Performance Comparison of Wavelet and FFT Based Multiuser MIMO OFDM over Wireless Rayleigh Fading Channel

Md. Mahmudul Hasan

Department of Electronics and Communication Engineering
University of Information Technology and Sciences, Dhaka 1212, Bangladesh
mahmudul.hasan@uits.edu.bd

Abstract

This article examines the performance of Wavelet based Multi-User MIMO OFDM (MU-MIMO WOFDM) systems and a comparison is made with classical FFT based multiuser MIMO OFDM system. Wavelet analysis has some strong advantages over fourier analysis, as it allows a time-frequency domain operation, allowing optimal resolution and flexibility. Wavelets have been satisfactorily applied in almost all the fields of wireless communication systems including OFDM which is a strong candidate for next generation of wireless system. In this paper, the performances of the ODFM system have been enhanced by using multiple antenna technique over rayleigh fading channel and an optimal beam-forming technique is used to deal with the multi user environment based on Signal-to-Leakage Ratio (SLR), but wavelet based OFDM outperforms FFT based OFDM in all the scenarios.

Keywords: *Wavelet based OFDM (WOFDM), Optimal Beam-forming, MIMO, and Signal to Leakage Ratio (SLR)*

1. Introduction

OFDM is a Multi-Carrier Modulation (MCM) scheme which converts a broadband frequency-selective channel into parallel flat-fading narrowband sub-channels. Cyclic Prefix (CP) is added to each symbol to mitigate the ISI (inter-symbol interference) caused by multipath wireless channel [1], and hence leads to spectral inefficiency. Cyclic prefix also causes ripples in the power spectral density (PSD) of the UWB (Ultra-Wideband) signal thus resulting in a transmit power back-off [4]. On the other hand, wavelet based modulation satisfies orthogonality criterion by orthogonal wavelet filter banks [2]. We can have all the benefits of OFDM even if we replace traditional sinusoid carriers of the fourier based OFDM with suitable wavelets. Wavelet based systems have been shown to have better immunity to impulse and narrowband noises than Fourier OFDM [3, 4] also the interference power can to a large extent, be mitigated [5]. Moreover, wavelet based OFDM doesn’t require any CP, hence increases spectral efficiency, does not produce the ripples, reduces complexity, leading to a better symbol rate, hence no power wastage for redundancy. Wavelet filter-bank approach brings some distinctive advantages for system implementation such as FPGAs (Field Programmable Gate Arrays). Also the performance of equalization in wavelet system is better than conventional OFDM [6]. Wavelet packet modulation will have much lower side lobes in transmitted signals which reduce the inter-carrier interference (ICI) and narrowband interference (NBI) [7]. In addition to spectral efficiency we also review an optimal beam-forming technique to maximize signal quality of individual user.

Recently the implementation of Multiple-Input and Multiple-Output (MIMO) has dramatically improved the channel capacity performance of wireless communication system. Here the basic principle is to exploit multipath signals in order to improve signal quality, increase in the range and throughput. MIMO systems are equipped with multiple number of antennas at both transmitter and receiver side to improve communication performance [8]. MIMO offers a significant improvement in data throughput without additional bandwidth requirement [8]. This is achieved by higher spectral efficiency and link diversity by reducing fading [8, 11].

In Multi-User MIMO (MU-MIMO) wireless systems, several co-channel users with multiple antennas intend to communicate with a base station designed with multiple antennas in the same frequency and time slot. In such cases, it becomes necessary to design a transmission system which is capable of suppressing the co-channel interference (CCI) at the user end. The interference arises due to the fact that the antenna might be serving multiple users.

