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Effective Mathematical Simulation for One-Dimensional Cutting Stock Problem using Heuristics Approach

P. L. Powar[1], Ravi Vishwakarma[2] and Chandrashekhar Meshram[3]

[1, 2, 3] Department of Mathematics and Computer Science,
R.D. University, Jabalpur, M.P., India,
[1]pvjrdvv@rediffmail.com, [2]ravirdvv@gmail.com, [3]cs_meshram@rediffmail.com

Abstract

Considering the cutting stock problem and lot sizing problem for tower transmission industry, we present the new effective mathematical simulation for One-Dimensional Cutting Stock Problem (1D-CSP) by applying heuristic approach. This approach works within pre-defined trim except at the end of cutting process. The pre-defined trim is called the sustainable trim which is based on average order lengths and given stock lengths. By considering the constraints like space, manpower etc., the main objective of this study is to minimize the trim loss. Powar et al. (One-Dimensional Cutting Stock Problem (1D-CSP) with First Order Sustainable Trim: A Practical Approach, International Journal of Computer Science Engineering and Information Technology Research (IJCSEITR), 3(3), 227-240, 2013) have been already designed a mathematical model in which cutting of at most two order lengths is considered in the cutting plan in such a way that no third order length can be cut before the cutting of two order lengths under consideration. In this paper, we have modified this model and relaxed the condition of exclusive cutting of only two order lengths. When out of two, only one order length is left to cut, we may club the third order length for cutting process. In order to justify our modification, we have discussed some numerical examples.

***Keywords**: Heuristics, 1D-CSP, Sustainable trim, simulation, mathematical modeling.*

1. Introduction

The cutting stock problem occurs mostly in production and planning of many industries like tower transmission, paper, steel, aluminium, readymade garments, woods and iron *etc*. (cf. [2], [3], [5], [8], [21], [26], [29]). When large items (stock lengths) are being cut to get small items (order lengths), the following two main problems arise (cf. [19]):

- To select proper number of pieces of stock lengths.
- Minimization of trim loss (scrap).

The combination of above two problems is called the Cutting Stock Problem (CSP).

The Cutting Stock Problem (CSP) was introduced as an integer linear programming problem (cf. [3]) initially, but many possibilities of cutting patterns made the problem complex. For reducing the complexity of patterns, many researchers have designed several algorithms (cf. [1], [5], [8], [17], [21], [26], [29]). This problem was initiated by Kantarovich [20] in 1960 later Gilmore and Gomory ([12], [13]) developed the approach to solve the problem by using the column generation approach. In this sequence of development, many researchers (Dyckoff [10], Stadtler [24], Haessler and Vonderembse [18], Dikili and Barlas [9], Gradisar *et al*. [15], Cui [6], Cui *et al*. [7], Vanderbeck [27], Wäscher *et al*. [28]) entered into

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this field and contributed different formulations and approaches to solve this problem.

A method for solving (1D-CSP) with usable leftover (CSPUL) has been presented by Gradisar *et al*. [16] for the cases where the ratio between the average stock and average order length is less than 3. Trkman *et al*. [25] described minimization of trim loss problem and costs incurred for handling the leftover after the completion of the cutting of required number of order lengths.

Mobashar and Ekici [22] have given new formulation for CSP by considering additional issue like setup cost and developed two local search algorithms and heuristic algorithm based on column generation. Using CSP, the routing problem for vehicle involving time constraint and lot size problem which is designed to act in a particular way considering setup time, Gondzio *et al*. [14] presented computational experiments and discussed the column generation technique for primal–dual.

Garraffa *et al*. [11] studied a new generation of the (1D-CSP) by using the cut losses dependent criterion on the cutting of items sequentially. Cui and Tang [7] have presented a value correction approach for One Dimensional Multiple Stock Size Cutting Stock Problem (1DMSSCSP) following the sequential steps. The algorithm designed for generating pattern set for 1DMSSCSP has been developed by Cui *et al*. (cf. [4]).

