Application of Iterative Singular Value Decomposition De-noising to Centrifugal Compressor Signal Analysis

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Abstract

Noise has great influence on the signal analysis for centrifugal compressor. In order to eliminate the noise, the iterative singular value decomposition (ISVD) de-noising is applied in this paper. Firstly, the algorithm of this method is introduced. It is based on singular value decomposition about the trajectory matrix of attractor which is reconstructed according to time delay embedding theory. Secondly, the accuracy of this method is tested by reconstructing the pseudo-phase portrait for the signal of Lorenz attractor. Comparing with the pseudo-phase portrait reconstructed from signal contained noise, the pseudo-phase portrait reconstructed after ISVD de-noising is more regular. Finally, this method is used in the centrifugal compressor signal analysis. By this method, the correlation dimension, which can reflect different fault condition for nonlinear system, is estimated accurately. It is proved that this method can reduce the noise effectively, which influences the correlation dimension computing a lot. This method can improve the nice rate of signal analysis.

Keywords: *iterative singular value decomposition, centrifugal compressor, de-noising, correlation dimension, pseudo-phase portrait*

1. Introduction

Centrifugal compressor is a large rotating machine in petrochemical enterprises. Because its structure is complex, centrifugal compressor usually functions showing greatly non-linear characteristics. In recent years, the theory of chaos and fractal has been widely applied in the fault diagnosis of the large, complex and non-linear system [1-3]. Pseudo-phase portrait can be used to extract qualitative feature of chaotic attractor, however, the noise contained in measured signal will make the pseudo-phase portrait irregular. The correlation dimension of the fractal theory is also a very important parameter for characterizing the chaotic attractor. It is usually used to quantitatively describe the behavior of the non-linear system. However, the correlation dimension is very sensitive to the presence of the noise. Kostelich and T. Schreiber [4] showed that, the noise contained in measured signal will strongly reduce the width of the scaling region on the log-log plot of correlation integrals, cause an increase of the correlation dimension and obscure the underlying fractal structure. Therefore, in order to make the pseudo-phase portrait regular and estimate the correlation dimension accurately, it is necessary to reduce noise from the measured signal.

At present, digital filter, time-average technique, wavelet de-noising, and so on are often used in reducing noise. If the noise has a lot of difference from the system signal, the noise can be eliminated by these methods. However, the noise and the system signal usually mix with each other for the non-linear system. The de-noising effect of the conventional methods is not obvious.

To reduce noise, the iterative singular value decomposition (ISVD) de-noising [5-7], which is based on the phase space reconstruction, is applied in this paper.

2. ISVD De-noising Algorithm

The procedure of the ISVD de-noising method is as follows (2.1~2.4)

2.1. Construction of the Trajectory Matrix

Given the measured one dimension time series $\{x_1, x_2, \cdots, x_N\}$, the trajectory matrix D can be constructed according to time delay embedding theory

$$D = \left\{ \begin{matrix} x_1 & x_{1+\tau} & \cdots & x_{1+(n-1)\tau} \\ x_2 & x_{2+\tau} & \cdots & x_{2+(n-1)\tau} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N_n} & x_{N_n+\tau} & \cdots & x_{N_n+(n-1)\tau} \end{matrix} \right. \tag{1}$$

where $N_n = N - (n-1)\tau$, τ is the time delay and n is the embedding dimension. The embedding theorem of Takens[8] states that the embedding dimension n should satisfy the inequality $n \geq 2d + 1$, where d is the dimension of a manifold containing the attractor.

2.2. Singular Value Decomposition

Apply singular value decomposition (SVD) to the trajectory matrix D.

$$D = S \sum C^T \tag{2}$$

where S is the eigenvectors of XX^T, C is the eigenvectors of $X^T X$, Σ is the diagonal matrix containing the singular values, $S \in \Re^{N_n \times n}$, $C \in \Re^{n \times n}$,and $\Sigma \in \Re^{n \times n}, N_n >> n$.

$$\Sigma = diag(\lambda_1, \lambda_2, \ldots, \lambda_n) \tag{3}$$

where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the singular values of D and $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n \geq 0$.