Multiuser detection (MUD) technique is used to design transmit beam-forming vectors for the end users while limiting the co-channel interference (CCI) from other users. Among the various MUDs, the classic MMSE (minimum mean square error) exhibits a low complexity at the cost of a limited performance. But optimum maximum likelihood (ML) is capable of achieving the best performance in MUD, though it increases the complexity. Several works have been proposed which almost nullifies the effect of CCI at each user [9, 10]. These methods show improved performance with a lot of assumptions, but they impose a condition on the system configuration by limiting the number of antennas, e.g. the number of transmitting antennas (nT) at the base station has to be larger than the sum of receiving antennas (nR) at all users. This condition may not be suitable for practical cases where users may want to use arbitrary number of antennas. In this paper, we consider an approach to design transmit beam-forming vectors based on maximizing the signal-to-leakage ratio (SLR) [17], which cancels CCI and does not impose any restriction on the number of available transmitting antennas.

Few works have been proposed in the recent years that compares the performances of wavelet and FFT based OFDM systems focusing on effects of noise, error performances, and computational complexity, etc. [13-16]. It is indeed needed to analyze the wavelet based OFDM systems in some practical multiuser scenarios. This article compares the performances of wavelet and FFT based OFDM systems in multiuser environment in terms of error performance and through an extensive computer simulation it is shown that wavelet outperforms the FFT based OFDM. The rest of this paper is organized as follows: A brief overview of FFT and wavelet, multiuser MIMO OFDM and equation formulation is given in Section 2. Simulation results are given in Section 3. Finally, Section 4 concludes the paper.

2. System Model

2.1 Fourier vs. Wavelet Based OFDM

Fourier based Conventional OFDM system has been a popular choice for wireless transmission over a long time for its transmission performances. In Fourier analysis we break up a signal into a set of an infinite sum of *Sines* and *Cosines* to exploit the Orthogonality relationship between them. On the other hand, using wavelet transform the signal is first decomposed by a low-pass (LP) and a high-pass (HP) filter. Half of the frequency components have been filtered out at filter outputs and hence can be down-sampled. We get approximation (1) and detail coefficients (2) from $g(n)$ and $h(n)$ filters respectively as shown in Figure 1.Where $g(n)$ and $h(n)$ are the wavelet's half-band low pass filter and high pass filter impulse responses [12].

In wavelet decomposition the details as well as the approximations can be split into a second level details and approximations. These two sets of coefficients are obtained by performing convolution between the input signals and wavelet filter coefficients.

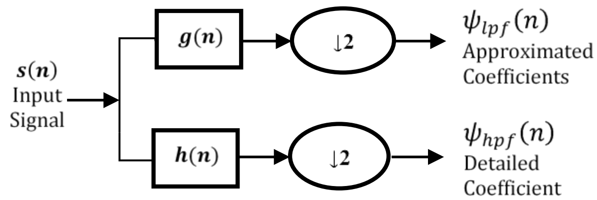

Figure 1. Block Diagram of Wavelet Decomposition

Decomposition process is repeated by a series of high and low pass filters until we are left with a coefficient sequence of wavelets that are orthogonal in nature, the original signal is then reconstructed by performing the reverse operation of this decomposition.

$$\psi_{lpf}(n) = \sum_{k=-\infty}^{k=+\infty} s(k)g(2n-k) \tag{1}$$

$$\psi_{hpf}(n) = \sum_{k=-\infty}^{k=+\infty} s(k)h(2n-k) \tag{2}$$

One thing about wavelet packet analysis that attracts communication system is “accurate reconstruction” using wavelet coefficients. We will see about accuracy in the simulation section of this paper.

2.2 Wavelet based Multiuser OFDM with MIMO

A Wavelet based OFDM system with beam former and MIMO configuration is explained in this section. Figure 2 shows the transmitter and receiver part respectively with $k = 8$ number of sub-carriers as an example. We consider this system is in a multiuser environment of K interfering users, where k^{th} user decorated with M_k number of antennas is communicating with a base station equipped with N number of antennas.

