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Compromise Ratio Method of MADM Problem Based on a New Interval-valued Intuitionistic Fuzzy Entropy

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Abstract

This article points out the shortcomings of the existing entropy measures, and puts forward a new improved interval-valued intuitionistic fuzzy (IVIF) entropy measure. The new entropy measure not only considers the absolute value of the deviation between membership and non-membership degrees, but also considers the effect of hesitancy degree of IVIF sets. For the two cases with attribute weights information unknown and partially known, new weighting methods are put forward by using the extended entropy method and by establishing the minimum entropy optimization model to solve the optimal weights. Further a compromise ratio method of intuitionistic fuzzy multi-attribute decision making (MADM) problems is put forward, and application examples proved the effectiveness and feasibility of the proposed methods.

Keywords: *Interval-valued intuitionistic fuzzy set; fuzzy entropy; multi-attribute decision making; compromise ratio method*

1. Introduction

With the complexity of the social economic environment and the incomplete understanding of human beings, there is a lot of ambiguity in real problems of management decision-making. In order to deal with the fuzzy phenomenon effectively, Zadeh [1] firstly proposed the concept of fuzzy set in 1965, and defined the corresponding algorithm. Since then, fuzzy sets have been widely applied in many fields such as automatic control, pattern recognition, medical diagnosis, management decision-making and so on. However, Zadeh's fuzzy sets only containing one membership which can only express two aspects of information, but it cannot be expressed the neutral state of neither support nor opposition. There are often many subjective and objective factors, such as the limited time, energy and the complete cognition of the objective things, which influence the decision maker’s judgement. In many situations, there often exist different hesitancy degree or show a certain degree of lack of knowledge. To this end, professor Atanassov defined the concept of intuitionistic fuzzy (IF) sets [1], by adding a non-membership degree, it can describe the three kinds of state of support, opposition and neutrality, can more comprehensive and delicate to describe the fuzzy nature of objective things. IF sets can well describe the decision makers to the hesitation and uncertainty of judgment through the addition of a non-membership parameters. And they can describe the fuzzy characters of things comprehensively, and thus are a powerful and effective tool in expressing uncertain or fuzzy information in actual applications.

Due to the complexity and uncertainty of objective things and the limitation of the knowledge of decision maker, the degree of membership and non-membership is sometimes very difficult to express with crisp numbers, and the interval numbers can be very good. For this reason, Atanassov and Gargov expanded the intuitionistic fuzzy sets,

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and propose the concept of IVIF sets [2]. By interval numbers depicting the membership and non-membership degree, IVIF sets are more attractive than IF sets and can easily be quantified and execrated by decision-makers. IVIF sets are already widely applied in management decision problems [3-5].

In information theory, entropy is a measure of the average uncertainty. Zadeh [6] first introduced the concept of entropy into the fuzzy events in 1968, and then developed and gave the definition of fuzzy entropy. Luca De and Termini [7] was firstly constructed the concept of fuzzy entropy. Fuzzy entropy is an important information measure, which is applied in many fields such as MADM, image segmentation and pattern recognition [8-10]. Intuitionistic fuzzy sets have to be considered in two aspects [11]: the degree of uncertainty and the unknown degree, and the degree of uncertainty is represented by the degree of hesitation. Since the IVIF sets are a natural generalization of IF sets, the construction study of interval valued intuitionistic fuzzy entropy needs to consider the uncertainty and the unknown degree.

By analyzing the existing interval valued intuitionistic fuzzy entropy measure, we find that some scholars have proposed the influence of the fuzzy entropy formula of the intuitionistic fuzzy entropy on the interval valued intuitionistic fuzzy uncertainty. In this section, we will develop a new IVIF entropy measure which not only considers the absolute value of the deviation between membership and non-membership degrees, but also considers the effect of hesitancy degree of IVIF sets. For the two cases with attribute weights information unknown and partially known, new weighting methods are put forward by using the extended entropy method and by establishing the minimum entropy optimization model to solve the optimal weights. Further a compromise ratio method of intuitionistic fuzzy MADM problems is developed.

The organization of this paper is as follows: In Section 2, we recall some concepts and notations of IVIF sets. Section 3 firstly analyses the shortcoming of existing entropy measures and then puts forward a new IVIF entropy measure. Section 4 develops a new MADM methods based on the proposed IVIF entropy and the concept of compromise ratio method. Section 5 illustrates the effectiveness and feasibility of the proposed method. Finally, the conclusions are provided in Section 6.

2. Preliminary Knowledge

In what follows, some basic concepts of IF sets and IVIF sets are introduced to facilitate the discussions. Atanassov proposed the concept of IF sets in 1986, which is defined as in Definition 1.

Definition 1 [2] Let $X=\{x_1,x_2,\cdots,x_n\}$ be a finite universe of discourse, then

$$U=\{<x_j,\mu_U(x_j),\upsilon_U(x_j)>|\,x_j\in X\} \tag{1}$$

is called an IF set, which assigns to each element x_j a membership degree $\mu_U(x_j)$ and a nonmembership degree $\upsilon_U(x_j)$.

Here $\mu_U(x_j)\in[0,1]$, $\nu_U(x_j)\in[0,1]$ and $\pi_U(x_j)=1-\mu_U(x_j)-\upsilon_U(x_j)$ is called the hesitation degree or intuitionistic index of an element x_j to U. It is the degree of indeterminacy membership of the element x_j to the IF set U. It is a good tool to reflect the uncertain information, thus it can also help the decision maker to descript the fuzzy information. Obviously, $0\le\pi_U(x_j)\le 1$ for every $x_j\in X$. And if $\pi_U(x_j)=0$, then the IF set U is reduced to a fuzzy set, *i.e.*, $U=\{<\mu_U(x_j),1-\mu_U(x_j)>|\,x_j\in X\}$.

In some situation, it is very difficult to use crisp numbers to express $\mu_U(x_j)$ and $\upsilon_U(x_j)$ precisely for the complexity and uncertainties of the objective things. But we can use intervals to express them. So, Atanassov and Gargov extended the IF set to the IVIF set in 1989, and gave the definition of the IVIFS as in Definition 2.