Shin et al. [6] showed that the noise will cause all the singular values to be non-zero. Especially, if the noise is white noise, the noise will cause all the singular values to be increased the same amount λ_{noise}^2, then they can be written as

$$\lambda_i^2 = \overline{\lambda}_i^2 + \lambda_{noise}^2 \qquad i = 1,2,\ldots,k$$

$$\lambda_{k+1}^2 =, \ldots, = \lambda_n^2 = \lambda_{noise}^2 \tag{4}$$

where k is the rank of D, λ_{noise} is caused by the noise and $\lambda_i^2 >> \lambda_{noise}^2$ $(\mathrm{i} = 1,2,\ldots,\mathrm{k})$

2.3. Acquisition of the Noise Reduced Signal

The trajectory matrix D can also be written as

$$D = D_m + W = \begin{bmatrix} S1 \; S2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} C_1^T \\ C_2^T \end{bmatrix} \tag{5}$$

where D_m is the deterministic part of the trajectory matrix that denotes the clean system signal, W denotes the noisy signal, $S_1 \in \Re^{N_n \times k}, \Sigma_1 \in \Re^{k \times k}$, $C_1 \in \Re^{k \times n}$ and k is the rank of D.

The optimal approximate of D_m is given by

$$\overline{D_m} = S_1 \Sigma_1 C_1^T \tag{6}$$

From eq.(6), we can see if we reserve the bigger singular values $\lambda_i (i = 1, \cdots, k)$ and set the others to be zero, $\overline{D_m}$ can be obtained by the inverse course of SVD.

Average each column of the matrix D_m, and we obtain the noise reduced signal.

2.4. Iteration

Because $\overline{D_m}$ is only an estimate of the deterministic part D_m, the recovered signal is not noise free after only one iteration. It needs to repeat the above steps several times before we obtain the satisfied signal.

2.5. An Important Question

It is an important question to determine the number of the bigger singular values k. It may affect the question whether the optimal approximate matrix $\overline{D_m}$ reserves the main information of the system signal. In this paper the contribution rate of the singular value is used to determine it.

The contribution rate of λ_i is given by

$$\varepsilon_i = \lambda_i^2 \Big/ \sum_i \lambda_i^2 \tag{7}$$

If $\sum_{i=1}^{p} \varepsilon_i^2 \Big/ \sum_{i=1}^{n} \varepsilon_i^2 \geq \beta$, then $k = p$, where β is the contribution rate criterion and k is the number of the bigger singular values. Application results show that this method can determine k correctly.

3. Numerical Simulation Experiment

In order to verify the validity of the program written according to the ISVD de-noising algorithm, the signal of Lorenz attractor is analyzed. The theoretic correlation dimension of Lorenz attractor is 2.06.

The Lorenz attractor is given by

$$\begin{cases} \dfrac{dx}{dt} = 10(y-x) \\ \dfrac{dy}{dt} = 28x - xz - y \\ \dfrac{dz}{dt} = xy - 8z/3 \end{cases} \tag{8}$$

To generate a time series, we set the initial condition $(x_0, y_0, z_0) = (0.05, 0.05, 0.05)$ and integrate eq. (8) using a fourth order Runge-Kutta method with a fixed integration step size of 0.01s, and after discarding the first 2000 steps as the transient regime, we collect 40960 data points of x.The signal is assumed to be clean and 10% Gaussian white noise is added to it.

3.1. Qualitative Experiment

The ISVD de-noising method is applied to the noisy signal. The iteration number is 2, the embedding dimension $n = 15$ and the time delay $\tau = 1$.

The pseudo-phase portraits [9] for noisy signal and by ISVD de-noising are shown in Figure 1 and the time waveforms are shown in Figure 2.

