# Nonlinear Channel Estimation Based on Multi-level PN Sequences in OFDM Systems

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**_Abstract_**

*Nonlinear distortion of power amplifier is one of the major factor that degrades the performance of OFDM systems .Instead of estimating the nonlinearity at transmitter such as ordinary pre-distortion (PD), this paper estimates the nonlinearity of amplifier at the receiver. The problem is that nonlinear and multipath dispersion mix together at receiver end. And their parameters are generally unknown, so conventional method doesn't work well. In this paper, an algorithm that estimates both the nonlinear parameters of the power amplifier and the impulse response of the wireless channel is proposed. Multi-level PN sequences are transmitted to construct a Vandermonde matrix, which separates the nonlinearity and linearity into different subspaces. The separation makes it easy to estimate the nonlinearity and linearity independently.*

## 1. Introduction

Orthogonal frequency division multiplexing system (OFDM) adopts a multicarrier modulation method with orthogonal subcarriers, and exhibits several advantages over single carrier modulation. OFDM is resistant to impulsive noise and fast fading due to longer symbols on separate sub-carriers. OFDM can also achieve a higher spectral efficiency than the single carrier scheme [1][2].Therefore, OFDM method is believed to be a promising technique in wireless high rate data transmission.

Although OFDM transmission exhibits many benefits for use in wireless communications, it nevertheless has some disadvantages such as high peak-to-average power ratio (PAPR) as well as strict time and frequency synchronization requirements. In particular, high PAPR causes OFDM systems to be sensitive to nonlinear distortion introduced by power amplifier (PA). This nonlinearity can't be ignored as it causes both in-band distortion and out-of-band spectrum re-growth. The in-band distortion degrades the bit error rate performance; and the out-of-band spectrum re-growth causes the adjacent channel interference (ACI). So the nonlinearity must be rightly estimated and effectively compensated to improve the performance of the system.

Many ordinary pre-distortion (PD) schemes have been utilized to compensate nonlinear distortion in OFDM systems [3][4][5][6]. The PD method improves system performance to some extent indeed. But it needs extra devices, such as linear attenuation, analog to digital convertor (ADC) etc. These devices would increase system cost and enlarge the size of circuit board.

If the nonlinearity can be estimated at the receiver end, no extra device is needed. It is more suitable for software defined radio. But the problem is that nonlinear PA and multipath channel mix together at receiver. And their parameters are generally unknown, so conventional method doesn't work well. Thus, in order to estimate them effectively, the combination of nonlinear PA and wireless channel can be considered as a nonlinear channel. Then the characteristic of this nonlinear channel may be estimated through some special methods.

Some work has been done to analyze characteristic of nonlinear channel. Volterra filters [7][8], neural networks [9][10]and support vector machine (SVM) [11][12]are the main techniques for nonlinear channel estimation. For volterra filters, dimension of the model grows exponentially as nonlinear order grows. For neural networks, a local optimum solution may be found out, instead of the global one. For SVM, it treats nonlinear channel as a black box and needs great calculation. Besides, performance of SVM depends on selection of its kernel function.

A relatively effective way to solve nonlinear problem is to separate nonlinearity and linearity into different subspaces. Fernando applied this way to estimate nonlinear optical fiber channel [13]. In fiber channel, nonlinearity exists in the radio-over-fiber (ROF) link. Transmitted Signal suffers from multipath dispersion first, then nonlinear distortion. Fernando's estimation is used to make equalization at receiver. But in OFDM systems, Transmitted signal suffers from nonlinear distortion first. In contrary to ROF link, the estimation proposed by this paper is used to make special pre-distortion at transmitter. Besides, Due to different location of nonlinearity, the mathematical analysis is somewhat different with Fernando's.

Once the nonlinearity and linearity are isolated, conventional methods can be used to estimate them independently. The complexity of this model just linearly grows with the order of nonlinearity. In this paper, Multi-level PN sequences are transmitted to generate a set of equations. Through solving the equations, linearity can be separated from the nonlinearity.

The rest of this paper is organized as follows: section 2 is the principle of nonlinear channel estimation based on Multi-level PN sequences. Section 3 is the simulation results. Finally in section 4, we give the conclusion about this paper.

## 2. Multi-level PN sequences for nonlinear channel estimation

A simplified nonlinear channel model composed of a nonlinear amplifier and a wireless channel is showing in Fig.1. $f(\cdot)$ is the nonlinear function of power amplifier. And it can be modeled with a $L$ order polynomial.

$$f(n) = C_1 x(n) + C_2 x^2(n) + \cdots + C_L x^L(n) \tag{1.1}$$

The polynomial represents a general nonlinear transfer function and its coefficients can be extracted from AM/AM and AM/PM measurements as described in [14].