Definition 2 [2] Let $X=\{x_1,x_2,\cdots,x_n\}$ be a finite universe of discourse, then

$$\tilde{U}=\{<x_j,\tilde{\mu}_{\tilde{U}}(x_j),\tilde{\upsilon}_{\tilde{U}}(x_j)>|\, x_j\in X\} \tag{2}$$

is called an IVIF set, where $\tilde{\mu}_{\tilde{U}}(x_j)$ and $\tilde{\upsilon}_{\tilde{U}}(x_j)$ are intervals, where $\tilde{\mu}_{\tilde{U}}(x_j)=[\mu^-_{\tilde{U}}(x_j),\mu^+_{\tilde{U}}(x_j)]$ and $\tilde{\upsilon}_{\tilde{U}}(x_j)=[\upsilon^-_{\tilde{U}}(x_j),\upsilon^+_{\tilde{U}}(x_j)]$. The element $<x_j,\tilde{\mu}_{\tilde{U}}(x_j),\tilde{\upsilon}_{\tilde{U}}(x_j)>$ is called an IVIF number [12]. The hesitation degree of an IVIF number $(\tilde{\mu}_{\tilde{U}}(x_j),\tilde{\upsilon}_{\tilde{U}}(x_j))$ can be defined as $\tilde{\pi}_{\tilde{U}}(x_j)=[\pi^-_{\tilde{U}}(x_j),\pi^+_{\tilde{U}}(x_j)]$, where $\pi^-_{\tilde{U}}(x_j)=1-\mu^+_{\tilde{U}}(x_j)-\upsilon^+_{\tilde{U}}(x_j)$ and $\pi^+_{\tilde{U}}(x_j)=1-\mu^-_{\tilde{U}}(x_j)-\upsilon^-_{\tilde{U}}(x_j)$, for all $x_j\in X$.

We briefly denote an IVIF number $<\tilde{\mu}_{\tilde{U}}(x_j),\tilde{\upsilon}_{\tilde{U}}(x_j)>$ by $\tilde{A}=<\tilde{\mu}_{\tilde{A}},\tilde{\upsilon}_{\tilde{A}}>$ or $\tilde{A}=<\tilde{\mu}_{\tilde{A}},\tilde{\upsilon}_{\tilde{A}},\tilde{\pi}_{\tilde{A}}>$, where

$$\tilde{\mu}_{\tilde{A}}=[\mu^-_{\tilde{A}},\mu^+_{\tilde{A}}]\subset[0,1],\tilde{\upsilon}_{\tilde{A}}=[\upsilon^-_{\tilde{A}},\upsilon^+_{\tilde{A}}]\subset[0,1],\mu^+_{\tilde{A}}+\tilde{\upsilon}^+_{\tilde{A}}\le 1 \tag{3}$$

$$\tilde{\pi}_{\tilde{A}}=[\pi^-_{\tilde{A}},\pi^+_{\tilde{A}}]\subset[0,1],\pi^-_{\tilde{A}}=1-\mu^+_{\tilde{A}}-\tilde{\upsilon}^+_{\tilde{A}},\pi^+_{\tilde{A}}=1-\mu^-_{\tilde{A}}-\tilde{\upsilon}^-_{\tilde{A}} \tag{4}$$

Definition 3 [2] Let $\tilde{A}_i=<\tilde{\mu}_{\tilde{A}_i},\tilde{\upsilon}_{\tilde{A}_i}>$ $(i=1,2)$ be two any IVIF number, then

(1) If $\mu^-_{\tilde{A}_1}\le\mu^-_{\tilde{A}_2},\mu^+_{\tilde{A}_1}\le\mu^+_{\tilde{A}_2}$ and $\upsilon^-_{\tilde{A}_1}\ge\upsilon^-_{\tilde{A}_2},\upsilon^+_{\tilde{A}_1}\ge\upsilon^+_{\tilde{A}_2}$, then $\tilde{A}_1$ is no larger than $\tilde{A}_2$, and noted by $\tilde{A}_1\le\tilde{A}_2$;

(2) If $\tilde{A}_1\le\tilde{A}_2$ and $\tilde{A}_1\ge\tilde{A}_2$, then $\tilde{A}_1$ is equal to $\tilde{A}_2$, and noted by $\tilde{A}_1=\tilde{A}_2$.

By Definition 3, $\tilde{A}^*=<[1,1],[0,0]>$ is the largest IVIF number; $\tilde{A}^-=<[0,0],[1,1]>$ is the smallest IVIF number.

Definition 4. Let $\tilde{A}=\{<x_i,\tilde{\mu}_{\tilde{A}}(x_i),\tilde{\upsilon}_{\tilde{A}}(x_i)>|\,x_i\in X\}$ and $\tilde{B}=\{<x_i,\tilde{\mu}_{\tilde{B}}(x_i),\tilde{\upsilon}_{\tilde{B}}(x_i)>|\,x_i\in X\}$ be two IVIF sets, then the following operations can be founded in [2]:

(3) The complementary set of $\tilde{A}$ denoted by $\tilde{A}^C$, is $\tilde{A}^C=\{<x_i,\tilde{\upsilon}_{\tilde{A}}(x_i),\tilde{\mu}_{\tilde{A}}(x_i)>|\,x_i\in X\}$;

(4) $\tilde{A}\preceq\tilde{B}$ called $\tilde{A}$ less fuzzy than $\tilde{B}$, *i.e.*, for $\forall x_i\in X$,

If $\tilde{\mu}_{\tilde{B}}(x_i)\le\tilde{\upsilon}_{\tilde{B}}(x_i)$, then $\tilde{\mu}_{\tilde{A}}(x_i)\le\tilde{\mu}_{\tilde{B}}(x_i),\tilde{\upsilon}_{\tilde{A}}(x_i)\ge\tilde{\upsilon}_{\tilde{B}}(x_i)$;

If $\tilde{\mu}_{\tilde{B}}(x_i)\ge\tilde{\upsilon}_{\tilde{B}}(x_i)$, then $\tilde{\mu}_{\tilde{A}}(x_i)\ge\tilde{\mu}_{\tilde{B}}(x_i),\tilde{\upsilon}_{\tilde{A}}(x_i)\le\tilde{\upsilon}_{\tilde{B}}(x_i)$.

