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Some Properties of One Dimensional Array of Isotropic Sensors

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Abstract

Sensor array processing is field of research devoted to the study of proper and mutual parameters of sensors such as the shapes of each antenna, the frequency bands, the interferences between radiation patterns and the geometrical arrangement of the array, all these parameters have an impact on the transmission and reception characteristics of the array. The far field angular beam scan using array of antennas relies on the geometrical description of the elements, where the visible region in polar or spherical coordinates depends on either one dimensional or two dimensional geometries. In this paper, we discuss some properties of one dimension uniform array of isotropic sensors, we present a detailed description of planar and conical ambiguities using wave vector projection. Next we present a particular type of broadside uniform array where we evaluate the half power beam width using the condition of non uniform input current distribution of antennas, this study is accomplished by numerical evaluation.

Keywords*: One dimensional array, planar ambiguity, conical ambiguity, array factor, half power beam width*

1. Introduction

In electronic engineering, an antenna [1] is device that converts the electric current into electromagnetic wave, it is classified based on many characteristics including the geometry such as dipole or patch element [1], its radiation pattern, directivity, gain and operational frequency. Measurements of these parameters are compared to the reference antenna which is considered isotropic, the radiated field is the same in all directions, its radiation pattern is represented by sphere. In many scientific and civil applications of antennas, it is beneficial to fix the characteristics of the electromagnetic field such as to construct a directional antenna towards the receiving devices, in this case, an array of antennas [2] is implemented instead of single element.

Using many elements increases the number of parameters to control such as the geometrical repartition of each antenna and the input parameters which are the excitation currents and phases, additionally, the array configuration yields to many possibilities where the elements can be identical or different. From this perspective, many experiments have been conducted to study and test the characteristics of the arrays of sensors, theirs applications are employed in different fields of research such as acoustical communications [3] using microphones, optics using arrays of light emitted diodes and electromagnetic communications using antennas.

In the other hand, we find the applications of arrays of sensors in many scientific and civil fields including telecommunications [4,5] by means of radio signals, underwater acoustics, radio astronomy and geophysics. The engineering of the arrays permits to exploit the interferometric properties of intercepted waves to measure the properties of radiating sources including the bearing estimation [1,2,5], source separation, interferences rejection and also the nature of propagation media such as the number of multipath and the types of reflections.

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The principle of superposition and constructive interferences is used to configure the parameters of radiating field by an array of sensors, indeed, the total field is simply the sum of the contributions from all sensors, which depends of the input parameters and geometry of each sensor. It is a similar case in optical physics where the intensity of diffraction grating is the sum of diffraction patterns of each aperture, the beam width of the central peak gets narrower with increased number of apertures.

In the context of bearing estimation, one of the familiar problems in array processing is the separation of angles of incidences for closely sources, a preliminary solution consists of adding more elements to the array to get narrow beam width and using high resolution beam scan spectra [1, 2, 5]. These procedures are the same for both one dimensional and two dimensional arrays of sensors [1]. For the first case, the objective is to estimation the azimuth angle of incidence of intercepted sources while for the second case with geometries such as rectangular or circular ones, both azimuth and elevation [6] are estimated via two dimensional beam scan.

In this paper, we present a detailed description of spatial ambiguities that one dimensional array of sensors presents in the context of incident angle estimation, precisely the planar and conical types. This study is based on the type of uniform linear array with identical and isotropic elements. We demonstrate that using one dimensional configuration, only half of the plane is available for angular beam scan, either azimuth or elevation angles. Next we numerically study the variation of array factor with non functioning single sensor.

2. Linear Array of Isotropic Sensors

In this section, we describe the relation between the geometry of one dimensional array and the spatial interferences produced by radiating sources, for simplicity, we consider the case of single source. Let an array of N elements intercepts a plane wave front coming from far field source with carrier frequency f_c.Given the relation between the speed of propagation c and the frequency, the wavelength is $\lambda = c / f_c$, the distance between the elements of the array is half the wavelength $d = \lambda / 2$. The array length is $D = (N-1)d$.