Recently, the working space for the industries has been squeezed remarkably in metropolitan cities as the industrial development is growing very fast globally. In almost all industries where the cutting of smaller lengths from the longer stock lengths of different dimensions is desired, maintenance of stock with different lengths is a major problem. Sorting of sufficiently large number of order lengths (approximately more than one thousand) after each stage of cutting and keeping them in the form of heaps till the entire cutting process is over, is another complex job carried over manually and is obviously space consuming.

Considering these issues, Power *et al*. [23] have designed a mathematical model for 1D-CSP and developed the algorithm for the heuristic approach which resolves these issues up-to some extent. As considered in [23], we also propose to cut not more than two order lengths at one stage of cutting in our heuristic method with a different approach; it takes care of space constraint and sorting problem. The cutting process adopted in [23] has been designed in such a way that at one step of cutting only two order lengths are cut and the process continues till the total required number of pieces of the selected order lengths are cut.

In the present paper, we have introduced a modification in designing the cutting plan, we have also considered the cutting of two order lengths at one stage of cutting but in case when during the cutting process one type of order length is left to cut, we shift it to the next set of remaining data instead of cutting alone. It is very interesting to note that this modification in the cutting plan results minimum trim loss up-to large extent, which is the major focus of 1D-CSP. Extracting the data from KEC International, we have explored the comparative study of these two approaches of cutting and concluded that this new method brings down the trim loss from 0.05888% to 0.03105% for the Data Set-1 and from 0.2128% to 0.0065% for Data Set-2 which is quite significant.

The rest of the contents of the article is organized as: the next section 2 covers pre-requisites; section-3 contains the modeling of the problem; section-4 proposes Heuristic approach; section-5 describes Numeric computation and the last section-6 concludes the article.

2. Pre-requisites

In this section, we introduce the notations and define the sustainable trim for given set of data.

2.1. Notations

The lengths are considered as integers throughout the paper, if the lengths are not integer values then all lengths can be converted into integers by multiplying them with 10^n, for suitable positive integer n. The notations are:

$\mathcal{B}(k)$- Block of integers 1, 2, 3, …, k,
o_i- Order lengths, i= 1, 2, 3, …, n,
p_i- Number of pieces of required order lengths o_i,
s_j- Stock lengths, j= 1, 2, 3, …, m.

2.2. Sustainable Trim by Using Average (cf. [23])

Considering order lengths with required number of pieces and stock lengths, we define the average of order lengths as follows:

$$\mathcal{L} = \frac{o_1 p_1 + \cdots + o_n p_n}{p_1 + \cdots + p_n} = \frac{\sum_{i=1}^{n} o_i p_i}{\sum_{j=1}^{n} p_j}$$

Next, we define

$\mathcal{L}_k = |s_k - i\mathcal{L}|$

($k = 1, 2, \ldots, m$ and i is an appropriate positive integer ≥ 1, for which $\mathcal{L}_k$ is minimum)

where $s_1, s_2, \ldots, s_m$ are the stock lengths. We finally define

$$t_s = \frac{\sum_{k=1}^{m} \mathcal{L}_k}{m} \tag{1}$$

which is the desired **sustainable trim**.

3. Mathematical Simulation of the Problem

In this section, we refer the mathematical model designed in [23] and modify it. The modified version has been discussed in Mathematical Model (b).

3.2. Mathematical Model (a) by Power et al. [23]

Considering the order lengths $o_1, o_2, \ldots, o_n$ with corresponding required pieces $p_1, p_2, \ldots, p_n$ respectively, we define for $i \neq j,\ i, j \in \mathcal{B}(n)$,

$$p_i = n_1 \alpha_{i1} + p_{i1}, \qquad p_j = n_1 \alpha_{j1} + p_{j1} \tag{2}$$

Consider the combination of o_i and o_j as,

$$y_{ij}^1 = \alpha_{i1} o_i + \alpha_{j1} o_j$$

by considering the appropriate values of n_1 in such a way that the y_{ij}^1 satisfy the following condition:

$$0 \leq s_r - y_{ij}^1 = w_r(\text{say}) \leq t_s$$

for some value of r ($r = 1, 2, \ldots, m$). The value of n_1 is chosen in such a way that w_r admits a minimum value of $(s_r - y_{ij}^1)$ lying within the stipulated range 0 to t_s.