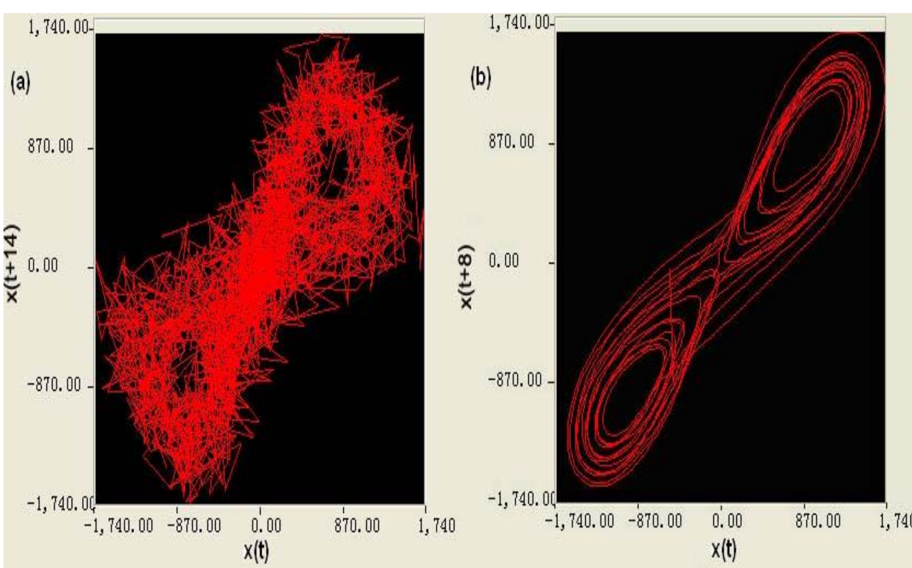

Figure 1. Pseudo-phase Portrait (a) for Noisy Lorenz Signal (b) by ISVD De-nosing

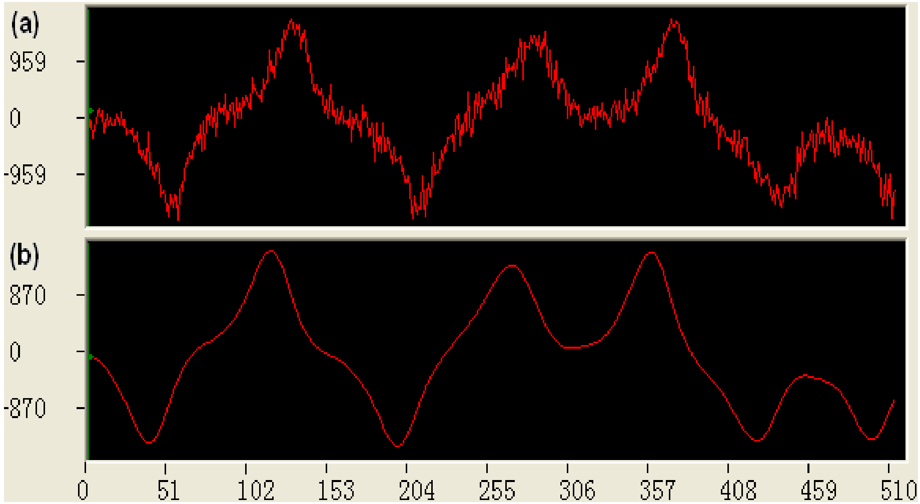

Figure 2. (a) Noisy Lorenz Signal (b) Recovered Time Series Obtained by Two Iterations of the ISVD De-nosing

Comparing Figure 1(a) with Figure 1(b), we can see that the pseudo-phase portrait by ISVD is more regular. Comparing Figure 2(a) with Figure 2(b), we can see that the waveform by ISVD is smoother and the characteristic waveform is well reserved.

3.2. Quantitative Experiment

In order to quantitatively describe Lorenz attractor, we can estimate the correlation dimension of Lorenz attractor. The ISVD de-noising method is applied to the noisy signal firstly. Then the correlation dimension of Lorenz attractor is estimated according to the modified G-P algorithm [10].

The parameters of ISVD de-noising are as follows, the iteration number is 2, the embedding dimension $n = 15$ and the time delay $\tau = 1$.

The parameters of G-P algorithm are as follows, the embedding dimension $n = 7$ and the time delay $\tau = 2$.

The log-log plot of correlation integrals and local slope [11] curve for noisy signal are shown in Figure 3. The curves by ISVD are shown in Figure 4.

According to the local slope curve, we can determine the scaling region of the log-log plot of correlation integrals. The correlation dimension is the slope of the scaling region which can be determined by using the least squares fit.

From Figure 3(b), we can see that for noisy Lorenz signal there is no range over which the slope is constant, so the scaling region doesn’t exist in Figure 3(a). The correlation dimension estimated tends to be infinite.

From Figure 4(b), we can see that when the value of $\ln(r)$ is between -2.7 and -3.8, the slope is constant, so the scaling region in Figure 4(a) is also between -2.7 and -3.8. By the least squares fit, the correlation dimension estimated is between 2.06 and 2.12. It is in accordance with the theoretic correlation dimension 2.06.

