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Object Recognition Viewed as an Application of the Mathematics of Partial Presence

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Abstract

In day-to-day life, we do not use algorithmic procedures to recognize objects. An exact quantification of parameters defining the shape of an object is not really important in object recognition. Further, while recognizing an object, we actually consider very few parameters. If approximate quantification of parameters is assumed to be possible, we should be able to express at least approximately our level of assurance regarding recognition of an object in terms of probability. In this article, we are going to put forward a procedure of how we can do that using the mathematics of partial presence.

Keywords and Phrases*: Normal fuzzy number, fuzzy membership function, fuzzy membership surface, normal fuzzy vector, probability measure*

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1. Introduction

When we see a mango tree, even from a long distance we can recognize that it is a mango tree. Indeed, as far as object recognition is concerned, there need not be anything mathematical about it. After all, even animals can recognize objects as we all know.

However, we can quantify, at least approximately, certain parameters defining the shape of an object. The human brain is supposed to be equipped to do that. Can we express the level of confidence that we have rightly recognized an object in terms of probability, at least approximately? In this article, an attempt would be made to answer this question.

We are interested to show that we can link probability theory with object recognition. One thing is true that to recognize an object, one must have a picture of that object in one's mind, and that picture has to be fuzzy in nature because it is necessarily subjective. If one has never seen a mango tree, never even seen a picture of a mango tree, it is not possible for one to recognize that a particular tree is a mango tree after all. If one has not met a person for years, one may still recognize that person if the old face that one remembers and the face of the person that one has met just now are in some way similar. This kind of object recognition related matters are not computer dependent procedures; we do not use pattern recognition procedures to recognize objects in day-to-day life.

There may be many parameters to be compared. However, for immediate recognition, which is what is very natural, there need not be too many parameters to be compared. For example, a particular dog may have just three legs, and we can still recognize that it is a dog. Therefore to recognize an animal as a dog, number of legs need not be a parameter. A mango tree can have twenty branches and five thousand leaves, while another mango tree can have just two leaves and a bud, and we can still recognize that both of them are mango trees. Therefore to recognize a mango tree, number of branches and number of leaves need not be parameters. Further, exact quantification of the parameters is not necessarily essential. Assuming that in the process of object recognition we can quantify certain necessary parameters at least approximately, in what follows, we are going to put

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forward a mathematical explanation of determining the assurance level of object recognition.

2. The Rationale Behind

To recognize an object, we need to compare the parameters describing it with shapes of objects that we already know. Such a comparison is not possible when we see an object for the first time ever. In such a situation, recognition is not possible anyway. It is obvious that in the process of object recognition, quantification of parameters defining the shape of the object is not necessary. After all, animals too can recognize objects, and in this process, quantification is out of question. However, for the human brain to recognize an object, quantification of parameters defining the shape of an object, at least approximately, is possible, although the process of recognition cannot be mathematical. We just recognize objects; there is nothing mathematical about it.

If we try to quantify a parameter defining the shape of an object, the approximate quantification has to be in terms of a normal fuzzy number. Therefore, for two or more parameters, the approximate quantification would have to be in terms of a normal fuzzy vector. Just as a normal fuzzy number is defined with reference to a membership function, a normal fuzzy vector should be defined with reference to a membership surface. From the approximate shape of the membership surface, the level of recognition of the object concerned can be found out at least approximately. This is the rationale behind the idea of finding an assurance level towards recognition of an object.

We are going to view the theory of fuzzy sets as mathematics of partial presence as described by Baruah ([1-3]). Construction of membership surfaces of normal fuzzy vectors was discussed by Baruah [4]. This was based on the Randomness-Fuzziness Consistency Principle for normal fuzzy numbers ([5, 6]). Our standpoint of defining a normal fuzzy number with reference to the mathematics of partial presence does not defy the Dubois – Prade definition of normal fuzzy numbers.

We need to specifically mention at this point that half a century ago, in 1965, Zadeh [7], while introducing the concept of fuzzy sets, did not mention how exactly we need to construct the membership function. Thereafter, in 1978, he put forward his possibility-probability consistency principle (Zadeh [8]) in which he proposed that a law of probability can be defined on the same interval on which a law of fuzziness was already defined. However, Baruah ([1-6]) has shown that not one single law in the entire interval $[a, c]$, but two independent laws of randomness, one in $[a, b]$ and the other in $[b, c]$ are necessary and sufficient to define a fuzzy law in $[a,b, c]$. Here is where the mathematics of partial presence differs from Zadeh's theory of fuzzy sets.

In the next Section, we shall discuss in short about our principle towards linking randomness with fuzziness. Our aim, as mentioned earlier, is to express an assurance level towards recognition of an object in terms of probability.

3. The Randomness-Fuzziness Consistency Principle for Normal Fuzzy Numbers

In the Dubois-Prade representation of a fuzzy number, the membership function $\mu(x)$ of a normal fuzzy number $[a, b, c]$ is given by

$$\begin{aligned} \mu(x) &= L(x),\ a \le x \le b, \\ &= R(x),\ b \le x \le c, \\ &= 0, \text{ otherwise,} \end{aligned}$$

where $L(x)$ is a continuous non-decreasing function and $R(x)$ is a continuous non-increasing function where

$$L(a) = R(c) = 0,$$

$$L\,(b) = R\,(b) = 1.$$

Dubois and Prade have not however gone forward to explain wherefrom these two functions appear.