Applying the same technique select a number n_2 for p_{i1} and p_{j1} satisfying the following condition:

$$p_{i1} = n_2 \alpha_{i2} + p_{i2}, \qquad p_{j1} = n_2 \alpha_{j2} + p_{j2} \tag{3}$$

$y_{ij}^2 = \alpha_{i2}\sigma_i + \alpha_{j2}\sigma_j$

$0 \le s_\omega - y_{ij}^2 = w_\omega(\text{say}) \le t_s$

for some value of ω ($\omega = 1,2,\dots,m$). The value of n_2 is chosen in such a way that w_ω admits a minimum value of $(s_\omega - y_{ij}^2)$ lying within the range 0 to t_s.

Proceeding this way, we finally define

$$p_{i,z-1} = n_z\alpha_{iz} + p_{iz}, \quad p_{j,z-1} = n_z\alpha_{jz} + p_{jz} \tag{4}$$

This process will continue till either $p_{iz} = 0$ or $p_{jz} = 0$ and in view of (2) - (4), we get the recurrence relations for p_i and p_j as follows:

$$p_i = \sum_{k=1}^{z} n_k\alpha_{ik} + p_{iz}, \ \ \alpha_{ik} > \alpha_{i,k+1} \tag{5}$$

$$p_j = \sum_{k=1}^{z} n_k\alpha_{jk} + p_{jz}, \ \ \alpha_{jk} > \alpha_{j,k+1} \tag{6}$$

$$p_{\mu z} = n_{z+1}\alpha_{\mu,z+1} + p_{\mu,z+1}, \quad (p_{\mu,z+1} < \delta_{\mu,z+1}) \tag{7}$$

where $\mu = i$ or j. In order to minimize the wastage, the positive integers $n_k, \alpha_{ik}, \alpha_{jk}$ may be chosen in accordance with the stock length.

Using the relations (5)-(7), we now define the combinations:

$$y_{ij}^r = \alpha_{ir}\sigma_i + \alpha_{jr}\sigma_j$$

$$y_\mu^{z+1} = \alpha_{\mu,z+1}\sigma_\mu, \tag{8}$$

$$y_\mu^{z+2} = p_{\mu,z+1}\sigma_\mu, \tag{9}$$

where $\mu = i$ or j and $r = 1,2,\dots,z$.

We define the sets of the combinations as follows:

$$\mathcal{Y} = \{y_{ij}^r, y_\mu^{z+1}, y_\mu^{z+2} : y_{ij}^r, y_\mu^{z+1}, y_\mu^{z+2} \le s_m\} \tag{10}$$

and $$\mathcal{Y}_k^r = \{y_{ij}^r, y_\mu^{z+1}, y_\mu^{z+2} : 0 < [s_k - (y_{ij}^r / y_\mu^{z+1} / y_\mu^{z+2})] < t_s \tag{11}$$

$$r = 1,2,\dots,z.$$

3.2. Proposed Mathematical Model (b)

Referring relations (5) and (6), let if possible $p_{iz} = 0$ and $p_{jz} \ne 0 \Rightarrow p_j > p_i$. Denote $p_j - p_i = p_j'$ which is the number of pieces of order length σ_j is left to cut. It is clear from Mathematical Model (a) (cf. relations (8) and (9)) that the cutting of single order length σ_j is continued till the p_j' is exhausted.

