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# Theoretical Development for Blind Identification of Non Linear Communication Channels

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***Abstract***

*In this paper, we present a theoretical analysis of non linear quadratic systems using higher order cumulants (HOC). In the one hand, we develop the equations linking the second and third order cumulants with the impulse response of quadratic non linear systems. In the other hand, these relationships are used to develop an extension of linear algorithm based on third order cumulants to non linear algorithm for identification of quadratic systems. The proposed algorithm is tested using different quadratic models for various values of signal to noise ratio (SNR).*

**Keywords:** *Higher order cumulants, Linear systems, Non linear quadratic systems, Communications channels*

## 1. Introduction

The problem of the blind identification of linear channels based on higher order cumulants (HOC), has received extensive attention in the literature [1, 2, 3, 6, 7]. There are several motivations behind this interest:

- The methods based on HOC are blind to any kind of a gaussian process, whereas autocorrelation function (second order cumulants) is not. Consequently, cumulants based methods boost signal to noise ratio (SNR) when signals are corrupted by gaussian measurement noise [3].
- The HOC methods are useful in identifying non minimum phase systems and in reconstructing non minimum phase signals when the signals are non gaussian.

Finite impulse response channels identification from output measurements only is a well defined problem in several science and engineering areas such as communications, speech signal processing, adaptive filtering, spectral estimation, radar Doppler, sonar, geophysical, biomedicine, blind equalization, plasma physics, seismic data processing, harmonic retrieval, time delay estimation and array processing [4].
When linear modelling of the channel is not adequate, the non linear modelling appeared like an alternative efficient solution in most real cases. However, blind identification of non linear systems has attracted much attention during the last years in the system theory [8, 9]. Quadratic models have been successfully used to represent non linear systems in a number of practical applications in the areas of chemical processes, biological systems, and communication and control.

In this work, we address the identification problem of single input single output (SISO) quadratic models using HOC techniques. Indeed the method developed by zidane, *et al.* in

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[1, 2] for linear systems, is extended to the general case of the non linear quadratic systems identification. Theoretical analysis and numerical simulation results, in noisy environment and for different signal to noise ratio (SNR), are presented to illustrate the performance of the proposed method.

## 2. Non linear Systems and Hypotheses

We consider a non linear channel (Fig. 1) described by the following equation:

$$y(k) = \sum_{i=0}^{q} h(i,i)x^2(k-i) + w(k)\,, \qquad (1)$$

where $x(k)$ is the input sequence, $h(i,i)$ and $q$ are the parameters and the order of non linear channel, respectively, $y(k)$ is the observed system output corrupted by additive gaussian noise $w(k)$.

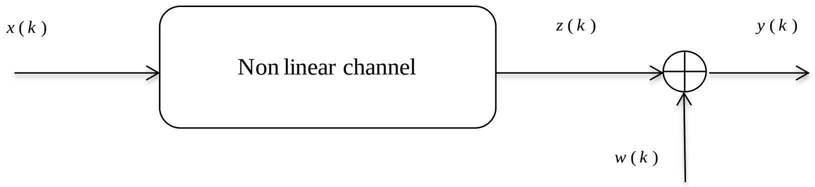


**Figure 1. Non Linear Channel Model**

For this system we assume that:

**A1:** The order $q$ is known;
**A2:** The input sequence $x(k)$ is independent and identically distributed (i.i.d) zero mean, stationary, non gaussian and with:

$$C_{k,x}(\tau_1, \tau_2, \dots \tau_{k-1}) = \begin{cases} \gamma_{k,x} & if\ \tau_1 = \tau_2 = \cdots = \tau_{k-1} = 0 \\ 0 & otherwise \end{cases}$$

where $\gamma_{k,x}$ denotes the $k^{th}$order cumulants of the input signal $x(k)$ at origin;
**A3:** The system is supposed causal and truncated, i.e. $\mathrm{h(i,i)} = 0$ if $\mathrm{i} < 0$ and $\mathrm{i} > q$ with $h(0,0) = 1$;
**A4:** The measurement noise sequence $w(k)$ is assumed to be zero mean, i.i.d, gaussian, independent of $x(k)$ with unknown variance.