Definition 5. Let $\tilde{A}=\{<x_i,\tilde{\mu}_{\tilde{A}}(x_i),\tilde{\upsilon}_{\tilde{A}}(x_i)>|\,x_i\in X\}$ and $\tilde{B}=\{<x_i,\tilde{\mu}_{\tilde{B}}(x_i),\tilde{\upsilon}_{\tilde{B}}(x_i)>|\,x_i\in X\}$ be two IVIF sets and the weight of x_i is w_i. Then the weighted Hamming distance of $\tilde{A}$ and $\tilde{B}$ is defined as[12]:

$$\begin{aligned} d(\tilde{A},\tilde{B})=\frac{1}{4n}\sum_{j=1}^{n}w_j\Big[&\left|\mu^-_A(x_j)-\mu^-_B(x_j)\right|+\left|\mu^+_A(x_j)-\mu^+_B(x_j)\right|+\left|\upsilon^-_A(x_j)-\upsilon^-_B(x_j)\right| \\ &+\left|\upsilon^+_A(x_j)-\upsilon^+_B(x_j)\right|+\left|\pi^-_A(x_j)-\pi^-_B(x_j)\right|+\left|\pi^+_A(x_j)-\pi^+_B(x_j)\right|\Big] \end{aligned} \tag{5}$$

In the following discussion, we always suppose that $\mathrm{IVIFS}s(Z)$ is the set of all IVIF sets defined in X.

Definition 6 Let $\tilde{A}=\{<x_i,\tilde{\mu}_{\tilde{A}}(x_i),\tilde{\upsilon}_{\tilde{A}}(x_i)>|x_i\in X\}$ and $\tilde{B}=\{<x_i,\tilde{\mu}_{\tilde{B}}(x_i),\tilde{\upsilon}_{\tilde{B}}(x_i)>|x_i\in X\}$ be two IVIF sets . A map $E:\mathrm{IVIFS}s(Z)\to[0,1]$ is called the IVIF entropy, if it satisfies the following properties [13]:

(i) $E(\tilde{A})=0$ if and only if $\tilde{A}$ is a crisp set;

(ii) $E(\tilde{A})=1$ if and only if $\tilde{\mu}_{\tilde{A}}(x_i)=\tilde{\upsilon}_{\tilde{A}}(x_i),\forall x_i\in X$;

(iii) $E(\tilde{A})=E(\tilde{A}^C)$;

(iv) If $\tilde{A}\preceq\tilde{B}$, then $E(\tilde{A})\le E(\tilde{B})$.

3. A New Effective IVIF Entropy

First we review several already existing IVIF entropy measures in reference. By analyzing the existing IVIF entropy measure, we find that some existing IVIF entropy measures without considering with the influence of the hestiation, such as Vlachos' IVIF entropy [14]:

$$E_1(A)=1-\sqrt{\frac{1}{2n}\sum_{i=1}^{n}(|\mu_A^-(x_i)-v_A^-(x_i)|^2+|\mu_A^+(x_i)-v_A^+(x_i)|^2)} \tag{6}$$

and Ye's IVIF entropy [15]:

$$E_2(\tilde{A})=\left\{\sin\frac{\pi\times[1+\mu_{\tilde{A}}^-(x_i)+p\Delta\mu_{\tilde{A}}(x_i)-\upsilon_{\tilde{A}}^-(x_i)-p\Delta\upsilon_{\tilde{A}}(x_i)]}{4}\right.$$
$$\left.+\sin\frac{\pi\times[1-\mu_{\tilde{A}}^-(x_i)-p\Delta\mu_{\tilde{A}}(x_i)+\upsilon_{\tilde{A}}^-(x_i)+p\Delta\upsilon_{\tilde{A}}(x_i)]}{4}-1\right\}\times\frac{1}{\sqrt{2}-1} \tag{7}$$

where $\Delta\mu_{\tilde{A}}(x_i)=\mu_{\tilde{A}}^+(x_i)-\mu_{\tilde{A}}^-(x_i)$, $\Delta v_{\tilde{A}}(x_i)=\upsilon_{\tilde{A}}^+(x_i)-\upsilon_{\tilde{A}}^-(x_i)$.

Example 1. Let $\tilde{A}$ =<[0.4, 0.5], [0.3, 0.4]>, $\tilde{B}$ = <[0.2, 0.3],[0.1, 0.2]>, $\tilde{C}$ =<[0.6, 0.6], [0.2, 0.2]> and $\tilde{D}$ = <[0.7, 0.7],[0.3, 0.3]> be four IVIF numbers. Intuitively, we can see that $\tilde{B}$ is more fuzzy than $\tilde{A}$, and $\tilde{C}$ is more fuzzy than $\tilde{D}$. But $E_1(\tilde{A})=E_1(\tilde{B})=0.9000$ and $E_2(\tilde{C})=E_2(\tilde{D})=0.8329$, which are not consistent with our intuition.

Another scholars considered the influence of hesitation degree in the construction ofIVIF entropy, such as Wei's IVIF entropy [13]:

$$E_3(\tilde{A})=\frac{1}{n}\sum_{i=1}^{n}\frac{2-|\mu_A^-(x_i)-\upsilon_A^-(x_i)|-|\mu_A^+(x_i)-\upsilon_A^+(x_i)|+\pi_A^-(x_i)+\pi_A^+(x_i)}{2+|\mu_A^-(x_i)-\upsilon_A^-(x_i)|+|\mu_A^+(x_i)-\upsilon_A^+(x_i)|+\pi_A^-(x_i)+\pi_A^+(x_i)}$$

and Jin et al's IVIF entropy [16] :

$$E_4(\tilde{A})=\frac{1}{n}\sum_{i=1}^{n}\frac{\min\{F_Q(\tilde{\mu}_{\tilde{A}}(x_i)),F_Q(\tilde{\upsilon}_{\tilde{A}}(x_i))\}+\pi_{F_Q(x_i)}}{\max\{F_Q(\tilde{\mu}_{\tilde{A}}(x_i)),F_Q(\tilde{\upsilon}_{\tilde{A}}(x_i))\}+\pi_{F_Q(x_i)}} \tag{8}$$

where $\pi_{F_Q(x_i)}=1-F_Q(\tilde{\mu}_{\tilde{A}}(x_i))-F_Q(\tilde{\upsilon}_{\tilde{A}}(x_i))\}$, $F_Q(\tilde{a})$ is the Yager's COWA operator of interval number $\tilde{a}=[a^-,a^+]$, which has the following form:

$$F_Q(\tilde{a})=\int_0^1\frac{dQ(y)}{dy}[a^+-y(a^+-a^-)]dy. \tag{9}$$

If $\lambda=\int_o^1 Q(y)dy$, then $F_Q([a^-,a^+])=\lambda a^++(1-\lambda)a^-$.