Let us study the problem in (x, y) plane, each sensor is considered isotropic, in polar coordinates, for the azimuth angle in the range $\theta \in [0, 2\pi]$, the gain equals $g(\theta) = 1$. Given the normal of the array, the plane wave passing through the array is characterized by the angle of incidence, per example $\theta = 0°$, the wave front is parallel to the array and no phase delay is observed in generated signals, if $\theta = 90°$, the wave font is perpendicular to the array and each delay between consecutive sensors is d / c. The generated signals are digitized using K samples, at instant $t = 1, ..., K$, the signal vector is given by the following equation:

$$x(t) = s(t)a(\theta) + n(t) \tag{1}$$

Where $x(t) \in \mathbb{C}^{N\times1}$ is the signal vector for N channels, $s(t)$ is a scalar that represents the envelope of source at t, $a(\theta) \in \mathbb{C}^{N\times1}$ is called steering vector that contains the information related to the array geometry and angle of incidence of source, if we consider the first sensor as the reference of the phase, the steering vector is written as the following

$$a(\theta) = \left[1, e^{-j2\pi d\lambda^{-1}\sin(\theta)}, ..., e^{-j2\pi d(N-1)\lambda^{-1}\sin(\theta)}\right]^T \tag{2}$$

$n(t) \in \mathbb{C}^{N\times 1}$ is the additive noise vector either generated in the circuit of the antenna such as electronic and thermal noise, or external such as interference signals from secondary sources or as result of diffuse reflection in propagation media.

The properties of one dimensional array, we are focused on, are related to the subject of angle of incidence detection of radiating source, the main relation that plays an important role in this context is the phase expression $e^{-j2\pi d\lambda^{-1}\sin(\theta)}$, we study some characteristics of this expression in the next section.

3. Planar Ambiguity of One Dimensional Uniform Array

Concerning the angular beam scan using high resolution spectra, each geometry of array is characterized by a set of ranges where the results of peak detection algorithms are correct. For one dimensional array we call this range the visible region Ω. If the parameters of the radiating source are outside Ω, the results of beam scan cannot describe the exact position of the planar wave fronts, in this case we consider that an ambiguity exists for such geometrical arrangement of sensors.

In order to determine Ω for one dimensional array, we consider two dimensional problem where the source and the base station are located in the same horizontal plan, let us associate a Cartesian reference (o, x, y, z) to the array such as the position of n^{th} sensor is $\vec{r}_n = -(n-1)d\vec{e}_y$, the radiating source is in the Fraunhofer region with angle of incidence θ relatively to the normal of the array, this configuration is given by Figure 1.

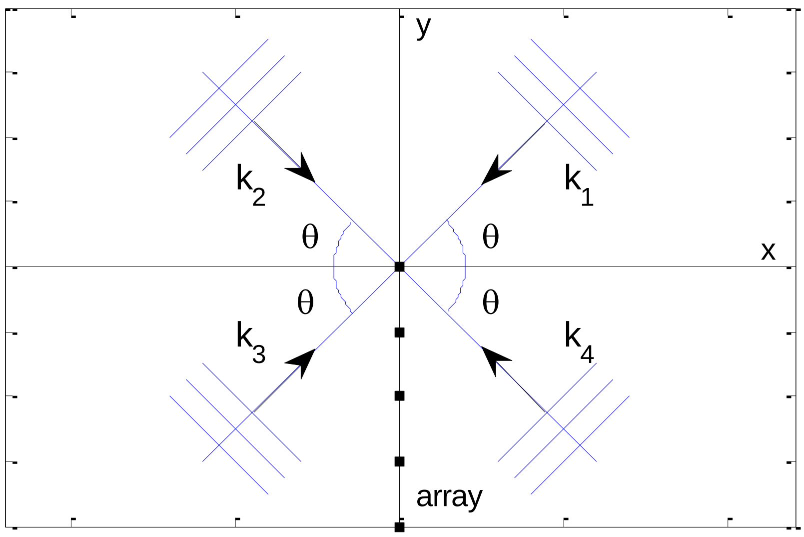


Figure 1. Planar Ambiguity of One Dimensional Array

Given that the plan (x, y) can be divided into four secondary plans, we study the four possibilities of angle θ, therefore we suppose that we have four radiating sources. Following the trigonometric sense, the four wave vectors that we have in Figure 1, are given by the relations:

$$\begin{cases} \vec{k}_1 = -\frac{2\pi}{\lambda}\cos\theta\vec{e}_x - \frac{2\pi}{\lambda}\sin\theta\vec{e}_y \\ \vec{k}_2 = +\frac{2\pi}{\lambda}\cos\theta\vec{e}_x - \frac{2\pi}{\lambda}\sin\theta\vec{e}_y \\ \vec{k}_3 = +\frac{2\pi}{\lambda}\cos\theta\vec{e}_x + \frac{2\pi}{\lambda}\sin\theta\vec{e}_y \\ \vec{k}_4 = -\frac{2\pi}{\lambda}\cos\theta\vec{e}_x + \frac{2\pi}{\lambda}\sin\theta\vec{e}_y \end{cases} \quad (3)$$

