factor: The artificial ants in ant colony algorithm have human's memory function, but with the elapse of time, the pheromone continuously volatilizes, the former information will gradually disappear. ρ represents the pheromone lasting factor, 1-ρ represents the pheromone volatilization factor. The size of pheromone lasting factor ρ reflects the strength of interact with each other among ant individuals, the size of pheromone volatilization factor 1-ρ is related to the global search ability of ant colony algorithm and convergence speed. In the experiment, ρ is set as 0.9, 0.8, 0.6, 0.4, 0.1, 0.05, and the rest parameters uses basic values. The experimental results are shown in Figure 3.

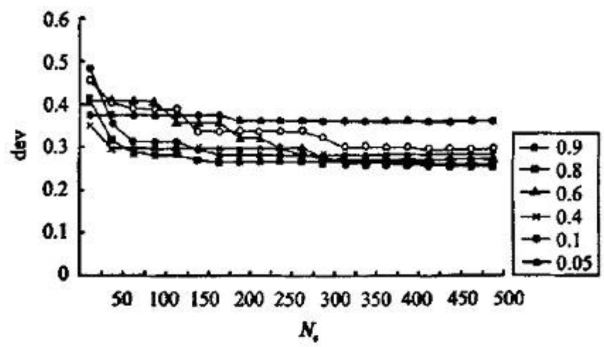

Figure 3. MMAS with various values of ρ

The Figure 3 shows that the size of pheromone lasting factor ρ has a very big effect on the convergence performance of ant colony algorithm. When ρ is very big, because in the path the residual pheromone is in dominance, the information positive feedback effect is relatively weak, searching randomness is enhanced, the convergence speed of ant colony algorithm is slow. Under the condition of algorithm convergence, ρ has little effect on the final value of the optimal solution, but when ρ is small, although the convergence speed is accelerated, but the positive feedback effect of information dominates, the searching randomness is abate, the calculation results are easy to fall into local optimum. Through the analysis of results, the scope of ρ can be set as [0.6, 0.9].

3.3.4. Path selection probability q_0: $q_0(0 \le q_0 \le 1)$ is a given parameter, the size of path selection probability q_0 determines the method of each ant choosing the next path at current node. When q_0 is bigger, the ants choose the best edges with large probability based on prior knowledge, the convergence speed is faster, but it is easy to fall into local optimum. When q_0 is less, the randomness of choosing another edge increases, the diversity of solution is also increased. In the experiment, q_0 is set as 0.99, 0.9, 0.75, 0.5, 0.25, and 0, the remaining parameters adopt basic values. The experimental results are shown in Figure 4.

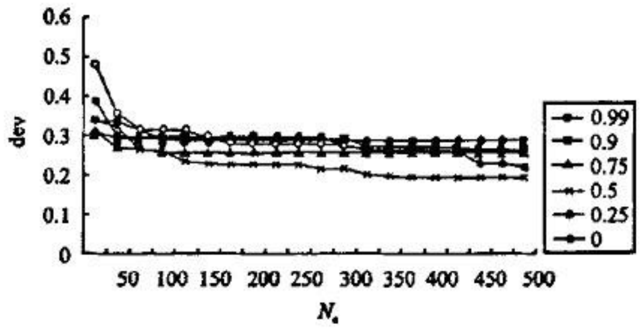

Figure 4. MMAS with various values of q_0

From Figure 4, the selection of path selection probability q_0 has a certain influence on the performance of ant colony algorithm. When q_0 value is too small, due to the randomness of selection, the result of initial solution is poor, convergence speed is also slower. When q_0 value is too large, the global search ability becomes poor; it is easy to appear premature stagnation phenomenon. Therefore, in the early stage of the algorithm implementation q_0 is set as a larger value to get a better initial solution, in the late stage of algorithm implementation, in order to improve the diversity of choice, q_0 is set as a smaller value.

