# Sensitivity Analysis of Atmospheric Dispersion Defined by the RIMPUFF Model

Supahi Mahanta[1*], Rituparna Chutia[2], D. Datta[3] and Hemanta K. Baruah[1]

*[1]Department of Statistics, Gauhati University, Guwahati, Assam, India*
*[2]Department of Mathematics, Gauhati University, Guwahati, Assam, India*
*Health Physics Division, BARC, Mumbai, India*
*supahi_mahanta@rediffmail.com, Rituparnachutia7@rediffmail.com,*
*ddatta@barc.gov.in, hemanta_bh@yahoo.com*
**Corresponding Author*

***Abstract***

*This article demonstrates the sensitivity of the input parameters of atmospheric dispersion defined by the RIMPUFF model. The sensitivity analysis was carried out based on the Randomness – Fuzziness Consistency Principle. According to this principle, a normal law of fuzziness can be defined using two different laws of randomness. For the two laws of randomness defined for every normal law of fuzziness, we can therefore have a pair of correlation coefficients. This leads to an analysis of sensitivity of the input parameters of the atmospheric dispersion defined by the RIMPUFF Model with reference to various fuzzy parameters defining concentration.*

***Keywords:*** *Correlation Coefficient, Randomness-Fuzziness Consistency Principle, RIMPUFF Model, Sensitivity Analysis*

## 1. Introduction

Atmospheric dispersion is a phenomenon based on uncertainties, and in general, the concentration of pollutants observed at any given time and location downwind of a source cannot be predicted precisely [1]. Atmospheric dispersion models are mathematical expressions or algorithms relating the quantity of a pollutant released to the atmosphere to its concentration at a given location. The concentration of an air pollutant at a given place is a function of a number of variables, including emission rate, distance of the receptor from the source and atmospheric conditions. The most important atmospheric conditions are wind speed, wind direction, and the vertical temperature structure of the local atmosphere [2]. It was long recognized that sensitivity and uncertainties in atmospheric dispersion models must be studied as part of any comprehensive model performance evaluation (See e.g., [3]). In applications of the mathematics of fuzziness in particular, the objective of sensitivity analysis is to identify the most important parameter, the variation of which results in the maximum of model output.

In this article, an emphasis has been given to find the most effective parameter among the input parameters - average wind speed ($\bar{u}$), boundary layer height ($h$), shears stress or friction velocity ($u_*$) and heat flux ($w_*$) with reference to horizontal and vertical dispersion coefficients, and pollutant concentration of the atmospheric dispersion defined by the RIMPUFF model. When a random variable is a function of certain other random variables, we can use the correlation co-efficient between the dependent variable and the independent variables taken one by one to find the most highly correlated independent variable. This would reflect the variable to the changes of which the dependent variable is most sensitive.

The sensitivity analysis was carried out in the line of [4]. In [4], we considered the Gaussian Plume Model, whereas in this present article, we have considered the RIMPUFF model. The correlation co-efficient of dispersion parameters as well as pollutant concentration is computed by evaluating concentration of pollutant taking all the parameters fuzzy once and then evaluating the dispersion coefficients and concentration of pollutant taking only that parameter as fuzzy for which we are going to find the coefficient of correlation by performing computer based simulation, repeating the experiments a large number of times. Simulation in this case is based on the classical assertion that for any random variable, the probability distribution function is again randomly distributed following the uniform law of randomness. This was done for all parameters and the parameter with the numerically highest correlation coefficients with reference to the left and right reference functions was taken to be the one for changes of which dispersion parameters and pollutant concentration is most sensitive.

Baruah [5, 6, 7 and 8] has established that two laws of randomness can together define one normal law of fuzziness, and that the left reference function of a fuzzy number is a distribution function and the right reference function is a complementary distribution function. Accordingly, we can define fuzzy correlation *probabilistically*, taking the left reference functions of two fuzzy numbers to find a correlation coefficient, and the right reference functions to find another correlation coefficient. This would give a pair of correlation coefficients, which would lead to measure randomness based fuzzy correlation. This however would need simulation of data based on the laws of randomness concerned. From the sample points for the left as well as for the right reference functions obtained by simulation, the pairs of correlation coefficients were evaluated.