As explained in the previous section, angular beam scan is based on phase delay expression $e^{-j\vec{k}.\vec{r}_n}$, applying the phase projection on the above quantities yields to the following result:

$$\begin{cases} \vec{k}_1.\vec{r}_n = +\frac{2\pi}{\lambda}(n-1)d\sin\theta \\ \vec{k}_2.\vec{r}_n = +\frac{2\pi}{\lambda}(n-1)d\sin\theta \\ \vec{k}_3.\vec{r}_n = -\frac{2\pi}{\lambda}(n-1)d\sin\theta \\ \vec{k}_4.\vec{r}_n = -\frac{2\pi}{\lambda}(n-1)d\sin\theta \end{cases} \quad (4)$$

We deduce that we cannot separate the sources $\vec{k}_1$ and $\vec{k}_2$ which is the same for sources $\vec{k}_3$ and $\vec{k}_4$, using two clusters of discernable sources we construct two sets $\{\vec{k}_1,\vec{k}_4\}$ and $\{\vec{k}_2,\vec{k}_3\}$, The visible region of one dimensional uniform array is therefore $\Omega = \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ ($\Omega = \left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ respectively). We can discuss this property using two particular cases, For horizontal propagation where the wave vector is perpendicular the array, we have $\theta = 0°$, the two possibilities of projections are $\vec{k} = \pm\frac{2\pi}{\lambda}\vec{e}_x$, the wave fronts are parallel to the array and the phase expression becomes $e^{-j\vec{k}.\vec{r}_n} = 1$, we cannot get information about the source if it is located in half-plan $(o,-x,y)$ or in $(o,+x,y)$. In the other case of vertical propagation where the wave vector is parallel to the array, the two possibilities are $\vec{k} = \pm\frac{2\pi}{\lambda}\vec{e}_y$ and the phase expression $e^{-j\vec{k}.\vec{r}_n}$ has two values, in this case, there is a possibility to estimate the angle of incidence in half plans $(o,x,-y)$ and $(o,x,+y)$. The property of Ω using uniform linear array is also known as front-back ambiguity [7]. If the objective is to estimate two parameters of source which are azimuth and elevation (θ,φ), we demonstrate in the next section that one dimensional array of sensors is characterized by conical ambiguity.

4. Conical Ambiguity of One Dimensional Uniform Array

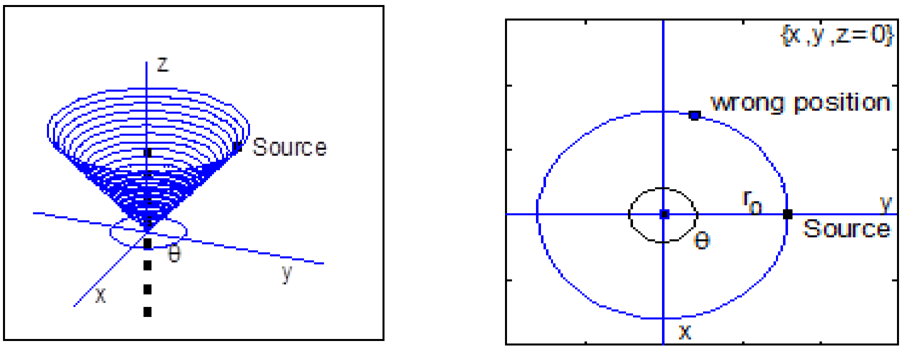
In spherical coordinates, the punctual source is generally described by three parameters (r_0,θ,φ) in ranges $r_0 \in [0,+\infty]$, $\theta \in [0,2\pi]$ and $\varphi \in [0,\pi]$. Let us consider two parameters (θ,φ), in order to make the study of the present case compatible with the previous result, we have to change the coordinates of the sensors and translate the visible region into $\Omega = [0, \pi]$. The sensors must be aligned with z axis of the reference (o, x, y, z), from geometrical viewpoint, this transformation is equivalent to multiplying the coordinates of the sensors by rotation matrix:

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \tag{5}$$

The new coordinates of isotropic sensors are $\vec{r}_n = (n-1)d\vec{e}_z$, we consider a case of single source, the projection of wave vector $\vec{k} = \vec{k}_r + \vec{k}_\theta + \vec{k}_\varphi$ is given by:

$$\vec{k} = -\frac{2\pi}{\lambda} \begin{pmatrix} \sin\varphi\cos\theta \\ \sin\varphi\sin\theta \\ \cos\varphi \end{pmatrix} \tag{6}$$