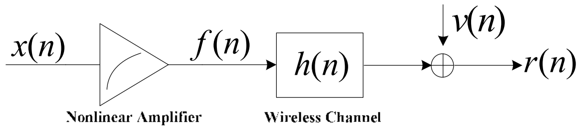


**Fig.1 the nonlinear channel model**

If there is no wireless channel, for example, in pre-distortion situation, the coefficients $C_i (i = 1, 2, \cdots, L)$ can be calculated through least squares polynomial fitting directly.

But once the wireless channel is involved in, the situation becomes much more complicated.

Let $h(n)$ be the impulse response of wireless channel. Then,

$$\begin{aligned} r(n) &= f(n) * h(n) + v(n) \\ &= s_1(n) + s_2(n) + \cdots + s_L(n) + v(n) \end{aligned} \tag{1.2}$$

Where

$$s_1(n) = C_1 \sum_{m=-\infty}^{\infty} h(m) x(n-m) \tag{1.3}$$

$$s_2(n) = C_2 \sum_{m=-\infty}^{\infty} h(m) x^2(n-m) \tag{1.4}$$

$$\vdots$$

Finally

$$s_L(n) = C_L \sum_{m=-\infty}^{\infty} h(m) x^L(n-m) \tag{1.5}$$

Therefore, for a given sequence $x(n)$, the received $r(n)$ consists of high order terms of $x(n)$ convoluted with channel impulse response $h(n)$.

### 2.1. Linear channel estimation

Compute the cross-correlation of received signal and transmitted PN sequences as follow,

$$\begin{aligned} R_{rx}(k) &= E[x^*(n) r(n+k)] \\ &= \tfrac{1}{N} \sum_{n=1}^{N} x^*(n) r(n+k) \\ &= \sum_{i=1}^{L} R_{s_i x}(k) + R_{vx}(k) \end{aligned} \tag{1.6}$$

Where,

$$R_{vx}(k) = \tfrac{1}{N} \sum_{n=1}^{N} x^*(n) v(n+k) \tag{1.7}$$

is the cross- correlation value of $x(n)$ and $v(n)$.

$$R_{s_i x}(k) = \tfrac{1}{N} \sum_{n=1}^{N} x^*(n) s_i(n+k) \tag{1.8}$$

is the cross-correlation value of $x(n)$ and $s_i(n)$.

Usually, $x(n)$ and $v(n)$ are uncorrelated, so $R_{vx}(k)$ can be treated as zero.

Then $R_{rx}(k)$ is simply written as,

$$R_{rx}(k) = \sum_{i=1}^{L} R_{s_i x}(k)$$

(1.9)

From (1.3) and(1.8), $R_{s_1 x}(k)$ can be written as,

$$\begin{aligned} R_{s_1 x}(k) &= \tfrac{1}{N}\sum_{n=1}^{N} x^*(n) s_1(n+k) \\ &= \tfrac{1}{N}\sum_{n=1}^{N} x^*(n) C_1 \sum_{m=-\infty}^{\infty} h(m) x(n-m) \\ &= C_1 \sum_{m=-\infty}^{\infty} h(m) R_{xx}(k-m) \end{aligned}$$

(1.10)

Where $R_{xx}(k)$ is the auto-correlation value of $x(n)$ .For PN sequences $x(n)$ , its auto-correlation $R_{xx}(k)$ is periodic and be determined as，

$$R_{xx}(k) = \begin{cases} 1 & if\ k = 0 \bmod N \\ -\frac{1}{N} & if\ k \neq 0 \bmod N \end{cases}$$

(1.11)

Assume synchronization is ideally achieved and $N ?\ 1$. If the correlation is computed within the time interval $1 \le n \le N$ , then $R_{xx}(k)$ can be approximately written as $R_{xx}(k) = \delta(k)$ .

Therefore, (1.10)can be rewritten as,

$$\begin{aligned} R_{s_1 x}(k) &= C_1 \sum_{m=-\infty}^{\infty} h(m) R_{xx}(k-m) \\ &= C_1 \sum_{m=-\infty}^{\infty} h(m) \delta(k-m) \\ &= C_1 h(k) \end{aligned}$$

(1.12)

Equation (1.12) shows that, the 1st order cross-correlation $R_{s_1 x}(k)$ equal to wireless channel response multiplied with a constant $C_1$ .So the linear channel estimation can be achieved if $R_{s_1 x}(k)$ is known. Note $C_1$ should be nonzero for effective estimation.