Example 2. Let us consider two candidates $\tilde{A}$ and $\tilde{B}$, the support ratio of $\tilde{A}=<[0.2,0.2],[0.067,0.067]>$ is 40%, while the support ratio $\tilde{B}=<[0.4,0.4],[0.2,0.2]>$ is 60%, then we will select $\tilde{B}$ is as the better candidate than $\tilde{A}$. This voting result is expressed that $\tilde{A}$ is more fuzzy than B. We note that, if $\lambda=0.5$, then $E_4(\tilde{A})=E_3(\tilde{A})$. In this case, we can get the result $E_4(\tilde{A})=E_4(\tilde{B})$, which is not consistent with the true situation.

The main task of this section is to put forward an effective IVIF entropy, which not only considers the absolute value of the deviation between membership and non-membership degrees, but also considers the effect of hesitancy degree of IVIF sets.

Let $A=\{<x_i,[\mu_{AL}(x_i),\mu_{AU}(x_i)],[\upsilon_{AL}(x_i),\upsilon_{AU}(x_i)]>|x_i\in X,i=1,2,...,n\}$ be an IVIF set, we define a new information measure of A as follows:

$$E(A)=\frac{1}{n}\sum_{i=1}^{n}\cos\frac{|\mu_{AL}^2(x_i)-\upsilon_{AL}^2(x_i)|+|\mu_{AU}^2(x_i)-\upsilon_{AU}^2(x_i)|}{4}\pi \tag{10}$$

Equation (8) can also be rewritten as

$$E(A)=\frac{1}{n}\sum_{i=1}^{n}\cos\frac{|(\mu_{AL}(x_i)-\upsilon_{AL}(x_i))(1-\pi_{AU}(x_i))|+|(\mu_{AU}(x_i)-\upsilon_{AU}(x_i))(1-\pi_{AL}(x_i))|}{4}\pi \tag{11}$$

From Equation (9), we can see that $E(A)$ not only considers the deviation between membership with nonmembership degrees(i.e. $\mu_{AL}(x_i)-\upsilon_{AL}(x_i)$, $\mu_{AU}(x_i)-\upsilon_{AU}(x_i)$), but also considers the hesitancy degree of the IVIF set(i.e. $\pi_{AL}(x_i)$, $\pi_{AU}(x_i)$).

Theorem 1. The measure given by Equation (9) is an IVIF entropy.

Proof. To prove the measure given by Equation (9) is an IVIF entropy, we only need to prove it satisfies the properties in definition 6. Obviously, for every x_i, we have: $0\le E(A)\le 1$.

(i) Let A be a crisp set, *i.e.,* for $\forall x_i\in X$, we have $\tilde{\mu}_A(x_i)=[0,0],\tilde{\upsilon}_A(x_i)=[1,1]$ or $\tilde{\mu}_A(x_i)=[1,1],\tilde{\upsilon}_A(x_i)=[0,0]$. It is obvious that $E(A)=0$.

If $E(A)=0$, *i.e.,* $E(A)=\frac{1}{n}\sum_{i=1}^{n}\cos\frac{|\mu_{AL}^2(x_i)-\upsilon_{AL}^2(x_i)|+|\mu_{AU}^2(x_i)-\upsilon_{AU}^2(x_i)|}{4}\pi=0$, then $\forall x_i\in X$, we have

$$\cos\frac{|\mu_{AL}^2(x_i)-\upsilon_{AL}^2(x_i)|+|\mu_{AU}^2(x_i)-\upsilon_{AU}^2(x_i)|}{4}\pi=0,$$

thus $|\mu_{AL}^2(x_i)-\upsilon_{AL}^2(x_i)|=1$, $|\mu_{AU}^2(x_i)-\upsilon_{AU}^2(x_i)|=1$, then we have $\tilde{\mu}_A(x_i)=[0,0],\tilde{\upsilon}_A(x_i)=[1,1]$ or $\tilde{\mu}_A(x_i)=[1,1],\tilde{\upsilon}_A(x_i)=[0,0]$, Therefore A is a crisp set.

(ii) Let $\tilde{\mu}_{\tilde{A}}(x_i)=\tilde{\upsilon}_{\tilde{A}}(x_i)$, $\forall x_i\in X$, from Equation (9), we have $E(A)=1$.

Now we assume that $E(A)=1$, then for all $x_i\in X$, we have:

$$\cos\frac{|\mu_{AL}^2(x_i)-\upsilon_{AL}^2(x_i)|+|\mu_{AU}^2(x_i)-\upsilon_{AU}^2(x_i)|}{4}\pi=1,$$

then $\frac{|\mu_{AL}^2(x_i)-\upsilon_{AL}^2(x_i)|+|\mu_{AU}^2(x_i)-\upsilon_{AU}^2(x_i)|}{4}=0$, we can obtain the conclusion $\mu_{AL}(x_i)=\upsilon_{AL}(x_i)$, $\mu_{AU}(x_i)=\upsilon_{AU}(x_i)$ for all $x_i\in X$.