3.3.5. The number of ants N_A : Ant colony algorithm is a kind of parallel randomly searching algorithm, it is through the group evolution process consisting of multiple candidate solutions to search the optimal solution, this process needs both ant individual adaptive ability and the internal mutual collaboration of ant colony. In the experiment, N_A is set as 150, 100, 50, 25, 5, and the rest parameters use basic values. The experimental results are shown in Figure 5.

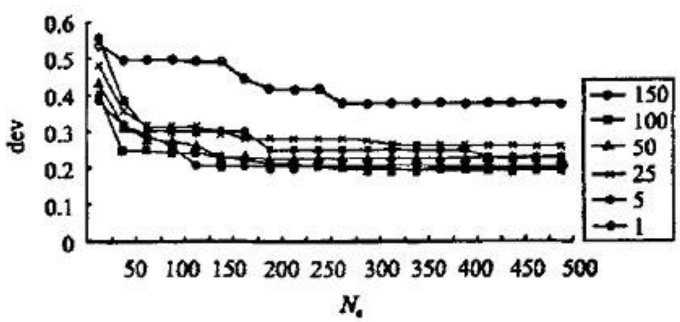

Figure 5. MMAS with various values of m

The Figure 5 shows, when the ant number N_A increases, the stability and global performance of search is improved; when the ant number N_A is much greater than the size of problem, continually increasing the ants number may improve the performance of algorithm, but it is not particularly evident. This is due to the parallelism of ant colony algorithm, the ant number N_A can improve the global search ability of ant colony algorithm and the stability of algorithm, when the ant number N_A increases to a certain degree, it can make a lot of information on former searched path tend to the average, the information positive feedback effect is not obvious, although search randomicity is strengthened, and the convergence speed slows down; On the other hand, if ant colony number N_A is very little, the randomness of search is abate; although convergence speed is fast, it can make the global convergence performance of algorithm lower down, it is easy to appear premature stagnation phenomenon. Considering the execution efficiency and effectiveness of algorithm, the ant number N_A can be set as the smaller value in the initial stage of algorithm, with the increase of iteration times, which gradually increases to improve the diversity of the solution, and avoids falling into local optimization. Due to the problem scale will have different size, for better implementation effect and execution efficiency, the appropriate maximum number of ants should be chosen.

3.4. Local search

Route for each ant is adopted 2-opt method to partially improve the sub-lines, and the longest sub-line adopts relocation method to do the line optimization.

3.4.1. 2-opt method: 2-opt method refers that two edges of a line are replaced by other two edges. As shown in Figure 6, in the line the edges $(i, i+1)$ and $(j+1, j)$ are replaced by edges (i, j) and $(i+1, j+1)$, and the client nodes between node i+1 and node j+1 are reversed to form a new line.

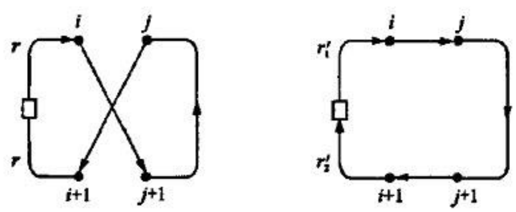

Figure 6. 2-opt operation

3.4.2. Relocation method: Relocation method is the optimization method among lines, which moves a client node from one route to another route. As shown in Figure 7, the node i is relocated from line r_1 to the interval between node j and node $j+1$ on line r_2, to form two new lines r_1' and r_2'. Here only the nodes from the longest sub-line are taken to do relocation.

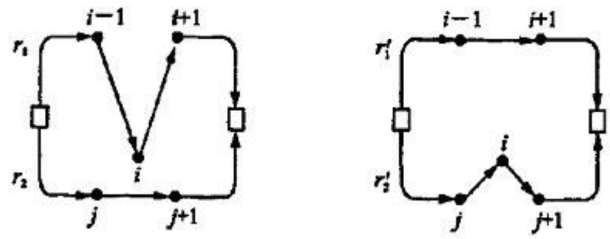

Figure 7. Relocation operation

4. Calculation results

The max-min ant system with self adaptive parameters in this paper is used in the examples put forward by Christofides, Mingozzi and Toth (CMT), which adopts seven questions (questions 1-5 and 11, 12) in the case to do calculation.