Where $\|\vec{k}\| = \sqrt{\vec{k}_r^2 + \vec{k}_\theta^2 + \vec{k}_\varphi^2} = 2\pi\lambda^{-1}$. The scalar product between the wave vector and n^{th} sensor is $\vec{k}.\vec{r}_n = -2\pi\lambda^{-1}d(n-1)\cos\varphi$. The only parameter of interest in this expression is elevation angle φ while the azimuth angle θ is ambiguous. Since we have $\theta \in [0, 2\pi]$, the signal received by the array that is coming from source defined by (θ, φ) is the same for any source with same elevation angle φ but with any azimuth angle θ, therefore, the uniform linear array is characterized by conical ambiguity. We present this configuration using the central element of the array as the origin of the phase, as illustrated in Figure 2.



(a) (b)

Figure 2. Conical Ambiguity of Uniform Linear Array for Source $\theta = 90°$ and $0 < \varphi < 90°$ (a) Tridimensional Representation, (b) Projection on (x,y) Plan

The one dimensional uniform array is valid for estimating one degree of freedom of radiating sources, either azimuth θ or elevation φ in half-plane, for two dimensional source characterization, it is mandatory to implement two dimensional geometry of sensors such as circular [6], rectangular or fractal configurations. In the next section, we study another property related to the array's response to the interfering plane waves.

5. Array Factor Variation with Non Uniform Input Current Distribution

In the same context of incident angle characterization, another important parameter that plays a crucial role in spatial interferences treatment is the array response to the incident plane waves, for example if wave field passing through the array is a combination of two sources such as the angular differences between them $\left|\theta_i - \theta_j\right|$ is small and if the number of sensors is relatively small, then the angular beam scan cannot isolate the two sources using the standard beam forming, this effect is explained by the fact that the array is characterized by Rayleigh angular resolution limit which is also called Half Power Beam Width that is a function of operational wavelength and array's length in the case of broadside array [8]:

$$\theta_{HPBW} \approx \frac{\lambda}{D} \tag{7}$$

This metric can be numerically evaluated from the array factor (see Appendix), given the previously cited condition of isotropic sensors, the array factor describes the repartition of the total electric field that is a superposition of each element's contribution, thus, array factor can be shaped by controlling the input parameters of the array which are the excitation current I and electric phase ϕ. In this part we study the property of array factor using non uniform condition of current distribution. Given an array of N isotropic sensors such as the gain is $g_n(\theta) = 1, \forall \theta \in [0, 2\pi]$ and $n = 1, ..., N$, the array factor in far field is given by the following expression [9] :

$$F(\theta) = \sum_{n=1}^{N} I_n e^{j((n-1)kd\cos(\theta)+\phi_n)} \tag{8}$$

Where $k = 2\pi / \lambda$, for simplicity we consider that $\phi_n = 0$. We evaluate $F(\theta)$ with respect to current values I_n. We consider the condition where the output voltage of single antenna element is null, physically the array is uniform with consecutive distance d, this configuration is different of the case of non uniform linear array, the output vector of the array has dimensions $N \times 1$ however the current of single element is $I_i = 0$ A while the remaining $N-1$ elements have $I_j = 1$ A. We study the effect of the position of non-functioning element on $F(\theta)$, therefore we compute N times $F(\theta)$ using input current matrix defined by the relation:

$$I = 1_{N \times N} - I_N \tag{9}$$

Where I_N is the identity matrix. For each non functioning sensor, we compute $F(\theta)$, the result is given in Figure.3, with $N = 15$ elements, $d = \lambda / 2$ and $\phi_n = 0$. We remark the variations of sidelobes of the factors $F(\theta)$ where the null current value is changed from $n = 1$ to 15. For each case, the amplitudes change, however for the main lobe, Rayleigh angular resolution limit is characterized by infinitesimal change, to quantify this last variation, for each case of current state I, we numerically measure the Half Power Beam Width of spectra $F(\theta)$ and compare the results as illustrated in Figure.4.