(iii) By $\tilde{A}^C = \{< x_i, \tilde{\upsilon}_{\tilde{A}}(x_i), \tilde{\mu}_{\tilde{A}}(x_i) > | x_i \in X\}$ and Equation (9), we have:

$$E(\tilde{A}^C) = \frac{1}{n}\sum_{i=1}^{n} \cos\frac{|\upsilon_{AL}^2(x_i) - \mu_{AL}^2(x_i)| + |\upsilon_{AU}^2(x_i) - \mu_{AU}^2(x_i)|}{4}\pi$$
$$= \frac{1}{n}\sum_{i=1}^{n} \cos\frac{|\mu_{AL}^2(x_i) - \upsilon_{AL}^2(x_i)| + |\mu_{AU}^2(x_i) - \upsilon_{AU}^2(x_i)|}{4}\pi .$$
$$= E(A)$$

(iv) If $\tilde{A} \preceq \tilde{B}$, then for $\forall x_i \in X$,

For the one case $\tilde{\mu}_{\tilde{B}}(x_i) \le \tilde{\upsilon}_{\tilde{B}}(x_i)$, $\tilde{\mu}_{\tilde{A}}(x_i) \le \tilde{\mu}_{\tilde{B}}(x_i), \tilde{\upsilon}_{\tilde{A}}(x_i) \ge \tilde{\upsilon}_{\tilde{B}}(x_i)$, that is

$$\mu_{BL}(x_i) \le \upsilon_{BL}(x_i), \mu_{BU}(x_i) \le \upsilon_{BU}(x_i),\ \mu_{AL}(x_i) \le \mu_{BL}(x_i), \mu_{AU}(x_i) \le \mu_{BU}(x_i),$$
$$\tilde{\mu}_{\tilde{A}}(x_i) \ge \tilde{\mu}_{\tilde{B}}(x_i), \tilde{\upsilon}_{\tilde{A}}(x_i) \le \tilde{\upsilon}_{\tilde{B}}(x_i).$$

Then

$$\mu_{AL}^2(x_i) - \upsilon_{AL}^2(x_i) \le \mu_{BL}^2(x_i) - \upsilon_{BL}^2(x_i) \le 0$$
$$\mu_{AU}^2(x_i) - \upsilon_{AU}^2(x_i) \le \mu_{BU}^2(x_i) - \upsilon_{BU}^2(x_i) \le 0$$

Thus we have

$$|\mu_{AL}^2(x_i) - \upsilon_{AL}^2(x_i)| \ge |\mu_{BL}^2(x_i) - \upsilon_{BL}^2(x_i)|$$
$$|\mu_{AU}^2(x_i) - \upsilon_{AU}^2(x_i)| \ge |\mu_{BU}^2(x_i) - \upsilon_{BU}^2(x_i)|$$

and

$$|\mu_{AL}^2(x_i) - \upsilon_{AL}^2(x_i)| + |\mu_{AU}^2(x_i) - \upsilon_{AU}^2(x_i)| \ge |\mu_{BL}^2(x_i) - \upsilon_{BL}^2(x_i)| + |\mu_{BU}^2(x_i) - \upsilon_{BU}^2(x_i)|$$

So, we can get

$$\cos\frac{|\mu_{AL}^2(x_i) - \upsilon_{AL}^2(x_i)| + |\mu_{AU}^2(x_i) - \upsilon_{AU}^2(x_i)|}{4}\pi$$
$$\le \cos\frac{|\mu_{BL}^2(x_i) - \upsilon_{BL}^2(x_i)| + |\mu_{BU}^2(x_i) - \upsilon_{BU}^2(x_i)|}{4}\pi$$

That is $E(A) \le E(B)$.

Simarily, we can prove that for the case of $\tilde{\mu}_{\tilde{B}}(z_i) \ge \tilde{\upsilon}_{\tilde{B}}(z_i)$, $\tilde{\mu}_{\tilde{A}}(z_i) \ge \tilde{\mu}_{\tilde{B}}(z_i)$ and $\tilde{\upsilon}_{\tilde{A}}(z_i) \le \tilde{\upsilon}_{\tilde{B}}(z_i)$, the result $E(\tilde{A}) \le E(\tilde{B})$ can also be proved.

Then we get the conclusion: If $\tilde{A} \preceq \tilde{B}$, then $E(\tilde{A}) \le E(\tilde{B})$.

4. A New MADM Method based on Proposed IVIF Entropy

Suppose that there exists an alternative set $A = \{A_1, A_2, \cdots, A_m\}$ consisting of n non-inferior alternatives from which the most preferred alternative is to be selected. A MADM method is to find the best alternative from a set of m alternatives with respect to a set $O = \{o_1, o_2, \cdots, o_n\}$ of n attributes. Suppose that the ratings of alternatives A_i on attributes o_j are expressed with the IVIF number $\tilde{a}_{ij} =< [\mu_{ijL}, \mu_{ijU}], [\upsilon_{ijL}, \upsilon_{ijU}] >$, where $[\mu_{ijL}, \mu_{ijU}]$ and $[\upsilon_{ijL}, \upsilon_{ijU}]$ are intervals, which express the membership (satisfactory) and nonmembership (nonsatisfactory) degree of the alternative A_i on the attribute o_j with respect to the fuzzy concept "excellence" given by the decision maker so that they satisfy the conditions: $\mu_{ijL}, \mu_{ijU}, \upsilon_{ijL}, \upsilon_{ijU} \in [0,1]$ and $0 \le \mu_{ijU} + \upsilon_{ijU} \le 1$ ($i = 1, 2, \cdots, m$; $j = 1, 2, \cdots, n$).

Thus, a MADM problem can be expressed with the following decision matrix:

$$D=(\tilde{a}_{ij})_{m\times n}=\begin{matrix} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix}\begin{matrix} o_1 & o_2 & \cdots & o_n \\ \left(\begin{matrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{matrix}\right) \end{matrix}$$

In real decision situations, evaluated attributes may have different importance for the decision process. Let $\boldsymbol{W}=(w_1,w_2,...,w_n)^T$ be the weight vector of all attributes, where $0\le w_j\le 1$ ($j=1,2,\cdots,n$) are weights of attributes $o_j\in O$, and satisfy $\sum_{j=1}^{n} w_j=1$. The information of attribute weight is usually completely unknown or partially known due to the insufficient knowledge or limitation of time of decision makers. Therefore, the determination of attribute weights is an important issue in MADM problems. Then in this section, we will develop two methods to determine the weights of attributes for the above-mentioned two cases, respectively.