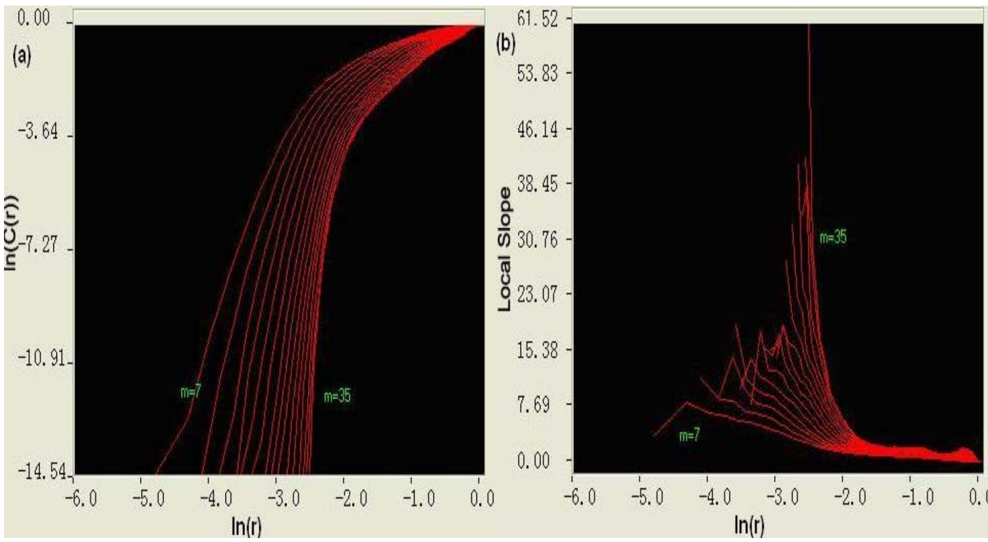

Figure 3. (a) Log-log plot of Correlation Integrals for Noisy Lorenz Signal (10% white noise added). Correlation Dimension tends to be Infinite. (b)Slope of Curves in (a)

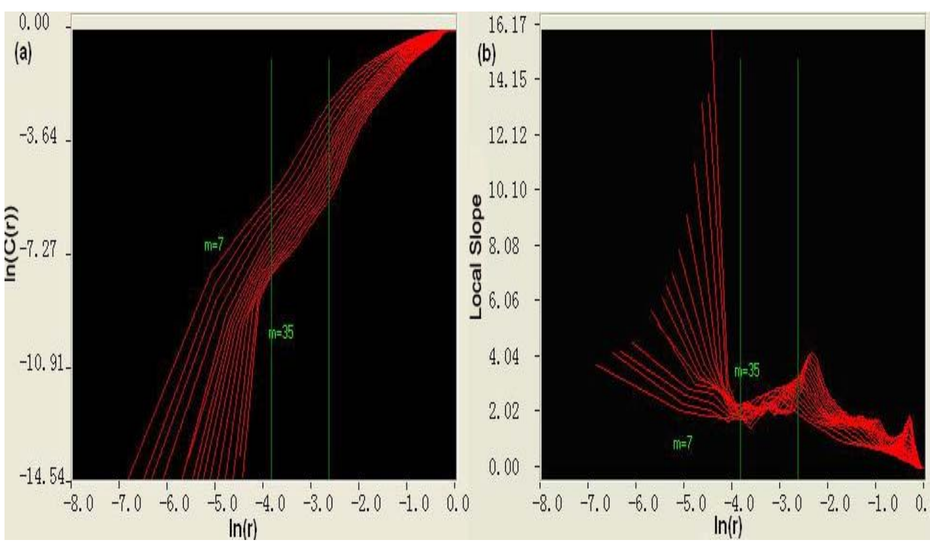

Figure 4. (a) Log-log Plot of Correlation Integrals for Lorenz Signal by ISVD De-nosing. Correlation Dimension D=2.06~2.12. (b) Slope of Curves in (a)

4. Actual Application

In this section, the ISVD de-noising method is applied to the centrifugal compressor signal analysis. We collect vibration signal when the centrifugal compressor is testing. The sampling frequency is 16000Hz and the sampling number of data is 40960.