It was explained by Baruah ([5], [6]) that $L(x)$ is nothing but a distribution function and that $R(x)$ is nothing but a complementary distribution function. A mathematical approach based on the mathematics of partial presence to link fuzziness with randomness leads to the Randomness-Fuzziness Consistency Principle for normal fuzzy numbers which can be stated as follows: for a normal fuzzy number $[a, b, c]$, the membership function $\mu\,(x)$ can be expressed as

$$\mu\,(x) = \theta\, L(x) + (1 - \theta)\, R(x),$$

with

$$\begin{aligned} \theta &= 1, \text{ if } a \le x \le b, \text{ and} \\ &= 0, \text{ if } b \le x \le c, \end{aligned}$$

where $L(x)$ is the distribution function of a random variable defined in $[a, b]$ and $R(x)$ is the complementary distribution function of another random variable defined in $[b, c]$, complementary in the sense that (1 - R(x)) is a distribution function. Accordingly, if we presume that our random variables concerned are indeed probabilistic, then *possibility* as defined by Zadeh [8] is nothing but probability defined by one law of randomness in $[a, b]$ and by another law of randomness in $[b, c]$.

In the next Section, we are going to show that we can extend the Randomness-Fuzziness Consistency Principle for normal fuzzy numbers (Baruah [5, 6]) to derive the Randomness-Fuzziness Consistency Principle for normal fuzzy vectors.

4. The Randomness-Fuzziness Consistency Principle for Normal Fuzzy Vectors

Let $\mu_X(x)$ and $\mu_Y(y)$ be the membership functions of the normal fuzzy numbers $[a, b, c]$ and $[d, e, f]$ respectively with $[a, b, c]$ defined on the horizontal axis and $[d, e, f]$ defined on the vertical axis. Let us define the membership functions as

$$\begin{aligned} \mu_X(x) &= L_X\,(x),\ a \le x \le b, \\ &= R_X\,(x),\ b \le x \le c, \\ &= 1, \text{ otherwise, and} \end{aligned}$$

$$\begin{aligned} \mu_Y\,(y) &= L_Y\,(y),\ d \le y \le e, \\ &= R_Y\,(y),\ e \le y \le f, \\ &= 1, \text{ otherwise.} \end{aligned}$$

Here,

$$\begin{aligned} L_X\,(x) &= \text{Prob}\,[a \le X \le x],\ a \le x \le b, \\ L_Y\,(y) &= \text{Prob}\,[d \le Y \le y],\ d \le y \le e, \\ R_X\,(x) &= 1 - \text{Prob}\,[b \le X \le x],\ b \le x \le c, \\ R_Y\,(y) &= 1 - \text{Prob}\,[e \le Y \le y],\ e \le y \le f \end{aligned}$$

for two random variables X and Y.

It was shown in [4] that the membership surface of a normal fuzzy vector $\{[a, b, c], [d, e, f]\}$ would be given by

$$\begin{aligned} \mu_{X,\,Y}\,(x, y) &= L_X\,(x)\, L_Y\,(y),\ a \le x \le b,\ d \le y \le e, \\ &= L_X\,(x)\, R_Y\,(y),\ a \le x \le b,\ e \le y \le f, \end{aligned}$$

$$= R_X(x)\, L_Y(y),\ b \le x \le c,\ d \le y \le e,$$
$$= R_X(x)\, R_Y(y),\ b \le x \le c,\ e \le y \le f.$$

We have

$$\mu_X(x) = \theta_X\, L_X(x) + (1 - \theta_X)\, R_X(x),$$

with

$$= 0, \text{ if } b \le x \le c,$$

where $L(x)$ is a distribution function and $R(x)$ is a complementary distribution function. Similarly, we have

$$\mu_Y(y) = \theta_Y\, L_Y(y) + (1 - \theta_Y)\, R_Y(y),$$

with

$$\theta_Y = 1, \text{ if } d \le y \le e, \text{ and}$$
$$= 0, \text{ if } e \le y \le f,$$

It is therefore clear that

$$\mu_{X,Y}(x, y) = \{\theta_X\, L_X(x) + (1 - \theta_X)\, R_X(x)\}\ \{\theta_Y\, L_Y(y) + (1 - \theta_Y)\, R_Y(y)\}$$

which is nothing but $\mu_X(x)\mu_Y(y)$. This is the Randomness-Fuzziness Consistency Principle for Normal Fuzzy Vectors. For vectors with more than two coordinates, the principle above can be extended. There would be three multipliers if there are three coordinates, four multipliers if there are four coordinates.