4.1. Weight Determining Method with Unknown Attribute Weights Information

When the attribute weights are completely unknown, we can use the proposed IVIF entropy to determine the attribute weights based on the concept of entropy weighting method. The calculate method is given as follows:

$$w_j=\frac{1-e_j}{n-\sum_{j=1}^{n} e_j},\quad j=1,2,...,n \tag{12}$$

where $e_j=\frac{1}{m}\sum_{i=1}^{m} E(\tilde{a}_{ij})$ ($j=1,2,\cdots,n$), and $E(\tilde{a}_{ij})=\cos\frac{|\mu_{ijL}^2-\upsilon_{ijL}^2|+|\mu_{ijU}^2-\upsilon_{ijU}^2|}{4}\pi$ is the IVIF entropy of $\tilde{a}_{ij}=<[\mu_{ijL},\mu_{ijU}],[\upsilon_{ijL},\upsilon_{ijU}]>$.

4.2. Weight Determining Method with Partial Attribute Weight Information

Generally, there will have more constraint conditions for the weight vector $\boldsymbol{W}=(w_1,w_2,...,w_n)^T$. We denote $\boldsymbol{H}$ as the set of the known weight information. To determine the attribute weights for MADM problem with attribute weights partially known under IVIF environment, Liu and Ren [17] determined the attribute weights by establishing a programming model according to the minimum entropy principle. In this paper, we will use the new IVIF entropy measure to determine the attribute weights and the method is similarly with Liu and Ren [17]. The specific process is given as follows.

To rank the alternatives according to the decision matrix $D=(\tilde{a}_{ij})_{m\times n}$, we propose a method to obtain the attribute weight vector by means of the proposed IVIF entropy measure. Entropy measure describes the degree of the fuzziness and intuitionism. The smaller the intuitionistic fuzzy entropy, the smaller of the fuzzy degree of attribute evaluation information, thus the more decision-making certainty information will be. Hence, we can utilize the principle of minimum entropy value to get the weight vector of attribute by computing the following programming:

$$\min E(A_i)=\sum_{j=1}^{n} w_j E(\tilde{a}_{ij})=\sum_{j=1}^{n} w_j \cos\frac{|\mu_{ijL}^2-\upsilon_{ijL}^2|+|\mu_{ijU}^2-\upsilon_{ijU}^2|}{4}\pi$$
$$s.t.\quad \sum_{j=1}^{n} w_j=1 \qquad (13)$$
$$\boldsymbol{W}\in H$$

Because each alternative is fair competition, the weight coefficient with respect to the same attribute should be also equal, thus we get the following optimization model:

$$\min E=\sum_{i=1}^{m} E(A_i)=\sum_{i=1}^{m}\sum_{j=1}^{n} w_j \cos\frac{(\mu_{ijL}^2-\upsilon_{ijL}^2)^2+(\mu_{ijU}^2-\upsilon_{ijU}^2)^2}{4}\pi$$
$$s.t.\quad \sum_{j=1}^{n} w_j=1 \qquad (14)$$
$$\boldsymbol{W}\in H$$

Hence, by solving the Equation (14), the optimal solution $\boldsymbol{W}^*=\arg\min E$ is chosen as the optimal attribute weights.

4.3. The New MADM method based on the Proposed IVIF Entropy

In this subsection, we put forward the new MADM method based on the above-mentioned work and the concept of compromise ratio method. The specific calculation steps are given as follows:

Step 1. Calculate the attribute weights according to Section 4.1 and Section 4.2;

Step 2. Determine the positive ideal solution (PIS) and negative ideal solution (NIS) of the intuitionistic fuzzy MADM problem.

The PIS is defined as: $A^*=(\tilde{a}_1^*,\tilde{a}_2^*,\cdots,\tilde{a}_n^*)$,where $\tilde{a}_j^*=<[1,1],[0,0]>$ ($j=1,2,\cdots,n$).

The NIS is defined as: $A^-=(\tilde{a}_1^-,\tilde{a}_2^-,\cdots,\tilde{a}_n^-)$, where $\tilde{a}_j^-=<[0,0],[1,1]>$ ($j=1,2,\cdots,n$).

Step 3. According to the weighted Hamming distance measure in Definition 3, the distance measures between alternative A_i with PIS and NIS are calculated respectively as follows:

$$\begin{aligned}d(A_i,A^*)&=\frac{1}{4}\sum_{j=1}^{n} w_j(|1-\mu_{ijL}|+|1-\mu_{ijU}|+|\upsilon_{ijL}-0|+|\upsilon_{ijU}-0|+|\pi_{ijL}-0|+|\pi_{ijU}-0|)\\&=\frac{1}{4}\sum_{j=1}^{n} w_j(|1-\mu_{ijL}|+|1-\mu_{ijU}|+|\upsilon_{ijL}|+|\upsilon_{ijU}|+|1-\mu_{ijU}-\upsilon_{ijU}|+|1-\mu_{ijL}-\upsilon_{ijL}|)\\&=\frac{1}{2}\sum_{j=1}^{n} w_j(2-\mu_{ijL}-\mu_{ijU})\end{aligned}$$

$$\begin{aligned}d(A_i,A^-)&=\frac{1}{4}\sum_{j=1}^{n} w_j(|\mu_{ijL}|+|\mu_{ijU}|+|1-\upsilon_{ijL}|+|1-\upsilon_{ijU}|+|\pi_{ijL}-0|+|\pi_{ijU}-0|)\\&=\frac{1}{4}\sum_{j=1}^{n} w_j(|\mu_{ijL}|+|\mu_{ijU}|+|1-\upsilon_{ijL}|+|1-\upsilon_{ijU}|+|1-\mu_{ijU}-\upsilon_{ijU}|+|1-\mu_{ijL}-\upsilon_{ijL}|)\\&=\frac{1}{2}\sum_{j=1}^{n} w_j(2-\upsilon_{ijL}-\upsilon_{ijU})\end{aligned}$$

Step 4. Calculate the compromise ratio value of each alternative.