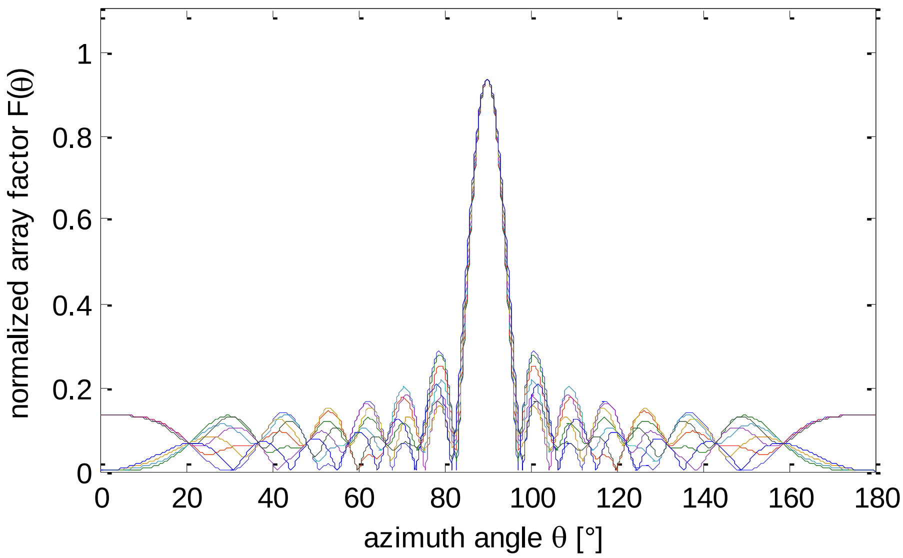


Figure 3. Array Factor $F(\theta)$ w.r.t Current Distribution $I_n = 0$ A, for $n = 1,...,15$

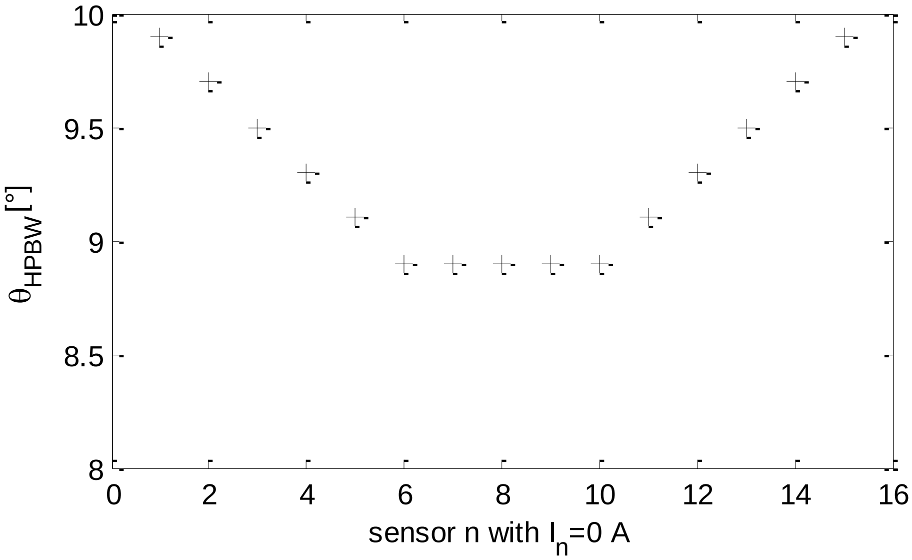


Figure 4. Half Power Beam Width of $F(\theta)$ w.r.t Current Distribution $I_n = 0$ A, for $n = 1,...,15$

According to the above result, the half power beam width gets narrower if the non functioning element $I_n = 0$, is in the central region of the array, for this case of $N = 15$, the difference between smallest and largest beam widths is approximately $\Delta\theta \approx 1°$.

6. Conclusion

In this paper, we have presented a detailed description of planar and conical ambiguities of one dimensional array of isotropic sensors. We have demonstrated the visible region of the array using wave vector analysis, in the context of incident angle estimation of far field radiating sources where the wave front is planar. Next, we have evaluated the effect of non uniform input current distribution of antennas on Rayleigh angular resolution limit, where the array is uniform and broadside, this particular case was evaluated numerically.

Appendix

The half power or half maximum beam width is evaluated numerically in the fifth section for the array factor $F(\theta)$. In this part we present the implemented algorithm.

Algorithm for Half Power Beam Width computation of $F(\theta)$.

```
+Input parameters: F[M], θ[M].
+Initialization:
  double Ma,Mi, dθ, s, L.
  double  Θ[].
  Integer V,ctr ⟵1.
+Parameters computation :
dθ  ⟵ (θ[M]- θ[1])/M.
Ma ⟵ Max{f}.
Mi ⟵ Min{f}.
s    ⟵ (Ma+Mi)/2.
+Procedure:
For i ⟵1 to M
      If f[i]>s then Θ[ctr]  ⟵ θ[i].
     ctr ⟵ ctr+1.
    End if.
End
+Width estimation:
V ⟵length(Θ).
L ⟵ Θ[V]- Θ[1]+dθ.
```

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