## 2. The RIMPUFF Model

The RIMPUFF (Riso Mesoscale Puff) model is a Lagrangian mesoscale atmospheric dispersion puff model [9, 10] designed for calculating the concentration and doses resulting from the dispersion of airborne materials. It is a local-scale puff diffusion model developed by Riso DTU (Technical University of Denmark) National Laboratory for Sustainable Energy, Denmark. It is a three-dimensional computer model used for the prediction and/or simulation of the diffusion and advection of atmospheric pollutants. The puff model technique is to simulate a plume with Gaussian shaped puffs with specified release rates within a specified grid [11]. The RIMPUFF model calculates the concentration at every grid point by summing the contributions from the surrounding puffs at each of the advection steps. The grid concentrations or doses can either accumulate or simply be updated with the latest instantaneous value calculated for average time. The model output can consist of time integrated air concentration and depositions in grid points at times specified in the input data. Updated grid concentrations $\chi(x_g, y_g, z_g)$ are evaluated at each of the grid points $(x_g, y_g, z_g)$ summing up all the contributions from the puffs in the grid. The concentration in a grid point $(x_g, y_g, z_g)$ from puff number $i$ is given by $(Bq/m^3)$:

$$\chi(x_g, y_g, z_g) = \frac{Q(i)}{(2\pi)^{\frac{3}{2}}(\sigma_{xy}(i))^2\sigma_z(i)} \exp\left[-\frac{1}{2}\left(\left(\frac{x_g - x_c(i)}{\sigma_{xy}(i)}\right)^2 + \left(\frac{y_g - y_c(i)}{\sigma_{xy}(i)}\right)^2\right)\right]$$

- (1)

$$\exp\left[-\frac{1}{2}\left(\frac{z_g - z_c(i)}{\sigma_z(i)}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{2z_{inv} - z_c(i)}{\sigma_z(i)}\right)^2\right]$$

where,

$Q(i)$ is puff inventory in puff number $(i)$

$x_c(i), y_c(i), z_c(i)$ are centre co-ordinates of puff number $(i)$.

$z_{inv}$ is height of inversion lid.

$\sigma_{xy}(i)$ $\sigma_z(i)$ are puff dispersion parameters in horizontal and vertical directions respectively and $\sigma_{xy}(i)$ $\sigma_z(i) > 0$.

In RIMPUFF, there are three parameterization schemes for the sigma parameters: Pasquill, Turbulence intensities and Similarity theory. The Pasquill-Gifford parameterization of plume spreads $\sigma_y$ and $\sigma_z$ could be replaced by the similarity scaling of atmospheric turbulence and diffusion. The basic concept is to base the calculations of plume spread on the physical parameters that govern the atmospheric boundary layer turbulence – this is parameters for heat flux $w_*$, shear stress $u_*$, the inversion height $z_{inv}$ and then from them derived Monin-Obukhov length $L$. With these the Pasquill's stability classes are replaced by the continuous non-dimensional parameter "$z/L$", where $z$ is the height above the ground or release height [10]. The standard deviation $\sigma_x$ in downwind direction is used and for simplicity $\sigma_x = \sigma_y$ is used, and this common value is marked as $\sigma_{xy}$, [12]. The spread parameter $\sigma_{xy}$ and $\sigma_z$ can be written as follows [10, 12]:

$$\left.\begin{aligned} \sigma_{xy} &= \sigma_\theta x f_y\left(\frac{x}{\bar{u}T_y}\right) \\ \sigma_z &= \sigma_e x f_z\left(\frac{x}{\bar{u}T_z}\right) \end{aligned}\right\} \quad -(2)$$

where,

$$\left.\begin{aligned} \sigma_\theta &= \frac{\sigma_v}{\bar{u}} \\ \sigma_e &= \frac{\sigma_w}{\bar{u}} \\ f_y\left(\frac{t}{T_y}\right) &= \sqrt{2}\frac{T_y}{t}\left[\frac{t}{T_y} - 1 + \exp\left(-\frac{t}{T_y}\right)\right]^{1/2} \end{aligned}\right\} \quad -(3)$$

and $t = x/\bar{u}$ is the diffusion time.