The compromise ratio value ξ_i represents the distances between the PIS and NIS simultaneously, and furthermore it can represent the subjective attitude of the decision maker by parameter ε. The compromise ratio value of each alternative is calculated as [18]:

$$\xi_i = \varepsilon \frac{d_-(A^*) - d_i(A_i, A^*)}{d_-(A^*) - d_+(A^*)} + (1-\varepsilon)\frac{d_i(A_i, A^-) - d_-(A^*)}{d_+(A^-) - d_-(A^-)} \quad (i = 1, 2, ..., m) \tag{15}$$

Where $d_-(A^*) = \max_{1 \le i \le m}\{d(A_i, A^*)\}$, $d_+(A^*) = \min_{1 \le i \le m}\{d(A_i, A^*)\}$, $d_-(A^-) = \min_{1 \le i \le m}\{d(A_i, A^-)\}$, $d_+(A^-) = \max_{1 \le i \le m}\{d(A_i, A^-)\}$ and $\varepsilon \in [0,1]$ is the attitude factor, which represents the decision maker's subjective attitude with respect to the relative distances of alternative with PIS and NIS, and generally $\varepsilon = 0.5$.

Step 5. Rank the alternatives according to the compromise ratio value (ξ_i) in decreasing order. The larger the value of ξ_i represents the better of the corresponding alternative A_i.

5. Numerical Examples

In order to illustrate the application of the proposed MADM method, two examples are given as follows:

Example 1. (This is the case of the attribute weights are complete unknown)

With the expansion of enterprise scale, enterprises need to introduce ERP management system to enhance competitiveness under the environment of economic globalization and fierce business competition,

ERP mainly lies in the integration of external resources within the enterprise, in a series of business process transformation, to improve the enterprise system, enhance the competitive ability and its content includes procurement, financial management, manufacturing, sales, research and development, human resources power module, and tts function can be further extended outward and supply chain management, customer relationship management, the combination of business intelligence. Thus ERP management system is a set of perfect function and the import process with highly complex, high risk, and import expensive suit software [8]. Based on the above characteristics, if enterprises want to conduct the software development, they must invest the massive human, physical strength, time and money *etc.*, which are completely not in line with the economic benefit. Therefore based on the consideration of economic benefit, enterprises will take with rich experience for guiding software package suppliers, and buy the current ERP software package, to save system development costs and shorten the time to implement, fast on-line operation, to grasp business opportunities.

The expansion of enterprise scale urges the enterprise to implement ERP management. After a preliminary investigation and screening, the enterprise finally determine five candidate ERP softwares A_i $(i = 1, 2, 3, 4, 5)$ to be choosen. Evaluated attributes ares ystem cost (o_1), functional satisfaction (o_2), system stability (o_3), software credibility (o_4) and service level (o_5). After the expert investigation and statistical analysis, the evaluation results of these 5 ERP softwares (alternatives) are obtained and expressed by IVIF numbers. Evaluated values are shown in Table 1.

Table 1. Interval-Valued Intuitionistic Fuzzy Decision Matrix

ERP Software	Attributes				
	o_1	o_2	o_3	o_4	o_5
A_1	<[0.4,0.5], [0.4,0.5]>	<[0.5,0.6], [0.1,0.2]>	<0.6,0.7], [0.2,0.3]>	<[0.7,0.8], [0.1,0.2]>	<0.7,0.8], [0.0,0.2]>
A_2	<[0.6,0.8], [0.1,0.2]>	<[0.5,0.6], [0.3,0.4]>	<0.4,0.5], [0.3,0.4]>	<[0.4,0.6], [0.3,0.4]>	<[0.4,0.7], [0.1,0.3]>
A_3	<[0.5,0.6], [0.3,0.4]>	<[0.5,0.7], [0.1,0.2]>	<[0.5,0.6], [0.3,0.4]>	<[0.3,0.4], [0.2,0.5]>	<[0.6,0.7], [0.2,0.3]>
A_4	<[0.5,0.6], [0.3,0.4]>	<[0.7,0.8], [0.0,0.1]>	<[0.4,0.5], [0.2,0.4]>	<[0.5,0.7], [0.1,0.2]>	<[0.5,0.7], [0.2,0.3]>
A_5	<[0.4,0.7], [0.2,0.3]>	<[0.5,0.6], [0.2,0.4]>	<[0.3,0.6], [0.3,0.4]>	<[0.6,0.8], [0.1,0.2]>	<[0.4,0.5], [0.2,0.3]>

In order to determine the best ERP management software, this paper uses the proposed method for sorting and selection of the 5 ERP management softwares.

Step 1. According to the Section 4.1, the attribute weights vector is obtained as:

$$\boldsymbol{W} = \left(0.1498, 0.2426, 0.0803, 0.2720, 0.2554\right)^T$$

Step 2.The PIS (A^*) and NIS (A^-) are respectively given as:

$$A^* = (\tilde{a}_1^*, \tilde{a}_2^*, ..., \tilde{a}_5^*) = (<[1,1],[0,0]>, <[1,1],[0,0]>, ..., <[1,1],[0,0]>)$$

$$A^- = (\tilde{a}_1^-, \tilde{a}_2^-, ..., \tilde{a}_5^-) = (<[0,0],[1,1]>, <[0,0],[1,1]>, ..., <[0,0],[1,1]>)$$

Step 3. The distance measures of each alternative from PIS and NIS are calculated as:

$$d(A_1, A^*) = 0.3515, d(A_2, A^*) = 0.4492, d(A_3, A^*) = 0.4667,$$

$$d(A_4, A^*) = 0.3831, d(A_5, A^*) = 0.4428$$

$$d(A_1, A^-) = 0.8098, d(A_2, A^-) = 0.7183, d(A_3, A^-) = 0.7241,$$

$$d(A_4, A^-) = 0.8067, d(A_5, A^-) = 0.7570$$

Step 4. Set $\varepsilon = 0.5$, the compromise ratio values are calculated as:

$$\xi_1 = 1.0000, \xi_2 = 0.0762, \xi_3 = 0.0316, \xi_4 = 0.8457, \xi_5 = 0.3157.$$

Therefore, the ranking order of all alternatives is $A_1 \succ A_4 \succ A_5 \succ A_2 \succ A_3$, and A_1 is the desirable alternative.

Example 2. (This is the case of the attribute weights are partially known.) The example is adopted from Wei [19], which considers a company's selection of outstanding management personnel.