Then, the rest question is how to isolate $R_{s_1 x}(k)$ from $R_{rx}(k)$ .

The easiest way is to repeat the sequences a few (say $N_a$) times with different amplitude. This way was originally proposed by Billings [15]. Multi-level PN sequences at the receiver generate simultaneously $N_a$ equations instead of a single equation. Then $R_{s_1 x}(k)$ can be solved out through the $N_a$ equations without computing the high order cross-correlation $R_{s_i x}(k)$ .Notice that $N_a$ should not be smaller than the order of nonlinearity.

Multi-level PN sequences are defined as $a_i x(n)$

Where $a_i \neq 0\ i = 1,2\mathrm{L}\ N_a$ and $a_i \neq a_j \forall i \neq j$

Firstly, assume $N_a = L$ , then

$$\begin{aligned} R_{ra_i x}(k) &= \sum_{j=1}^{L} a_i^j R_{s_j x}(k) \\ &= \sum_{j=1}^{N_a} a_i^j R_{s_j x}(k) \end{aligned} \tag{1.13}$$

Expanding equation (1.13)in matrix form gives,

$$A \times \begin{bmatrix} R_{s_1 x}(k) \\ R_{s_2 x}(k) \\ \vdots \\ R_{s_{N_a} x}(k) \end{bmatrix} = \begin{bmatrix} R_{ra_1 x}(k) \\ R_{ra_2 x}(k) \\ \vdots \\ R_{ra_{N_a} x}(k) \end{bmatrix} \tag{1.14}$$

Where,

$$A = \begin{pmatrix} a_1 & a_1^2 & \cdots & a_1^{N_a} \\ a_2 & a_2^2 & \cdots & a_2^{N_a} \\ \vdots & \vdots & \cdots & \vdots \\ a_{N_a} & a_{N_a}^2 & \cdots & a_{N_a}^{N_a} \end{pmatrix}$$

As described in [13], the determinant of A is given below, which is nonzero for $a_i \neq 0$ and $a_i \neq a_j \forall i \neq j$

$$\det(A) = \prod_{1 \le i \le N_a} a_i \prod_{1 \le j < i \le N_a} (a_i - a_j) .$$

Thus, for every value of $m$ , equation (1.14) has a unique solution for $R_{s_1 x}(k)$ .

Usually, the nonlinear order $L$ is unknown in advance. For better estimation, $N_a$ should not be smaller than $L$ . Otherwise, high order interference would be introduced in. It is worth mention that Fernando's conclusion is somewhat improper as he considered $N_a$ is independent of $L$ .

### 2.2. Nonlinear polynomial fit

From(1.12) the estimated channel response can be written as,

$$\begin{aligned} h^*(n) &= R_{s_1 x}(n) \\ &= C_1 h(n) \end{aligned} \tag{1.15}$$

After compensating channel dispersion with $h^*(n)$ , the signal out of power amplifier $f(n)$ can be written as,

$$\begin{aligned} f^*(n) &= \frac{1}{C_1} f(n) + v'(n) \\ &= x(n) + \frac{C_2}{C_1} x^2(n) + \cdots + \frac{C_L}{C_1} x^L(n) + v'(n) \end{aligned} \tag{1.16}$$

With $x(n)$ and $f^*(n)$, $C$ can be estimated through the least squares polynomial fitting.

$$\hat{C} = \arg\min_{\hat{C}} \sum_{n=1}^{N_T} \left| \hat{f}(n) - f^*(n) \right|^2 \tag{1.17}$$

Where $\hat{f}(n) = \hat{C}_1 x(n) + \hat{C}_2 x^2(n) + \cdots + \hat{C}_L x^L(n)$, and $N_T$ is the number of samples used to polynomial fitting.

Define matrix $X$ as below,

$$X = \begin{pmatrix} x^L(1) & x^{L-1}(1) & \cdots & 1 \\ x^L(2) & x^{L-1}(2) & \cdots & 1 \\ \vdots & \vdots & \cdots & \vdots \\ x^L(N_T) & x^{L-1}(N_T) & \cdots & 1 \end{pmatrix}$$

Then $\hat{C}$ can be easily fitted as:

$$\hat{C} = (X^T X)^{-1} X^T y \tag{1.18}$$

Where, $y$ is the estimated output of amplifier.

## 3. Simulation results

To evaluate this nonlinear channel estimation algorithm, simulation is run under Matlab/simulink environment.