## 3. Theoretical Development

In this section, we use the Leonov Shiryaev formula [5] and the definition of the cumulants to demonstrate the equations linking the higher order cumulants and the diagonal parameters of non linear systems.
Thus, the cumulants, order $r$, and the moments of the stochastic signal are linked by the following relationships of the Leonov and Shiryayev formula [5]:

$$Cum[z_1, z_2, \dots, z_r] = \sum (-1)^{k-1}(k-1)!\, E\left[\prod_{i\epsilon v_1} z_i\right].E\left[\prod_{j\epsilon v_2} z_j\right] \dots E\left[\prod_{k\epsilon v_p} z_k\right], \qquad (2)$$

where the addition operation is over all the set of $v_i$ $\{1 \le i \le p \le r\}$ and $v_i$ compose a partition of $\{1, 2, \dots, r\}$. In Eq. (2) $k$ is the number of elements compose a partition.

For the second order cumulants, we are $r = 2$ and $1 \le p \le 2$. The possible partitions are: (1,2) and (1)(2):

 

$$Cum[z_1,z_2] = (-1)^{1-1}(1-1)!\,E[z_1z_2] + (-1)^{2-1}(2-1)!\,E[z_1]E[z_2] = E[z_1z_2] - E[z_1]E[z_2] \quad (3)$$

$$C_{2,z}(\tau) = Cum(z_1,z_2) = Cum[z(k),z(k+\tau)] = E\left[\sum_{i=0}^{q} h(i,i)x^2(k-i)\sum_{j=0}^{q} h(j,j)x^2(k+\tau-j)\right] - E\left[\sum_{i=0}^{q} h(i,i)x^2(k-i)\right]E\left[\sum_{j=0}^{q} h(j,j)x^2(k+\tau-j)\right] \quad (4)$$

$$C_{2,z}(\tau) = \sum_{i=0}^{q} h(i,i)h(i+\tau,i+\tau)\,E[x^2(k-i)x^2(k-i)] - \sum_{i=0}^{q} h(i,i)h(i+\tau,i+\tau)\,E[x^2(k-i)]E[x^2(k-i)] \quad (5)$$

Under the assumption A2 and Eq. (5), the second order cumulants becomes:

$$C_{2,z}(\tau) = \left(\gamma_{4,x} - \gamma_{2,x}^2\right)\sum_{i=0}^{q} h(i,i)h(i+\tau,i+\tau) \quad (6)$$

For the third order cumulants, we are $r = 3$ and $1 \le p \le 3$. The possible partitions are: (1, 2, 3), (1)(2, 3), and (1)(2)(3):

$$Cum[z_1,z_2,z_3] = (-1)^0(0)!\,E[z_1z_2z_3] + (-1)^{2-1}(2-1)!\,[3]E[z_1]E[z_2z_3] + (-1)^{3-1}(3-1)!\,E[z_1]E[z_3]E[z_2] = E[z_1z_2z_3] - E[z_1]E[z_2z_3] - E[z_2]E[z_1z_3] - E[z_3]E[z_1z_2] + 2E[z_1]E[z_3]E[z_2] \quad (7)$$

Our objectif is to identify the non linear parameters of quadratic models defined by Eq. (1).

The third order cumulants becomes:

$$C_{3,z}(\tau_1,\tau_2) = Cum(z_1,z_2,z_3) = Cum[z(k),z(k+\tau_1),z(k+\tau_2)] = E\left[\sum_{i=0}^{q} h(i,i)x^2(k-i)\sum_{j=0}^{q} h(j,j)x^2(k+\tau_1-j)\sum_{l=0}^{q} h(l,l)x^2(k+\tau_2-l)\right] - [3]E\left[\sum_{i=0}^{q} h(i,i)x^2(k-i)\right]E\left[\sum_{j=0}^{q} h(j,j)x^2(k+\tau_1-j)\sum_{l=0}^{q} h(l,l)x^2(k+\tau_2-l)\right] + [2]E\left[\sum_{i=0}^{q} h(i,i)x^2(k-i)\right]E\left[\sum_{j=0}^{q} h(j,j)x^2(k+\tau_1-j)\right] \times E\left[\sum_{l=0}^{q} h(l,l)x^2(k+\tau_2-l)\right] \quad (8)$$

$$C_{3,z}(\tau_1,\tau_2) = \sum_{i=0}^{q} h(i,i)h(i+\tau_1,i+\tau_1)\,h(i+\tau_2,i+\tau_2)E[x^2(k-i)x^2(k-i)x^2(k-i)] - [3]\sum_{i=0}^{q} h(i,i)h(i+\tau_1,i+\tau_1)\,h(i+\tau_2,i+\tau_2)E[x^2(k-i)]E[x^2(k-i)x^2(k-i)] + [2]\sum_{i=0}^{q} h(i,i)h(i+\tau_1,i+\tau_1)\,h(i+\tau_2,i+\tau_2)E[x^2(k-i)]E[x^2(k-i)] \times E[x^2(k-i)] \quad (9)$$