Instead of cutting a single order length σ_j, in our rectified version, we now consider σ_j with p_j' along with the remaining set of data $\sigma_1, \sigma_2, \dots, \sigma_{j-1}, \sigma_{j+1}, \dots, \sigma_{i-1}, \sigma_{i+1}, \dots, \sigma_n$ with required number of pieces $p_1, p_2, \dots, p_{j-1}, p_{j+1}, \dots, p_{i-1}, p_{i+1}, \dots, p_n$. Hence, our new data is

$\sigma_1, \sigma_2, \dots, \sigma_{j-1}, \sigma_j, \sigma_{j+1}, \dots, \sigma_{i-1}, \sigma_{i+1}, \dots, \sigma_n$ → Order lengths

$p_1, p_2, \dots, p_{j-1}, \boldsymbol{p_j'}, p_{j+1}, \dots, p_{i-1}, p_{i+1}, \dots, p_n$ → Required number of pieces of above respective order lengths

Applying the similar technique as described in Mathematical Model (a) up-to the step (6), we continue the process until a single order length may be left to cut. In this case, we follow the procedure described in (8) and (9) of Mathematical Model (a).

4. Heuristic Approach for Cutting

In this section, we have discussed cutting plans for mathematical model (a) and mathematical model (b). We present the algorithms for:

- The computation of sustainable trim,
- Mathematical model (a),
- Modified version of cutting plan for Mathematical Model (b).

4.1. Cutting Plan for Mathematical Model (a)

It has been noticed practically that with the preference of starting from the largest order length to the smaller one, the cutting process has been executed in general as the smaller order lengths left behind can be combined easily amongst them such that one can get the minimum trim loss.

Cutting of σ_n:

Setting $r = 1$, we first consider $y^1_{nj} \in \mathcal{Y}^1_k$ for fixed n, and arbitrary j and, choose y^1_{nj} as:

$$|y_n| = \max_{j \in \mathcal{B}(n)_{\sim n}} y^1_{nj}$$

such that $|y_n| \leq s_j$ for some $j \in \mathcal{B}(m)$.

Now, there are sets $\mathcal{Y}^1_\eta, \mathcal{Y}^1_\zeta, \mathcal{Y}^1_\xi, \ldots$ containing $|y_n|$ which are associated with $s_\eta, s_\zeta, s_\xi, \ldots$ and we choose the set $\mathcal{Y}^1_\eta$ which corresponds to the smallest stock length s_η. Referring (11), we get

$$y^1_{nj} = \alpha_{n1}\sigma_n + \alpha_{j1}\sigma_j \quad \text{for} \quad j \in \mathcal{B}(n)_{\sim n}.$$

Referring relation (2), it may be noted that $n_1.\alpha_{n1}$ pieces of σ_n and $n_1.\alpha_{j1}$ pieces of σ_j may be considered to cut by cutting n_1 bars of available stock s_η. The main objective of our cutting process is to finish first the cutting of σ_n and σ_j. During this cutting procedure the following cases may occur:

Case 1. Either (a) $p_n - n_1.\alpha_{n1} < \alpha_{n1}$
or (b) $p_j - n_1\alpha_{j1} < \alpha_{j1}$
or (c) (a) and (b) may hold together.

Referring relation (11), consider

$$y^2_{nj} = \alpha_{n2}\sigma_n + \alpha_{j2}\sigma_j \quad \text{(fixed } j\text{)}$$

By the definition of $\mathcal{Y}^r_k$, corresponding to each $\mathcal{Y}^2_\eta, \mathcal{Y}^2_\zeta, \mathcal{Y}^2_\xi, \ldots$, the stock lengths $s_\eta, s_\zeta, s_\xi, \ldots$ respectively are being considered and $y^2_{nj} \in \mathcal{Y}^2_\eta$ or $\mathcal{Y}^2_\zeta$ or $\mathcal{Y}^2_\xi$ or … . We select $\mathcal{Y}^2_\zeta$(say) corresponding to the smallest stock length s_ζ. Referring relation (3), it may be noted that $n_2.\alpha_{n2}$ more pieces of σ_n and $n_2.\alpha_{j2}$ more pieces of σ_j have been cut from n_2 bars of stock length s_ζ. The process will continue till either p_n or p_j is completely exhausted *i.e.* either p_n or p_j is of the form

$$p_n = \sum_{k=1}^{z} n_k\alpha_{ik} \text{ or } p_j = \sum_{k=1}^{z} n_k\alpha_{jk}.$$