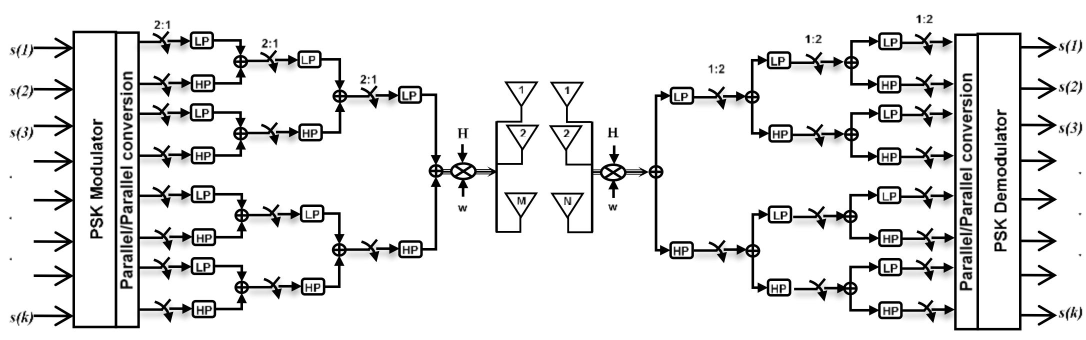

Figure 2. WOFDM Transmitter and Receiver

On the transmitter side, first a binary phase shift keying (BSPK) modulator is used for mapping $s(k)$ data stream to the symbol stream $x(n)$. After the mapping process a parallel-to-parallel (P/P) converter reshapes the modulated data stream $x(n)$ into, for example, N = 8 parallel data streams. This P/P converter makes sure that $N = 2^n$, where n is an integer, so that the transmitter can perform inverse discrete wavelet transform (IDWT) and produce one final sequence in "n" stages. Sequential two $x(n)$ symbol streams are up-sampled by the up-sampling factor 2, filtered by the wavelet filter $g(n)$ or $h(n)$, respectively, and then summed. Output streams are up-sampled by 2, filtered and summed again. The up-sampling and filtering processes continue until one single output stream is obtained. To satisfy the Orthogonality criterion of the wavelet filters we use Quadrature Mirror Filter (QMF) bank. According to QMF, the relationship between both filters is given by$h(n) = (-1)^n g(L - 1 - n)$, where L is the filter length of the impulse response g (n). After obtaining the output stream (or wavelet coefficients), $\psi(n)$ is multiplied by $N \times 1$ Beamforming vector $w_k(n)$ and weighted signals are added to form the output signal [17]. Mathematically it can be given as

$$y_{nT}(n) = \sum_{k=1}^{N} w_k(n)\psi_k(n) = \boldsymbol{w^H}\boldsymbol{\psi} \tag{3}$$

Where subscript $\boldsymbol{H}$ represents $Hermitian$ of a vector, and $\boldsymbol{\psi}$ represents vector representation of $\psi(n)$ and $\boldsymbol{w^H} = [w_1^* \, w_2^* \, w_3^* \, \dots \dots w_N^*] = [w^T]^*$ in (3). At the receiver side in Fig. 2, the received signal by i^{th} user is given by

$$y_i(n) = H_i \sum_{k=1}^{N} w_k(n)\psi_k(n) + n_i(n) \tag{4}$$

Where H_i is an $M_i \times N$ channel matrix and $n_i(n)$ is additive white Gaussian noise in (4). Elements of H_i are complex Gaussian variables with zero-mean and unit-variance and it is assumed that the channel matrices H_i are available at the base station as well as at the i^{th} receiver end, To be more specific, (4) can be broken down into two terms in (5), first term is the desired signal for i^{th} user and second term is the CCI caused by other users present in the system [17].

$$y_i(n) = \underbrace{H_i w_i \psi_i}_{\text{Desired Signal}} + \sum_{k=1, k\neq i}^{K} \underbrace{H_i w_k(n)\psi_k(n)}_{\text{CCI}} + n_i(n) \tag{5}$$

Beamforming weight w_i in (4) is chosen such that i^{th} user can extract ψ_i from $y_i(n)$ by mitigating the effect of CCI. Conventional MUDs techniques aim to cancle the effect of second term, in this aproach of SLR maximization we try to maximize the desired signal power [17].