Under the assumption A2 and Eq. (9), the third order cumulants becomes:

$$C_{3,z}(\tau_1,\tau_2) = (\gamma_{6,x} - 3\gamma_{2,x}\gamma_{4,x} + 2\gamma_{2,x}^3)\sum_{i=0}^{q} h(i,i)h(i+\tau_1,i+\tau_1)h(i+\tau_2,i+\tau_2) \quad (10)$$

## 4. Extension of Linear Algorithm to Non Linear Algorithm using Third Order Cumulants

The Fourier transforms of the second and third order cumulants are given respectively by the following equations:

$$S_{2,y}(\omega) = TF\{C_{2,y}(\tau)\} = (\gamma_{4,x} - \gamma_{2,x}^2)H(-\omega,-\omega)H(\omega,\omega) \quad (11)$$

$$TF\{C_{3,y}(\tau_1,\tau_2)\} = (\gamma_{6,x} - 3\gamma_{2,x}\gamma_{4,x} + 2\gamma_{2,x}^3)H(-\omega_1-\omega_2,-\omega_1-\omega_2)H(\omega_1,\omega_1)H(\omega_2,\omega_2) \quad (12)$$

If we suppose that $\omega = (\omega_1 + \omega_2)$, Eq. (11) becomes:

$$S_{2,y}(\omega_1 + \omega_2) = (\gamma_{4,x} - \gamma_{2,x}^2)H(-\omega_1 - \omega_2, -\omega_1 - \omega_2)H(\omega_1 + \omega_2, \omega_1 + \omega_2) \tag{13}$$

Then, from Eqs. (12) and (13) we obtain the following equation:

$$S_{3,y}(\omega_1, \omega_2)H(\omega_1 + \omega_2, \omega_1 + \omega_2) = \frac{(\gamma_{6,x} - 3\gamma_{2,x}\gamma_{4,x} + 2\gamma_{2,x}^3)}{(\gamma_{4,x} - \gamma_{2,x}^2)} H(\omega_1, \omega_1)H(\omega_2, \omega_2)S_{2y}(\omega_1 + \omega_2,\ \omega_1 + \omega_2)$$

The inverse Fourier transform of Eq. (14) demonstrates that the third order cumulants, the autocorrelation function (second order cumulants) and the impulse response channel parameters are combined by the following equation:

$$\sum_{i=0}^{q} C_{3,y}(\tau_1 - i, \tau_2 - i)h(i,i) = \frac{(\gamma_{6,x} - 3\gamma_{2,x}\gamma_{4,x} + 2\gamma_{2,x}^3)}{(\gamma_{4,x} - \gamma_{2,x}^2)} \sum_{i=0}^{q} h(i,i)h(\tau_2 - \tau_1 + i, \tau_2 - \tau_1 + i)\, C_{2,y}(\tau_1 - i) \tag{15}$$

If we use the autocorrelation function property of the stationary process such as $C_{2,y}(\tau) \neq 0$ only for $-q \leq \tau \leq q$ and vanishes else where.
If we suppose that $\tau_1 = 2q$ the Eq. (15) becomes:

$$\sum_{i=0}^{q} C_{3,y}(2q - i, \tau_2 - i)h(i,i) = \frac{(\gamma_{6,x} - 3\gamma_{2,x}\gamma_{4,x} + 2\gamma_{2,x}^3)}{(\gamma_{4,x} - \gamma_{2,x}^2)} h(q,q)h(\tau_2 - q, \tau_2 - q)\, C_{2,y}(q), \tag{16}$$

else, if we suppose that $\tau_2 = 2q$, Eq. (16) will become:

$$C_{3y}(q,q)h(q,q) = \frac{(\gamma_{6,x} - 3\gamma_{2,x}\gamma_{4,x} + 2\gamma_{2,x}^3)}{(\gamma_{4,x} - \gamma_{2,x}^2)} h^2(q,q)C_{2,y}(q) \tag{17}$$

Using Eqs. (16) and (17) we obtain the following relation:

$$\sum_{i=0}^{q} C_{3,y}(2q - i, \tau_2 - i)h(i,i) = C_{3,y}(q,q)\, h(\tau_2 - q, \tau_2 - q) \tag{18}$$