Suppose a company wants to choose a good management personnel from 5 candidates A_i $(i = 1, 2, ..., 5)$. Through discussion of company managers, the final determined evaluation attributes are professional skills (o_1), interpersonal skills (o_2), rational skills (o_3) and design skills (o_4). Assuming that the evaluation information of the potential managers is given by the IVIF numbers, the corresponding IVIF decision matrix $D = (\tilde{a}_{ij})_{m \times n}$ is shown in Table 2.

Table 2. Interval-Valued Intuitionistic Fuzzy Decision Matrix

Candidates	Attributes			
	o_1	o_2	o_3	o_4
A_1	<[0.4,0.5], [0.3,0.4]>	<[0.4,0.6], [0.2,0.4]>	<[0.3,0.4], [0.4,0.5]>	<[0.5,0.6], [0.1,0.3]>
A_2	<[0.5,0.6], [0.2,0.3]>	<[0.6,0.7], [0.2,0.3]>	<[0.5,0.6], [0.3,0.4]>	<[0.4,0.7], [0.1,0.2]>
A_3	<[0.3,0.5], [0.3,0.4]>	<[0.1,0.3], [0.5,0.6]>	<[0.2,0.5], [0.4,0.5]>	<[0.2,0.3], [0.4,0.6]>
A_4	<[0.2,0.5], [0.3,0.4]>	<[0.4,0.7], [0.1,0.2]>	<[0.4,0.5], [0.3,0.5]>	<[0.5,0.8], [0.1,0.2]>
A_5	<[0.3,0.4], [0.1,0.3]>	<[0.7,0.8], [0.1,0.2]>	<[0.5,0.6], [0.2,0.4]>	<[0.6,0.7], [0.1,0.2]>

Assume the attribute weights are partially known, and the weights satisfies the set $H=\{0.13\le w_1\le 0.15, 0.35\le w_2\le 0.40, 0.15\le w_3\le 0.20, 0.30\le w_4\le 0.35, w_3\le w_4\}$.

Then the calculation steps of the proposed decision making method are given as follows:

Step 1. According to the Equation (19), we can establish the following programming model:

$$\min E = 4.9064w_1 + 4.2861w_2 + 4.8951w_3 + 4.3749w_4$$

$$s.t.\quad \begin{cases} 0.13\le w_1\le 0.15 \\ 0.35\le w_2\le 0.40 \\ 0.15\le w_3\le 0.20 \\ 0.30\le w_4\le 0.35 \\ w_3\le w_4 \\ w_1+w_2+w_3+w_4+w_5=1 \end{cases}$$

We use MATLAB software to solve this model, and get the optimum attribute weight vector:

$$\boldsymbol{W}=\left(0.13, 0.40, 0.15, 0.32\right)^T.$$

Step 2. The PIS (A^*) and NIS (A^-) are respectively given as:

$$A^*=(\tilde{a}_1^*,\tilde{a}_2^*,\tilde{a}_3^*,\tilde{a}_4^*,\tilde{a}_5^*)=(<[1,1],[0,0]>,<[1,1],[0,0]>,...,<[1,1],[0,0]>)$$

$$A^-=(\tilde{a}_1^-,\tilde{a}_2^-,\tilde{a}_3^-,\tilde{a}_4^-,\tilde{a}_5^-)=(<[0,0],[1,1]>,<[0,0],[1,1]>,...,<[0,0],[1,1]>)$$

Step 3. The weighted Hamming distance measures of each alternative from PIS and NIS are calculated as

$$d(A_1,A^*)=0.5130, d(A_2,A^*)=0.4100, d(A_3,A^*)=0.7355,$$

$$d(A_4,A^*)=0.4590, d(A_5,A^*)=0.3640$$

$$d(A_1,A^-)=0.7030, d(A_2,A^-)=0.7670, d(A_3,A^-)=0.5070,$$

$$d(A_4,A^-)=0.7865, d(A_5,A^-)=0.8210$$

Step 4. Set $\varepsilon=0.5$,the compromise ratio values are calculated as:

$$\xi_1=0.6116, \xi_2=0.8266, \xi_3=0, \xi_4=0.8172, \xi_5=1.0000$$

Step 5. Based on C_i values, the ranking of the alternatives in descending order are $A_5\succ A_2\succ A_4\succ A_1\succ A_3$, and A_5 is the best desirable candidate. This result is in agreement with the result in Wei [19]. However, the method presented in this paper can reflect the subjective attitude of the decision maker, and it is more consistent with the objective reality.

9. Conclusion

In order to measure the fuzzy degree of IVIF sets, a new fuzzy entropy is proposed. It includes not only the absolute value of the deviation between membership and non-membership degrees, but also the uncertainty of the intuitionistic fuzzy sets. Thus the new IVIF can better describe the degree of uncertainty and the degree of uncertainty of the IVIF sets. On this basis, the MADM problems of two kinds of situations, which the weights information of attribute are completely unknown and partially known, are sloved by developing a new MADM method on the basis of concept of compromise ratio method.

The advatage of the new proposed MADM method are descriped as follows:

(1) Reference [20] pointed out that TOPSIS method exists some shortcomings, such as reverse problem, comprehensive evaluation value reflects only the evaluation schemes in the relative proximity, which does not reflect and ideal of optimal solution is really close to the degree. To overcome these drawbacks, Li [18] proposed a compromise ratio method for fuzzy MADM problems. In this paper, we propose a compromise ratio method for MADM problems under IVIF environment based on IVIF entropy. The method can not only reflect the alternatives is close to the ideal point away from the negative ideal point, and through introducing attitude factor which is introduced as weight of the decision making strategy of "closeness to the positive ideal solution" and more in line with the actual situation.

(2) For the two cases with attribute weights information unknown and partially known, new weighting methods are put forward by using the extended entropy method and by establishing the minimum entropy optimization model to solve the optimal weights. Two examples are used and shown the proposed method is feasible and effective.

(3) The entropy measure can be applied to image processing, pattern recognition and medical diagnosis and so on, the proposed multi attribute decision making method can be applied to such as venture investment project selection, the site selection of multi attribute decision making problem, has good theoretical value and practical value.

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