To maximize SLR we define two terms, desired signal power in (5) may be given as $\|H_i w_i\|^2$, and interference power received by k^{th} user caused by this i^{th} user is $\|H_k w_i\|^2$ which appears as intereference to k^{th} user.

$$SLR = \frac{\|H_i w_i\|^2}{\|\widetilde{H}_i w_i\|^2} = \frac{w_i^* H_i^* H_i w_i}{w_i^* \widetilde{H}_i^* \widetilde{H}_i w_i} \tag{6}$$

where $\widetilde{H}_i = [H_1^* \, H_2^* \, H_{i-1}^* \, H_{i+1}^* \dots \dots H_N^*]$ is a modified channel matrix of H which excludes H_i only. Here we design w_i for i = 1,2,3 ... K such that SLR for every user is maximized [17].

$$w_i^0 = \underset{w_i \in C^{N\times 1}}{\arg\ max} \frac{\|H_i w_i\|^2}{\sum_{k=1,k\neq i}^{K}\|H_k w_i\|^2} \tag{7}$$

Rayleigh-Ritz quotient we can solve this in terms of eigenvector and can be given as

$$\frac{w_i^* H_i^* H_i w_i}{w_i^* \widetilde{H}_i^* \widetilde{H}_i w_i} \leq \lambda_{max}\left(H_i^* H_i, \widetilde{H}_i^* \widetilde{H}_i\right) \tag{8}$$

Where λ_{max} is the largest generalized eigenvalue of $H_i^* H_i$ and $\widetilde{H}_i^* \widetilde{H}_i$. Equality occurs if w_i is proportional to a generalized eigenvector that corresponds to the largest generalized eigenvalue, and can be written as

$$w_i^0 \propto \lambda_{max}\left(H_i^* H_i, \widetilde{H}_i^* \widetilde{H}_i\right) \tag{9}$$

$$or\ w_i^0 \propto \lambda_{max}\ (\widetilde{H}_i^* \widetilde{H}_i)^{-1} H_i^* H_i)\ \text{, If } \widetilde{H}_i^* \widetilde{H}_i \text{ is invertible} \tag{10}$$

$\widetilde{\mathrm{H}}_{\mathrm{i}}^* \widetilde{\mathrm{H}}_{\mathrm{i}}$ in (10) is found to be invertible if $nT = nR$, otherwise we can always use (9) for non-invertible case, i.e. $nT \neq nR$. Symbols are transmitted using M_i antenna over wireless channel using w_i. For detection, a classical single-user maximum-likelihood (ML) detection scheme is used [17]. According to the (ML), estimated coefficient from received $y_i(n)$ signal yields

$$\tilde{\psi}(n) = \frac{w_i^* H_i^* y_i}{\|H_i w_i\|^2} = \frac{w_i^* H_i^* H_i w_i \psi_i}{w_i^* H_i^* H_i w_i} + \frac{w_i^* H_i^* \sum_{k=1,k\neq i}^{K} H_i w_k \psi_k}{\|H_i w_i\|^2} + \frac{w_i^* H_i^*}{\|H_i w_i\|^2} \tag{11}$$

3. Simulation

The WOFDM system given in Figure 2 is simulated using MATLAB, Considering a multiuser environment with $K = 2{,}3$ and 4 users. The user data stream is first BPSK modulated and then level 3 wavelet decomposition is done in this approach with a "$Haar$" wavelet function. Hence the low-pass and high-pass decomposition filter coefficients are ([0.7071, 0.7071] and [0.7071,-0.7071]), whereas reconstruction filter coefficients are ([0.7071, 0.7071] and [-0.7071,-0.7071]). During the simulation we also checked the system with some other wavelet functions like $bior3.5, bior5.5\ and\ db1 - 10$. But $Haar$ function shows better performance in all the cases. Based on 802.11g, other simulation parameters are: system works in 2.4GHz, symbol period 4.6 µs, OFDM symbols are created with 64 subcarriers. Symbols are transmitted over Wireless Rayleigh fading channel. Simulation results are shown in Figure 3 and Figure 4. For instance in Figure 3, considering the base station as well as all the end users are equipped with 4 antennas. Here the multi-user WOFDM system with $K = 2{,}3$ and 4 performs better than FFT based OFDM system. The WOFDM system with K=2 even performs better than the single user system, i.e. theoretical bit-error-rate (BER). Also this system doesn't put any restriction on the number of antennas we use. Figure 4 shows the performance where receiver wanted to use 8 antennas, i.e. $nT = 4$ and $nR = 8$.