Else, if we suppose that the system is causal and truncated, i.e. $h(i,i) = 0$ if $i < 0$ and $i > q$. Thus, for $\tau_2 = q, \ldots, 2q$, the system of Eq. (18) can be written in matrix form as:

$$\begin{bmatrix} C_{3,y}(2q-1, q-1) & \cdots & C_{3,y}(q,0) \\ C_{3,y}(2q-1, q) - C_{3y}(q,q) & \cdots & C_{3,y}(q,1) \\ \vdots & \ddots & \vdots \\ C_{3,y}(2q-1, 2q-1) & \cdots & 0 \end{bmatrix} \begin{bmatrix} h(1,1) \\ \vdots \\ h(i,i) \\ \vdots \\ h(q,q) \end{bmatrix} = \begin{bmatrix} C_{3,y}(q,q) - C_{3,y}(2q,q) \\ -C_{3,y}(2q, q+1) \\ \vdots \\ -C_{3,y}(2q, 2q) \end{bmatrix} \tag{19}$$

## 5. Simulation Results and Discussion

In this section, we test the proposed approach to identify the non linear parameters of the non linear channels. For this reason, we select the models characterizing a non linear quadratic system, with known parameters and then we try to recover these parameters using proposed algorithm.

### 5.1 Model 1

$$y(k) = x^2(k) - 0.35\, x^2(k-1) - 0.95\, x^2(k-2) \tag{20}$$

The simulation results are shown in the Table 1 for different values of signal to noise ratio (SNR) and an small sample size N=500, over 50 Monte Carlo runs.

 

**Table 1. Estimated Parameters of Model 1 in Noise Case for N=500, different SNR and for 50 Monte Carlo Runs**

| SNR (dB) | $\hat{h}(1,1) \pm \sigma$ | $\hat{h}(2,2) \pm \sigma$ | NMSE |
|---|---|---|---|
| 0 | $-0.4061 \pm 0.2027$ | $-0.9803 \pm 0.1799$ | $267 \times 10^{-4}$ |
| 8 | $-0.3626 \pm 0.0851$ | $-0.9321 \pm 0.1234$ | $16 \times 10^{-4}$ |
| 16 | $-0.3605 \pm 0.0652$ | $-0.9346 \pm 0.0861$ | $12 \times 10^{-4}$ |
| 24 | $-0.3406 \pm 0.0664$ | $-0.9466 \pm 0.0805$ | $7.3672 \times 10^{-4}$ |

From Table 1 we can conclude that:

The NMSE values obtained using the proposed algorithm demonstrate that the estimates impulse response parameters of non linear channel are approximately closer to real values. This may be due to the fact that the algorithm is based on the higher order cumulants, which are zero for gaussian process. Indeed the proposed algorithm give a good values the (std) of estimates parameters, this implies a small variance around the mean value.

In the Figure 2 we have presented the estimation of the magnitude and the phase of the impulse response using the proposed algorithm for data length N = 500 and different SNR.

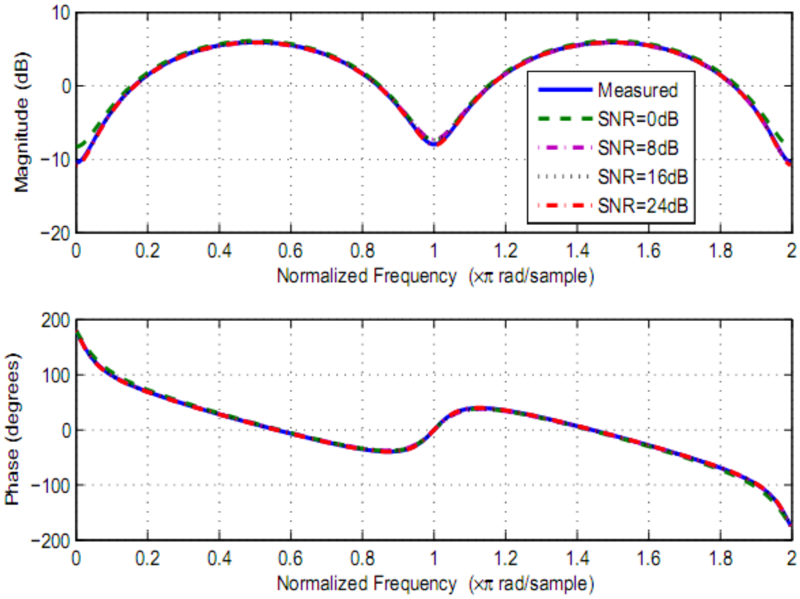


**Figure 2. Estimated Magnitude and Phase of the Model 1, for N = 500 and Various SNR**

This figure gives us a good idea about the precision of the proposed algorithm. We observe parfait accord between the estimated and measured non linear channel.