The values of $\sigma_{xy}$ and $\sigma_z$ are completely determined from $\sigma_\theta$ and $\sigma_e$, the standard deviations of the wind direction fluctuations in the horizontal and the vertical directions, respectively. But the universal function for $f_z$ is very difficult to determine since few data are available on the vertical concentration distribution. Standard deviations of wind direction fluctuations in the horizontal and the vertical directions are implemented based on the equations by Carruthers [13]:

$$\left.\begin{aligned} \sigma_u^2 &= 0.3{w_*}^2 + 6.25{T_{W_N}}^2(z)u_*^2 \\ \sigma_v^2 &= 0.3{w_*}^2 + 4.0{T_{W_N}}^2(z)u_*^2 \\ \sigma_w^2 &= \left\{0.4{T_{W_C}}^2 + \left(1.3T_{W_N}(z)\frac{u_*}{w_*}\right)^2\right\}{w_*}^2 \end{aligned}\right\} \quad -(4)$$

where

$$\left.\begin{aligned} T_{W_C}(z) &= 2.1\left(\frac{z}{h}\right)^{1/3}\left(1-0.8\frac{z}{h}\right) \\ T_{W_N}(z) &= \left(1-0.8\frac{z}{h}\right) \end{aligned}\right\} \quad -(5)$$

where,

$z$ is puff height $(m)$

$h$ is boundary layer height $(m)$

$w_*$ is heat flux $(W/m^2)$

$\bar{u}$ is average wind speed $(m/\sec)$

$u_*$ is friction velocity or Shear stress $(m/\sec)$

$T_y, T_z$ are averaging time $(\sec)$

## 3. The Input Parameters and their Fuzziness

The parameters with inherent uncertainty have been taken as triangular normal fuzzy numbers. Average wind speed, roughness length, shears stress or friction velocity, boundary layer height and heat flux are such parameters. In case of neutral condition of air, the logarithmic wind profile [14] is

$$\bar{u} = \frac{u_*}{k_a}\ln\frac{z}{z_o},\ z \ge z_0 \quad -(6)$$

Here $k_a$ is the von Karman constant. The value of this constant varies from 0.35 – 0.43 and usually the most likely value 0.40 is used. The term $z_0$ is the value of puff height $z$ at which average wind speed $\bar{u}$ vanishes. It represents turbulent eddy size at the surface and is a measure of roughness, thus termed as surface roughness length, and $u_*$ is the friction velocity.

Table 1 shown below [12] lists typical values of $z_0$ and $u_*$. Friction velocity $u_*$ (m/sec) has been taken as a normal fuzzy number < 0.23, 0.4, 0.65 > from Table 1, and so the surface roughness length $z_0$ (cm) is again a normal fuzzy number < 0.03, 1.0, 9.0 >. Thus the average wind speed $\bar{u}$ (m/sec) obtained from the equation (6) at puff height 10 m is < 2, 5, 17 >.

Boundary layer height $h$ (m) was taken as a triangular normal fuzzy number found from the equation

$$h = 0.2 \frac{u_*}{2\Omega \sin \phi} \qquad -(7)$$

where $\Omega = 7.29 \times 10^{-5}\,(\sec^{-1})$ is the rate of the earth's rotation and $\varphi$ is the latitude [12], and was found to be < 673.5, 1171.3, 1903.46 >.

Heat flux $w_* \,(W/m^2)$ was obtained from

$$w_* = 0.4(R_s - 100) \qquad -(8)$$

where $R_s$ is the solar radiation, and was found to be < 178.84, 186.12, 195.46 > for Indian atmospheric conditions.