Let if possible $p_n = \sum_{k=1}^{z} n_k\alpha_{ik}$ holds, then p_j would be of the form

$$p_j = \sum_{k=1}^{z} n_k\alpha_{jk} + p_{jz} \tag{12}$$

Relation (12) shows that p_{jz} more pieces of order length σ_j are left to cut. We now express p_{jz} as

$$p_{jz} = n_{z+1}\alpha_{j,z+1} + p_{j,z+1}\ (p_{j,z+1} < \alpha_{j,z+1}).$$

Referring the relation (8), we consider

$$y^{z+1}_j = \alpha_{j,z+1}\sigma_j$$

Now, $y^{z+1}_j \in \mathcal{Y}^{z+1}_\eta$ or $\mathcal{Y}^{z+1}_\zeta$ or $\mathcal{Y}^{z+1}_\xi$ or … and corresponding to each $\mathcal{Y}^{z+1}_\eta, \mathcal{Y}^{z+1}_\zeta, \mathcal{Y}^{z+1}_\xi, \ldots$ the stock lengths $s_\eta, s_\zeta, s_\xi, \ldots$ respectively are being considered. Now, we choose y_{ξ}^{z+1} which corresponds the smallest stock length s_{ξ}. Referring (5) - (7), it may be noted that $p_{j,z+1}$ pieces of σ_j are remain to cut out of p_j. Considering $p_{j,z+1}$ in place of p_{jz}, the process may be continued till $(s_k - p_{j,z+\theta}\sigma_j)$ for some k and θ is positive (k and θ are positive integers). We next choose the stock length s_t such that $s_t - p_{j,z+\theta}\sigma_j$, is minimum. In view of aforesaid, cutting of the order length σ_j is completed.

Remark: At the last stage of cutting process, if $p_{j,z+\theta}$ pieces of order length σ_j are left to cut but the difference $s_t - p_{j,z+\theta}\sigma_j$ exceeds the bound t_s of sustainable trim, we are compel to cut it.

Case 2. Either (a) $p_n = n_1.\alpha_{n1}$ or (b) $p_j = n_1\alpha_{j1}$ or (c) both (a) and (b) hold together. When (c) holds then cutting of order lengths σ_n and σ_j is exhausted by cutting n_1 bars of the stock length s_η.

Considering order lengths σ_n and σ_j, we are now in a position to calculate the trim loss t'_{r_1}:

$$t'_{r_1} = \left(s_\eta - y_{nj}^1\right)n_1 + \left(s_\zeta - y_{nj}^2\right)n_2 + \cdots$$
$$= \sum_{r=1}^{z}\left(s_\ell - y_{nj}^r\right)n_r + (s_t - p_{j,z+1}\sigma_j) \quad ; \quad \ell = \eta, \zeta, \ldots.$$

Thus, the cutting of σ_n and σ_j is totally exhausted. The rest of order lengths are now arranged in ascending order $\sigma_1, \sigma_2, \ldots, \sigma_{\mathcal{N}}$ (say) with respect to their lengths. We first consider

$$|y_{\mathcal{N}}| = \max_{j \in \mathcal{B}(\mathcal{N})_{\sim\mathcal{N}}} y_{\mathcal{N}j}^1 \quad , j \in \mathcal{B}(\mathcal{N})_{\sim\mathcal{N}}$$

such that $|y_{\mathcal{N}}| \le s_j$ for some $j \in \mathcal{B}(m)$.

Proceeding in a similar manner, we get

$$t'_{r_2} = \sum_{r=1}^{z}\left(s_\ell - y_{nj}^r\right)n_r + (s_t - p_{j,z+1}\sigma_j) \quad ; \quad \ell = \eta', \zeta', \ldots.$$

The process is continued till the cutting of last order length is over. We now define

$$\mathcal{T} = t'_{r_1} + t'_{r_2} + \cdots$$

With reference to sustainable trim t_s, the percentages of total trim lose may be computed easily by counting the total length of stock used for cutting process.