Algorithm is described as:

(1) The geographical location of nodes in problem CMT-1 to CMT-5 is randomly generated, in the route building the parallel method is adopted; in CMT-11 and CMT-12 the node location is clustering distribution. Sequential method is used in line building.

(2) The pheromone setting: $r_0 = 1/D_0$, D_0 is the initial longest sub-line generated by nearest domain method, $\tau_{\max} = \tau_0$, $\tau_{\min} = 0.005\tau_0$.

(3) The parameters setting: the size of candidate set $Allowed_l$ is limited as $n/6$; the information heuristic factor α is set as constant 1; the initial value of expectation heuristic factor β is 10, for every 150 iteration which descends 1, until β is 3 ; the initial value of q_0 is 0.9, for every 20 iterations which descends 0.01, until q_0 is 0; the initial value of pheromone continuous parameter ρ is 0.9, for every 100 iterations which descends 0.1, until ρ is 0.6; The initial value of ant number N_A is 1, for every 100 iterations which increases 1, until N_A is 40. The setting of above parameter makes the algorithm quickly get a good solution in the initial stage, to speed up the convergence. In the latter stage the diversity of solution can be enhanced to avoid falling into local optimization.

(4) The number of iterations is set as 2000; when the best solution repeats 1000 times, the algorithm can enter stagnation, the pheromone and the parameters are initialized again.

The adaptive parameters of MMAS are applied to min-max vehicle routing problem of CMT, each problem needs to solve 10 times, the calculation results are shown in Table 1, D_n and D_m are used to express the average value and optimal value of the longest sub-line length.

Table 1. Calculation Results

Problems		n	Q	N_A	D_0	D_m	D_{krw}
CMT-1	c50	50	160	5	116.375	111.370	87.864
CMT-2	c75	75	140	10	106.928	97.773	86.533
CMT-3	c100	100	200	8	120.801	114.421	99.860
CMT-4	c150	150	200	12	110.475	106.569	99.860
CMT-5	c199	199	200	16	131.020	120.985	99.860
CMT-11	c120	120	200	7	214.450	209.646	198.565
CMT-12	c100b	100	200	10	120.532	120.532	117.047

Using adaptive parameter max-min ant system to solve the optimal results of c50, the route includes five sub-lines, respectively are 0-46-32-2-20-35-32-2-20-31-22-1-0, 0-12-38-9-30-34-21-29-50-16-11-0, 0-27-48-8-26-7-23-43-24-14-6-0, 0-18-25-13-41-19-42-17-0, 0-47-4-37-44-15-45-33-39-10-49, which are shown in Figure 8.

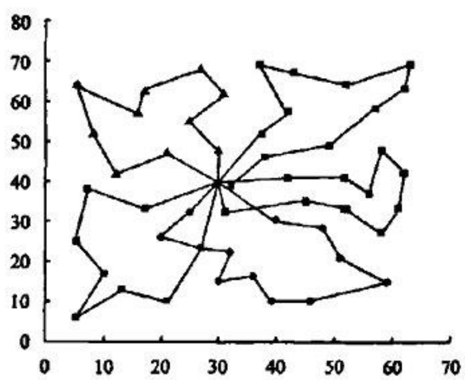

Figure 8. Optimal routes of CMT-1 resulted from MMVRP

5. Conclusion

Based on the description of min-max vehicle routing problem, this paper uses the improved max-min ant system to solve the problem. In route construction, according to the randomness and the clustering of different data sets, parallel method and serial method are used to generate lines; aiming to the line selection characteristics of ant colony algorithm, the pheromone update strategy is put forward correspondingly; analysis information heuristic factor α , expectation heuristic factor β ,pheromone continuous factor ρ , path selection probability q_0, ant number N_A etc., these main parameters influence the performance of max-min ant system, and do adaptive adjustment to parameters in the algorithm execution process, which enhance the searching near the optimal solution, to speed up the convergence of algorithm, and also to some extent to ensure the diversity of solution to avoid falling into local optimization. This algorithm is applied to some examples and gets good results.

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