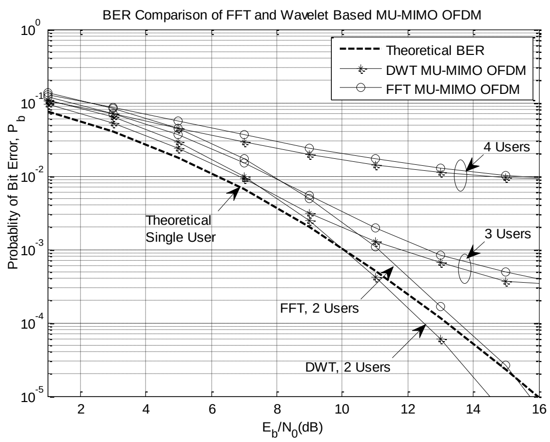

Figure 3. BER Comparison of FFT Based and Wavelet Based MU-MIMO 4x4 OFDM over Rayleigh fading channel considering 2, 3 and 4 users. $(\boldsymbol{nT = 4, nR = 4})$

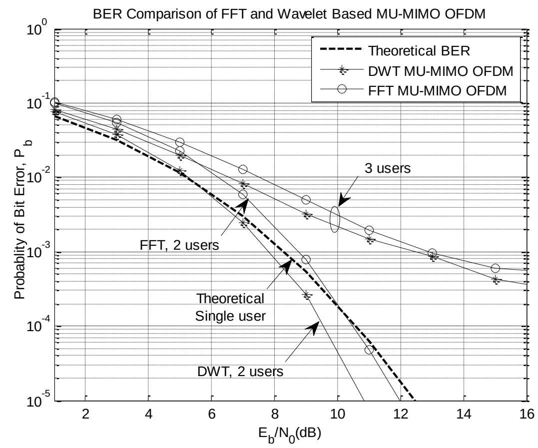

Figure 4. BER Comparison of FFT Based and Wavelet Based MU-MIMO 4x8 OFDM over Rayleigh fading channel considering 2 and 3 users. $(\boldsymbol{nT = 4, nR = 8})$

4. Conclusion

This article introduces an effective implementation of wavelet packet transform instead Fourier transform in multi-carrier communication considering practical cases. Wavelet based analysis is more immune to impulse and narrowband noises than conventional Fourier based OFDM system, also improves spectral efficiency and saves transmission power. In this paper, a beamforming technique is used based on Signal-to-leakage ratio maximization to analyze the multiuser system. It is found that WOFDM system performs better than FFT based OFDM without putting any restriction on the number of antennas we are using at the base station as well as at the receiver ends.