Algorithm for sustainable trim

```
Insert n order lengths σ_i's with required number of pieces p_i's
  Arrange σ_i's in ascending order w.r.t. their respective lengths
    Insert m stock lengths s_k's
      Arrange s_k's in ascending order w.r.t. their respective lengths
         Let L = Σ_{i=1}^{n}(σ_i * p_i)/ Σ_{i=1}^{n} p_i
           For k = 1 to m
        Int i = 1, L_k2 = 0
      do
         { i++;
            D_ki = s_k − i * L
              If D_ki < 0
                 L_ki = (−1) * D_ki
              Else
                 L_ki = D_ki
```


15 End if (Line 11)
16 }
17 while $(L_{ki} \geq L_{k,i+1})$
18 Let $L_k = L_{ki}$
19 Let $t_s = (L_1 + L_2 + \cdots + L_m)/m$
20 End For (Line 6)

Algorithm for cutting plan

1 Insert order lengths σ_i's with required number of pieces p_i's
2 Arrange σ_i's in ascending order w.r.t. their respective lengths
3 Insert stock lengths s_k's
4 Arrange s_k's in ascending order w.r.t. their respective lengths
5 Consider sustainable trim t_s
6 For i=n to 1
7 For $j = n - 1$ to 1
8 If $i \neq j$
9 Let $b = \min\{p_i, p_j\}$
10 Else goto Line 7
11 For n_1 = 1 to b
12 For α_{1i} = 1 to p_i
13 For α_{1j} = 1 to p_j
14 Let $y_{ij}^1 = \alpha_{i1} * \sigma_i + \alpha_{j1} * \sigma_j$
15 For k = 1 to m
16 If $y_{ij}^1 \leq s_m$
17 Else goto line 11
18 End If (Line 16)
19 If $0 \leq s_k - y_{ij}^1 \leq t_s$
20 $w_k^{n_1} = \min_{\alpha_{1i},\alpha_{1j}}\{s_k - y_{ij}^1\}$
21 Calculate $p_{i1} = p_i - n_1 * \alpha_{i1}$, $p_{j1} = p_j - n_1 * \alpha_{j1}$
22 Else goto line 11
23 End If (Line 19)
24 End For (Line 13)
25 End For (Line 12)
26 Calculate $w_k^j = \min_{n_1}\{w_k^{n_1}\}$
27 Let $w_k = n_1 * w_k^j$
28 End For (Line 11)
29 Recall Line 8 to Line 28
30 Continue p_i = p_{i1} and $p_j = p_{j1}$
31 Break If (p_i = 0 or $p_j = 0$) or (p_i = $p_j = 0$)
32 If p_i = 0 or $p_j = 0$
33 Let $p_\mu = p_i$ or p_j
34 Continue p_i = 0 and $p_j = p_\mu$
35 Recall Line 8 to Line 30
36 Else goto line 6 (by excluding old p_i and p_j)
37 Break If (conditions on Line 16 or Line 19 not hold)
38 Let $p_j = p_\theta$
39 For $l = 1$ to p_θ
40 $y_l = l * \sigma_j$
41 For k = 1 to m

42 Calculate $t_{kl} = s_k - y_l$
43 Let $t_k^l = \min_k\{t_{kl}\}$
44 End For (Line 41)
45 Calculate $t_k = \min_l\{t_k^l\}$
46 Let $p_j = p_j - l$
47 Break if $p_j = 0$
48 End For (Line 39)
49 End For (Line 7)
50 End For (Line 6)

4.2. Cutting Plan for Proposed Mathematical Model (b)

Under this plan of cutting, we follow the process as described in the Mathematical Model (a) up-to the step (12). It may be noted that up-to this stage of cutting the order length σ_i and σ_j were under consideration. L*et al*l the pieces of σ_i have been exhausted and p_{jz} pieces of the order length σ_j are left to cut.