The concentration was evaluated at downwind distance $x$ = 100 m away from the source at an averaging time 1 hour at centre co-ordinates (10, 0, 0) with inversion lid height 2000 m.

**Table 1: Values of $z_0$ and $u_*$ for Use in Vertical Wind Speed Profiles**

| Type of Surface | $z_0$ in cm | $u_*$ in m/sec |
|---|---|---|
| Smooth mud flats, ice | 0.001 | 0.16 |
| Smooth snow | 0.005 | 0.19 |
| Smooth sea | 0.02 | 0.22 |
| Level desert | 0.03 | 0.23 |
| Snow surface, lawn grass to 1.0 cm height | 0.1 | 0.26 |
| Lawn, grass to 5 cm | 1-2 | 0.38-0.43 |
| Lawn, grass to 60 cm | 4-9 | 0.51-0.65 |
| Fully grown root crops | 14 | 0.75 |
| Pasture land | 20 | 0.87 |
| Suburban housing | 60 | 1.66 |
| Forest, cities | 100 | 2.89 |

## 4. Sensitivity Analysis of the Horizontal Dispersion Co-efficient

The membership function approximated by Lagrange's interpolation of degree four and the membership curve of horizontal dispersion co-efficient ($\sigma_{xy}$) taking all the input parameters as fuzzy are found as follows

$$\mu_{\sigma_{xy}}(X) = \begin{cases} F(X), & 36.01145 \le X \le 628.4325 \\ G(X), & 628.4325 \le X \le 9009.373 \\ 0, & otherwise. \end{cases}$$

where

$$F(X)=\frac{(X-36.01145)(X-127.9303)(X-257.1932)(X-628.4325)0.25}{-203463587.4}+\frac{(X-36.01145)(X-68.1293)(X-257.1932)(X-628.4325)0.5}{355625583}$$
$$+\frac{(X-36.01145)(X-68.1293)(X-127.9303)(X-628.4325)0.75}{-2006716671}+\frac{(X-36.01145)(X-68.1293)(X-127.9303)(X-257.1932)}{61675629728},$$
$$36.01145\le X\le 628.4325$$

and

$$G(X)=\frac{(X-9009.373)(X-2438.979)(X-1304.297)(X-628.4325)0.25}{-1.178\times10^{14}}+\frac{(X-9009.373)(X-4529.177)(X-1304.297)(X-628.4325)0.5}{2.82139\times10^{13}}$$
$$+\frac{(X-9009.373)(X-4529.177)(X-2438.979)(X-628.4325)0.75}{-6.70479\times10^{12}}+\frac{(X-9009.373)(X-4529.177)(X-2438.979)(X-1304.297)}{6.93141\times10^{12}},$$
$$628.4325\le X\le 9009.373$$

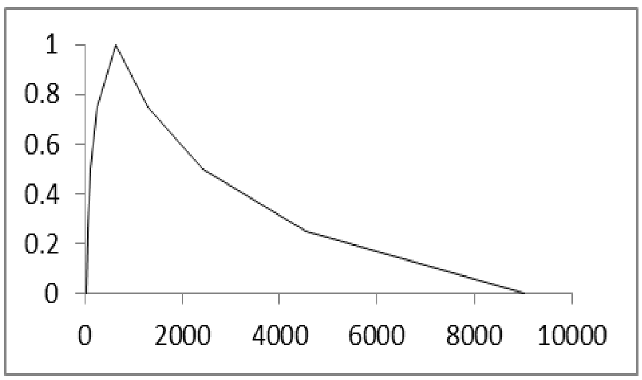


**Figure 1. Membership Curve of Horizontal Dispersion Co-efficient Taking all Input Parameter as Fuzzy**

The membership function and the membership curve of horizontal dispersion co-efficient ($\sigma_{xy}$) with the parameter boundary layer height ($h$) fuzzy are as follows