Firstly, the ISVD de-noising method is applied to the signal to reduce the noise. The iteration number is 2, the embedding dimension $n = 15$ and the time delay $\tau = 1$. Then the correlation dimension is estimated according to the modified G-P algorithm. The parameters of G-P algorithm are $n = 3$ and $\tau = 25$. The log-log plot of correlation integrals and local slope curve for noisy signal are shown in Figure 5.The curves by ISVD are shown in Figure 6.

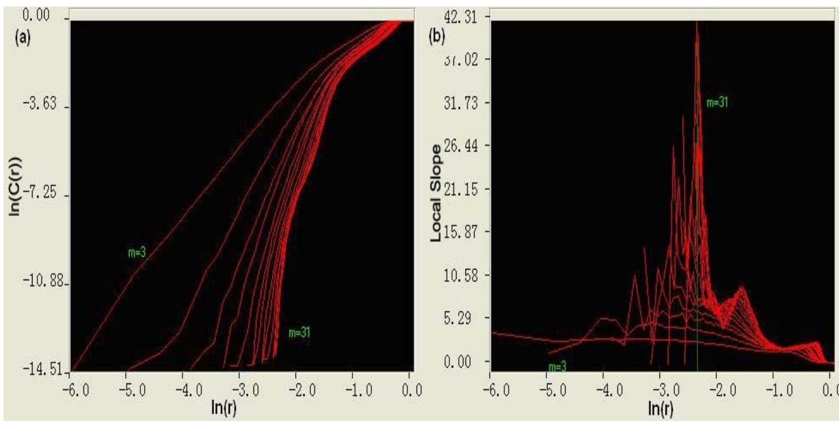

Figure 5. (a) Log-log Plot of Correlation Integrals for Data of Centrifugal Compressor. Correlation Dimension tends to be Infinite. (b) Slope of Curves in (a)

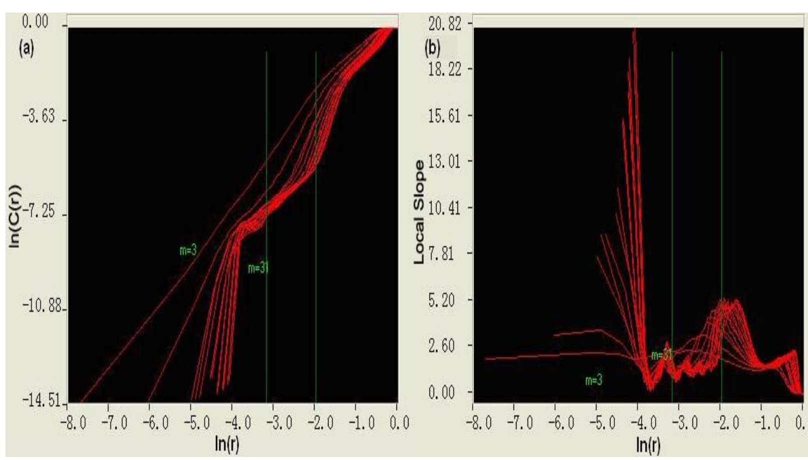

Figure 6. (a) Log-log Plot of Correlation Integrals for Centrifugal Compressor Signal by Two Iterations of the ISVD De-nosing. Embedding Dimension is Fixed at n=31.Correlation Dimension D=1.53~1.57. (b) Slope of Curves in (a)

From Figure 5, we can see that there is rarely scaling region and the correlation dimension tends to be infinite for noisy signal. From Figure 6, we can see that when the value of $\ln(r)$ is between -1.9 and -3.2, the scaling region exists. Then the correlation dimension is estimated between 1.53 and 1.57.

By spectrum analysis, it is estimated that this centrifugal compressor has the fault of rotor unbalance. Wang et al. [10] pointed out that the rotor unbalance belongs to the fault whose characteristic concentrated on the power frequency, and their correlation dimension should be near to 1. Therefore, the correlation dimension estimated in the above is reasonable.

5. Conclusion

Noise is always contained in the measured vibration signal of centrifugal compressor. It has great influence on the signal analysis. To reduce the noise, the iterative singular value decomposition (ISVD) de-noising is applied in this paper. Its algorithm is simple and it's easy to implement. Numerical experiment and actual application show that, its de-noising effect is obvious. The pseudo-phase portrait by ISVD de-nosing is more regular, and the correlation dimension is estimated accurately. The ISVD de-noising method can improve the nice rate of signal analysis.

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