References

[1] J. A. C Bingham, "Multicarrier modulation for data transmission: An idea whose time has come", IEEE Communication Magazine, vol. 28, no. 5, **(1990)**, pp. 5-14.
[2] B. G. Negash and H. Nikookar. "Wavelet-Based Multicarrier Transmission Over Multipath Wireless Channels", IEE Electronics Letters, vol. 36, **(2000)** October, pp. 1787-1788.
[3] C. V. Bouwel, J. Potemans, S. Schepers, B. Nauwelaers and A. V. de Capelle, "Wavelet Packet Based Multicarrier Modulation", Proc. IEEE Benelux Symposium on Communications and Vehicular Technology, Leuven, Belgium, (2000) October 19.
[4] A. R. Lindsey, "Generalized Orthogonally Multiplexed Communication via Wavelet Packet Bases", Ph.D. Thesis, Ohio University, **(1995)**.
[5] A. Batra, J. Balakrishnan, G. R. Aiello, J. R. Foerster and A. Dabak, "Design of a multiband OFDM system for realistic UWB channel environments", IEEE Transactions on Microwave Theory and Techniques, vol. 52, no. 9, **(2004)** September, pp. 2123–2139.
[6] K. Ohno, T. Ikebe and T. Ikegami, "A Proposal for an Interference Mitigation Technique Facilitating the Coexistence of Bi-Phase UWB and Other Wideband Systems", CD-ROM Proc. (Joint UWBST & IWUWBS 2004), WA2-5, **(2004)**.
[7] H. Sakakibara, E. Okamoto and Y. Iwanami, "A wavelet packet modulation method for satellite communications", IEEE TENCON 2005, **(2005)** November.
[8] D. Gesbert, M. Shafi, D. -S. Shiu, P. J. Smith and A. Naguib, "From theory to practice: an overview of MIMO space-time coded wireless systems", IEEE Journal on Selected Areas in Communications, vol. 21, Issue 3, **(2003)** April, pp. 281 -302.

[9] S. Mallat, "A Wavelet Tour of Signal Processing", ISBN: 0-12-466606-X; Academic Press, **(1999)**.
[10] R. Chen, J. G. Andrews and R. W. Heath, "Multiuser spacetime block coded MIMO system with unitary downlink precoding", in Proc. IEEE International Conference on Communications, Paris, France, **(2004)** June, pp. 2689–2693.
[11] K. K. Wong, R. D. Murch and K. B. Letaief, "Performance enhancement of multiuser MIMO wireless communication systems", IEEE Transactions on Communications, vol. 50, **(2002)** December, pp. 1960-1970.
[12] G. Strang and T. Nguyen, "Wavelets and Filter Banks", Wellesley, MA: Wellesley-Cambridge Press, **(1996)**.
[13] K. Abdullah and Z. M. Hussain, "Studies on DWT-OFDM and FFT-OFDM Systems", IEEE International Conference on Communication, Computer and Power, **(2009)** February 15-18.
[14] W. Saad, N. El-Fishawy, S. EL-Rabaie and M. Shokair, "An Efficient Technique for OFDM System Using Discrete Wavelet Transform", Springer-Verlag Berlin Heidelberg, **(2010)**, pp. 533–541.
[15] K. Abdullah and Z. M. Hussain, "Impulsive Noise Effects on DWT- and WPTOFDM versus FFT-OFDM", International Conference on Communication, Computer and Power (ICCCP'09), **(2009)** February 15-18.
[16] D. Gupta, T. Harris, V. B. Vats, K. K. Garg, "Performance Analysis of DFTOFDM, DCT-OFDM and DWT-OFDM Systems in AWGN Channel", The fourth ICWMC 2008, IEEE Computer Society Washington, USA, **(2008)**.
[17] A. Tarighat, M. Sadek and A. Sayed, "A multi user beamforming scheme for downlink MIMO channels based on maximizing signal-toleakage ratios", Proc. IEEE Int. Conf. on Acoustics, Speech and Signal Processing, **(2005)** March.

Authors

Md. Mahmudul Hasan has been an Asst. Professor at University of Information Technology & Sciences, Dhaka, Bangladesh since 2010 and before that he was an Asst. Professor at KIIT University since 2009. He holds a Masters of Technology degree in Electronics and Telecommunication with specialization on Communication Systems Engineering, and a Bachelor of Technology degree in Electronics and Telecommunication Engineering from KIIT University, India. He has several national and international publications in his area of expertise. His research area addresses the issues related to wireless communication and network; WLAN, multicarrier and multiuser communications like CDMA, OFDM, MIMO; Wireless sensor networks; Smart and Microstrip antenna.