The block diagram of simulation model is shown in Figure.2. PN sequence $x(n)$ is generated by a PN generator. A gain block is used to get multi-level sequences. Nonlinear is modeled by a polynomial function:

$$f(n) = x(n) + 0.5x^2(n) + 0.2x^3(n)$$

Wireless channel is described by a discrete tap delay line filter with complex coefficient:

$$\begin{aligned} h(n) = &(-0.04 - 0.53j)\delta(n-1) \\ &+(0.25 + 0.57)\delta(n-4) \\ &+(-0.26 + 0.22j)\delta(n-6) \\ &+(0.08 + 0.45j)\delta(n-11) \end{aligned}$$

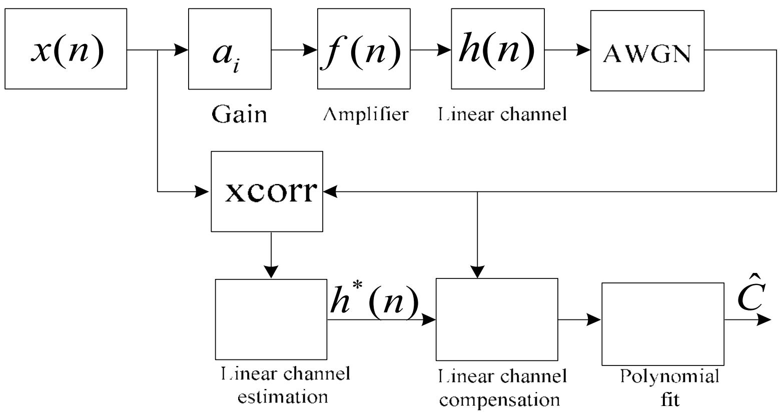


**Figure.2 block diagram of simulation model**

The levels of PN sequences are respectively set to be 1, 1.2 and 1.4. And length of PN sequences is 255.

The actual channel impulse response and estimated channel impulse response are depicted in Figure.3. It can be seen that the proposed method is quite efficient for wireless channel estimation.

Figure. 4 compares the MSE performance of channel estimation under linear and nonlinear amplifier. MSE performance under nonlinear amplifier is just a little worse than under linear amplifier. This indicates that nonlinearity of amplifier has little effect on estimation of wireless channel. The degradation is caused by the correlation properties of PN sequence, as the auto-correlation function is not ideal $\delta$ function. If better orthogonal sequence is used, performance will be improved.

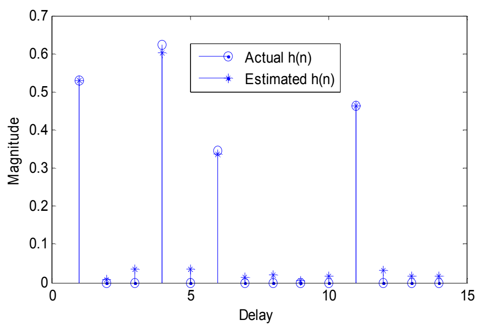


**Figure.3 Actual and estimated channel response**

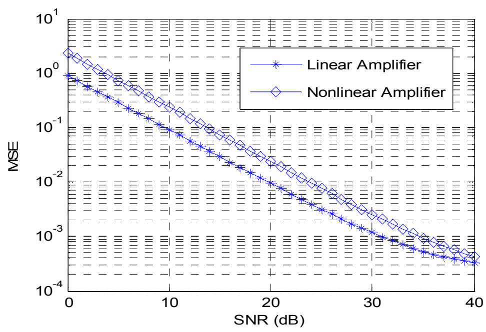


**Figure. 4 MSE performance of channel estimation**

By polynomial fitting, coefficients of nonlinear function can be easily estimated as,

$$\hat{f}(n) = 0.9985x(n) + 0.5286x^2(n) + 0.2051x^3(n)$$

It is very close to actual nonlinear function $f(n)$. It is necessary to mention that coefficient $C_1$ is set to 1 for convenience. In fact, $C_1$ can be an arbitrary nonzero number.

## 4. Conclusion

This paper proposed a nonlinear channel estimation algorithm based on multi-level PN sequences. The combination of linearity and nonlinearity can be separated into two independent parts. Then, estimation is achieved through two independent steps. Simulation results show that parameters of both nonlinearity and linearity can be efficiently estimated.

This paper is mainly concerned about the estimation of nonlinearity. Details of compensation are going to be proposed in the next article. Estimated parameters are sent back to transmitter for pre-distortion through a feed-back link. Then, transmitter compensates nonlinearity with the right estimated parameters, which gradually lowers down the order of polynomial. After convergence, the channel becomes a nearly linear one.

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