Instead of cutting order length σ_j separately as described in section 4.1, we club it to the remaining data and tackle our problem with $\sigma_1, \sigma_2, \dots, \sigma_{j-1}, \sigma_j, \sigma_{j+1}, \dots, \sigma_{i-1}, \sigma_{i+1}, \dots, \sigma_n$ order lengths for which the number of pieces to cut are $p_1, p_2, \dots, p_{j-1}, p_{jz}, p_{j+1}, \dots, p_{i-1}, p_{i+1}, \dots, p_n$.

The process is continued till at most one order length is left to cut and it would be completely cut by the process described after (12) in Mathematical Model (a).

Algorithm for cutting plan

1 Insert n order lengths σ_i's with required number of pieces p_i's
2 Arrange σ_i's in ascending order w.r.t. their respective lengths
3 Insert m stock lengths s_k's
4 Arrange s_k's in ascending order w.r.t. their respective lengths
5 Consider sustainable trim t_s
6 For i=n to 1
7 For $j = n - 1$ to 1
8 If $i \neq j$
9 Let $b = \min\{p_i, p_j\}$
10 Else goto Line 6
11 For n_1 = 1 to b
12 For α_{1i} = 1 to p_i
13 For α_{1j} = 1 to p_j
14 Let $y_{ij}^1 = \alpha_{i1} * \sigma_i + \alpha_{j1} * \sigma_j$
15 For k = 1 to m
16 If $y_{ij}^1 \leq s_m$
17 Else go to line 11
18 End If (Line 16)
19 If $0 \leq s_k - y_{ij}^1 \leq t_s$
20 $w_k^{n_1} = \min_{\alpha_{1i}, \alpha_{1j}}\{s_k - y_{ij}^1\}$
21 Calculate $p_{i1} = p_i - n_1 * \alpha_{i1}$, $p_{j1} = p_j - n_1 * \alpha_{j1}$
22 Else go to line 11
23 End If (Line 19)
24 End For (Line 13)
25 End For (Line 12)


```
26      Calculate w_k^j = min_{n_1} { w_k^{n_1} }
27      Let w_k = n_1 * w_k^j
28    End For (Line 11)
29          Continue ℘_i = ℘_{i1} and ℘_j = ℘_{j1}
30          Recall the process Line 8 to Line 28
31        Break If (℘_i = 0 or ℘_j = 0) or (℘_i = ℘_j = 0)
32          If ℘_i = 0 or ℘_j = 0
33            Let ℘_μ = ℘_i or ℘_j
34            Continue ℘_i = ℘_μ and for another value of ℘_j goto line 6
35            Recall the process Line 6 to Line 28
36          Else goto line 5 (by excluding old ℘_i and ℘_j)
37        Break If (conditions on Line 16 or Line 19 not hold)
38      End For (Line 7)
39        Let ℘_i = ℘_θ
40          For l = 1 to ℘_θ
41            y_l = l * σ_i
42              For k = 1 to m
43                Calculate t_{kl} = s_k − y_l
44                  Let t_k^l = min_k{ t_{kl} }
45              End For (Line 41)
46            Calculate t_k = min_l{ t_k^l }
47            Let ℘_i = ℘_i − l
48          Break if ℘_i = 0
49      End For (Line 39)
50    End For (Line 6)
```

5. Numeric Computation

We consider the following sets of data for testing of algorithms and computation of total trim loss as discussed in Section 4.

Consider the available stock for data set-1 and data set-2:

Table 1. Stock for Data Set-1 and Data Set-2

Stock lengths	Stock lengths
700	1050
750	1100
800	1150
850	1200
900	1250
950	1300
1000	

Data-1:

Table 2. Data Set-1

Order lengths	Required pieces	Order lengths	Required pieces
115	96	440	40
170	80	720	32
210	64	830	24

Sustainable trim by using weighted means (t_s) of **Data-1 is 50.1130**

- By using the algorithm of Mathematical Model (a), we have calculated the total trim loss for Data set-1. Finally computed the total trim loss in percentage which is given by

$$T_1 = 0.05888 \%$$

- Also, by using algorithm of Mathematical Model (b), we have calculated the total trim loss for Data set-1. Finally computed the total trim loss in percentage which is given by

$$T_2 = 0.03105 \%$$

Data-2:

Table 3. Data Set-2

Order lengths	Required pieces	Order lengths	Required pieces
310	40	640	23
660	16	750	24
800	13	400	47
890	36	230	32
550	39	460	21

Sustainable trim by using weighted means (t_s) of **Data-2 is 85.3854**

- By using the algorithm of Mathematical Model (a), we have calculated the total trim loss for Data set-2. Finally computed the total trim loss in percentage which is given by

$$T_1 = 0.2128 \%$$

- Also, by using algorithm of Mathematical Model (b), we have calculated the total trim loss for Data set-2. Finally computed the total trim loss in percentage which is given by

$$T_2 = 0.0065 \%$$

Table 4. Comparison of Two Approaches

Data Set	Trim loss of model (a)	Trim loss of model (b)
Data set-1	0.05888 %	0.03105 %
Data set-2	0.2128 %	0.0065 %

6. Conclusion

Mathematical Model (a) which was described in [23] has been designed in such a way that it covers the constraints of space and minimization of manpower. The modified Mathematical Model (b) described in the present paper detains the characterization of Mathematical Model (a) with modification. It may be noted significantly that the modified version of Mathematical Model reduces the trim loss up-to high extent.

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Authors

P. L. Powar, she received her M.Sc. and Ph.D. in Mathematics form Rani Durgavati University, Jabalpur, India. Currently, she is a Professor in Deparment of Mathematics and Computer Science, Rani Durgavati University, Jabalpur, India. Her research areas are Cutting Stock Problem, Spline Approximation Theory, Finite Element Methods and Topology.

Ravi Vishwakarma, he received his M.Sc. and M.Phil in Mathematics from Rani Durgavati University, Jabalpur, India. Currently, he is persuing Ph.D. and is a National Junior Research Fellow by UGC, New Delhi, India in Deparment of Mathematics and Computer Science, Rani Durgavati University, Jabalpur, India. His research area is Cutting Stock Problem.

Chandrashekhar Meshram, he received the M.Sc and M.Phil degrees, from Pandit Ravishankar Shukla University, Raipur (C.G.) in 2007 and 2008, respectively and PhD from R.T.M. Nagpur University, Nagpur (M.S.) India. Presently he is Post-Doctoral Fellow under Dr. D S Kothari postdoctoral fellow New Delhi, India. His research interested in the field of Cryptography and its Application, Cutting Stock Problem, Statistics, Raga (Music and Statistics), Neural Network , Ad hoc Network, Number theory, Environmental chemistry, Mathematical modeling, Solid Mechanics and Chaos Theory. He is a member of International Association of Engineers (IAENG), Hong Kong, World Academy of Science, Engineering and Technology (WASET), New Zealand , Computer Science Teachers Association (CSTA), USA, Association for Computing Machinery (ACM), USA, International Association of Computer Science and Information Technology (IACSIT), Singapore, European Association for Theoretical Computer Science (EATCS), Greece, International Association of Railway Operations Research (IAROR), Netherland, International Association for Pattern Recognition (IAPR), New York , International Federation for Information Processing (IFIP) ,Austria, Association for the Advancement of Computing in Education (AACE),USA, International Mathematical Union (IMU) Berlin, Germany, European Alliance for Innovation (EAI), International Linear Algebra Society (ILAS) Haifa, Israel, Science and Engineering Institute (SCIEI), Machine Intelligence Research Labs (MIR Labs) , USA, Society: Intelligent Systems, KES International Association, United Kingdom, Universal Association of Computer and Electronics Engineers (UACEE), The Society of Digital Information and Wireless Communications (SDIWC) and Life –time member of Internet

Society (ISOC),USA ,Indian Mathematical Society , Cryptology Research Society of India and Ramanujan Mathematical Society of India (RMS) and editor in chief of IJRRWC, UK and managing editor of IJCMST, India. He is regular reviewer of sixty International Journals and International Conferences and published 60 papers in International and National journals.