It may be noted that multiplication such as $L_X(x)\ R_Y(y)$ is meaningful only because they are distribution functions. Otherwise, multiplying $L_X(x)$ by $R_Y(y)$ would not make any sense, and in that case, defining the membership surface mathematically would not be possible. This is perhaps why no one else from the fuzzy mathematics fraternity has yet raised the question of how to describe the membership surface of a normal fuzzy vector. It is another matter that even without describing the membership surface, fuzzy vectors can be defined. But following Zadeh's definition of the membership function of a normal fuzzy number, to describe the membership surface of a normal fuzzy vector is not possible. In the next Section, we are going to discuss how to use the principle stated above to recognize an object.

5. Object Recognition

Suppose the normal fuzzy vector {[a, b, c], [d, e, f]} approximately represents two independent parameters describing an object. We can approximately quantify the membership values of the object to be recognized. Suppose x and y are the numerical values of the fuzzy variables defined in [a, b, c] and [d, e, f] respectively. Then

$$\mu_{X,Y}(x, y) = \{\theta_X\, L_X(x) + (1 - \theta_X)\, R_X(x)\}\ \{\theta_Y\, L_Y(y) + (1 - \theta_Y)\, R_Y(y)\}$$

defined in Section – 4 above would give us an approximate value in terms of probability.

We have discussed a rather simple case. Indeed, there can be a large number of parameters defining the shape of an object. Accordingly, the membership surface concerned would be far more complex than what could perhaps be imagined. In the process of object recognition, it is another matter that no one goes for quantification of parameters. But if we want to define the process mathematically, we have shown that it can be done using the mathematics of partial presence.

Now suppose we are considering just two parameters towards recognition of an object. Fuzzy quantification of such a parameter gives us the number $X = [2, 3, 4]$, say. Let the other parameter be quantified as $Y = [10, 11, 12]$. It is obvious that in such cases, randomness laws must necessarily be probabilistic, and an assumption that the concerned laws of probability are uniform is not unrealistic. What we mean is that the fuzzy numbers can be assumed to be triangular. Now for example, if the observed value of the first parameter is 3.5 and if that of the second parameter is 10.5, at least approximately, then

$$\mu_{X,Y}(3.5, 10.5) = R_X(3.5).\, L_Y(10.5)$$
$$= (0.5)\,.\,(0.5) = 0.25.$$

This means, the probability that the object is indeed of the type concerned is approximately 0.25. How small a value of such a probability we should possibly accept to recognize an object may be a question. We may perhaps go for the statistical convention in this regard: if we are not even 5% sure about a hypothesis, then it is better to reject it.

6. Discussions

Object recognition is a day to day affair for everyone. We recognize a mango tree as a mango tree, and in the process we do not use the algorithms of pattern recognition. Even animals can recognize objects. Accordingly, quantification of parameters that define the shape of an object is not quite important. However, if we try to express the process of object recognition mathematically, we have shown in this article how to do that.

The concept that we have used towards defining object recognition is nothing but the theory of fuzzy sets. However, in the original definition of fuzzy sets forwarded by Zadeh [7], there was no mention as to how to construct the membership function of a fuzzy number. According to Baruah ([1-3]), fuzziness can be described as mathematics of partial presence of an element in a set. This leads to an assertion that two laws of randomness are necessary and sufficient to define a normal law of fuzziness. Most of the researchers may not agree with our views. This is why we have put forward the topic as though it is an application of the mathematics of partial presence, whereas it actually is an application of the mathematics of fuzziness if we define fuzziness from our standpoint.

Three years after introducing the theory of fuzzy sets, and ten years before introducing his possibility-probability consistency principle, in 1968, Zadeh put forward what he termed as probability measures of fuzzy events. In the process, he defined mean of the membership function of a fuzzy set with respect a probability measure. The probability measure could actually be the uniform probability measure (Eq. 3, Zadeh [9]), in which case the mean of the membership function would simply be the integral of the membership function $\mu_X(x)$ with respect to x

for the fuzzy set *X*. In other words, what Zadeh proposed was that the membership function can be integrated too. However, in the entire range of statistical literature, one would not find even a single instance of integrating the probability distribution function; this is never done simply because this has no physical significance. Now, in our definition of the membership function, which does not defy the Dubois-Prade definition, the left reference function is a distribution function and the right reference function is a complementary distribution function. Accordingly, integration of the membership function does not really have any measure theoretical sense.

Since the beginning, since 1965, researchers have been following Zadeh's concept of fuzziness. It cannot be expected that our standpoint would be immediately acceptable universally. Baruah [10] has put forward an axiomatic approach to define normal fuzziness; if at least that is found acceptable to the world of mathematics, it would not be necessary to continue to believe that the concept of fuzziness must be heuristic in nature.

The notion of probability does not enter into the definition of the probability measure, just as (Rohatgi and Saleh [11], page - 43) the notion of probability does not enter into the definition of a random variable. Indeed, the Probability Measure should have initially been given another name. The very mention of the word *probability* in the name *Probability Measure* led to all sorts of confusions. Zadeh [7] tried and failed to link fuzziness with probability. This has led the researchers to believe that the probability measure just cannot have anything to do with fuzziness. Random variables, both probabilistic and deterministic, follow the probability measure theoretic formalisms. In the present context, we have presumed that the concerned laws of randomness are probability laws, for otherwise expressing the assurance level in terms of probability would not be possible.

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