### 5.2 Model 2

The proposed method in this paper is applied to non linear channel of order three describe by the following equations:

$$y(k) = x^2(k) + 1.1\, x^2(k-1) - 0.35\, x^2(k-2) - 0.95 x^2(k-3) \tag{21}$$

In order to test the robustness of the proposed algorithm we increase the order of the model. For 50 Monte Carlo runs, the proposed algorithm was compared for various SNR: 0dB-30dB and for input data length N=2000.

**Table 2. Estimated Parameters of Model 2 in Noise Case for N=2000, Different SNR and for 50 Monte Carlo Runs**

| SNR(dB) | $\hat{h}(1,1) \pm \sigma$ | $\hat{h}(2,2) \pm \sigma$ | $\hat{h}(3,3) \pm \sigma$ | NMSE |
|---|---|---|---|---|
| 0 | $1.2995 \pm 0.2568$ | $-0.1675 \pm 0.1133$ | $-1.3087 \pm 0.3247$ | $4475 \times 10^{-4}$ |
| 10 | $1.1182 \pm 0.0845$ | $-0.3587 \pm 0.0595$ | $-1.0353 \pm 0.1433$ | $89 \times 10^{-4}$ |
| 20 | $1.0836 \pm 0.0774$ | $-0.3689 \pm 0.0642$ | $-0.9643 \pm 0.1420$ | $34 \times 10^{-4}$ |
| 30 | $1.0787 \pm 0.0749$ | $-0.3511 \pm 0.0597$ | $-0.9198 \pm 0.1170$ | $14 \times 10^{-4}$ |

From the simulation results, presented in Table 1 we can note that:

- The impulse response parameter estimates of the non linear channel are almost independent of the noise principally if $SNR \geq 10dB$, because the proposed algorithm is based only on the third order cumulants, which are zeros for gaussian signal. In addition, the values of the estimated parameters using proposed algorithm are very close to the true ones, same if we increase the model order.

- Concerning the values of standard deviation (std), we can say that the proposed algorithm is more precise for all SNR.

In the Figure 3 we have presented the estimation of the magnitude and the phase of the impulse response using the proposed algorithm for data length N = 2000 and various SNR.

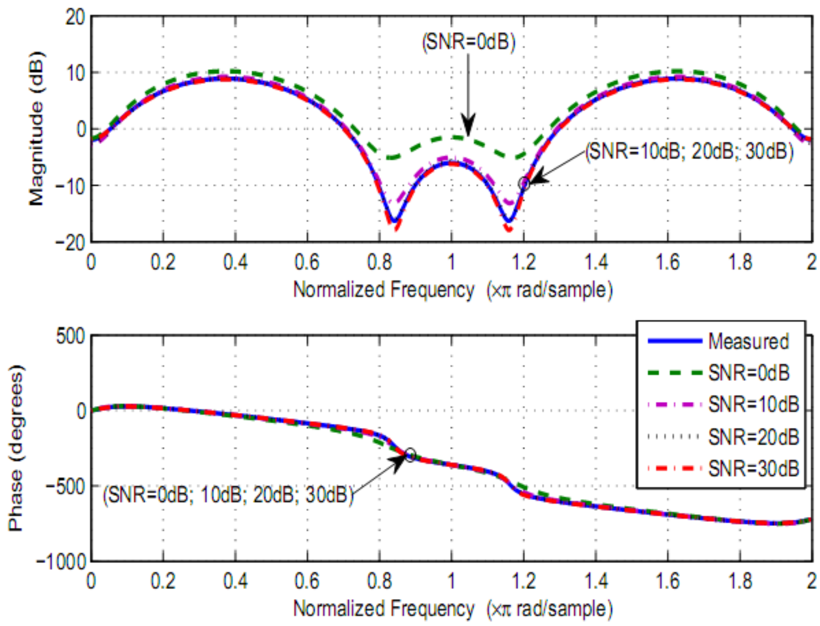


**Figure 3. Estimated Magnitude and Phase of the Model 2, for N = 2000 and Different SNR**

From the Figure 3 we observe parfait accord between the estimated and measured channel principally if the $SNR \geq 10\ dB$.

 

## 6. Conclusion

In this paper, we have developed a theoretical analysis for non linear quadratic systems using HOC. Using this theoretical we have proposed an algorithm for blind identification of non linear communication channels. Simulation results using different SNR and small sample sizes show that the proposed approached is adequate for estimating diagonal quadratic channel.

In the perspective, we will test the efficiency of the proposed algorithm for the blind identification and equalization of the non linear Broadband Radio Access Network (BRAN) channels especially MC-CDMA system.

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