$$\mu_{\sigma_{xy(h)}}(X)=\begin{cases}\dfrac{\sqrt{(27571.40625X^2-1.08887\times10^{10})^2-4(10189.34375X^2-4024056334)(18651.40831X^2-7365951175)}-(27571.40625X^2-1.08887\times10^{10})}{2(10189.34375X^2-4024056334)}, & 628.4324509\le X\le 628.4326444\\ \dfrac{(114608.3284X^2-4.52620\times10^{10})-\sqrt{(114608.3284X^2-4.52620\times10^{10})^2-4(22041.86947X^2-8704949664)(148978.6173X^2-5.88358\times10^{10})}}{2(22041.86947X^2-8704949664)}, & 628.4326444\le X\le 628.4327455\\ 0, & otherwise\end{cases}$$

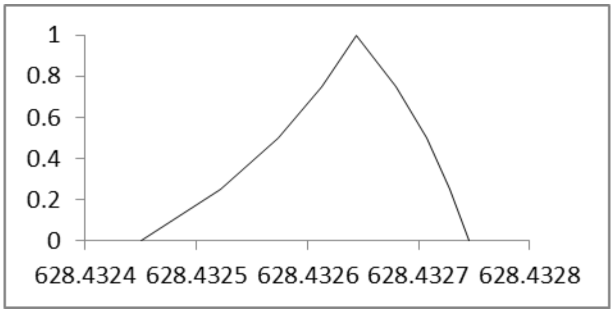


**Figure 2. Membership Curve of Horizontal Dispersion Co-efficient with Boundary Layer Height (h) Fuzzy**

Considering the parameter heat flux ($w_*$) as fuzzy and all the other input parameters as fixed the membership function and the membership curve of horizontal dispersion co-efficient are as follows

$$\mu_{\sigma_{xy(w_*)}}(X) == \begin{cases} \dfrac{-2603.9104 + \sqrt{6780349.371 - 211.9936\left(31985.84989 - \dfrac{X^2}{11.13}\right)}}{105.9968}, & 603.853201 \le X \le 628.432533 \\ \dfrac{3651.1928 - \sqrt{13331208.86 - 348.9424\left(38206.71589 - \dfrac{X^2}{11.13}\right)}}{174.4712}, & 628.432533 \le X \le 659.967091 \\ 0, & otherwise \end{cases}$$

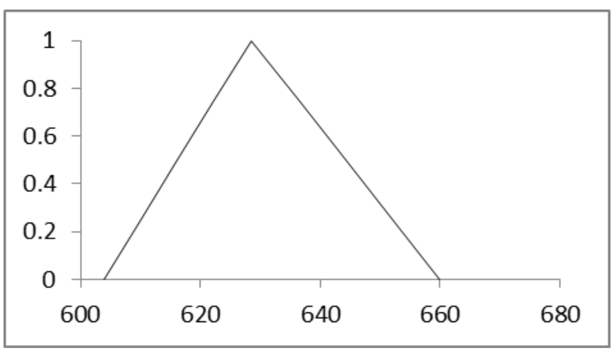


**Figure 3. Membership Curve of Horizontal Dispersion Co-efficient with Heat Flux ($w_*$) Fuzzy**

The membership function and the membership curve of horizontal dispersion coefficient with the input parameter friction velocity ($u_*$) fuzzy and all the other parameters as fixed are as follows

$$\mu_{\sigma_{xy(u_*)}}(X)=\begin{cases}\dfrac{-0.308542+\sqrt{0.095198-0.456104\left(10392.40504-\dfrac{X^2}{37.1}\right)}}{0.228052}, & 628.419757\le X\le 628.432533\\ \dfrac{1.282302-\sqrt{1.644298-0.986388\left(10393.86331-\dfrac{X^2}{37.1}\right)}}{0.493194}, & 628.432533\le X\le 628.463846\\ 0, & otherwise\end{cases}$$

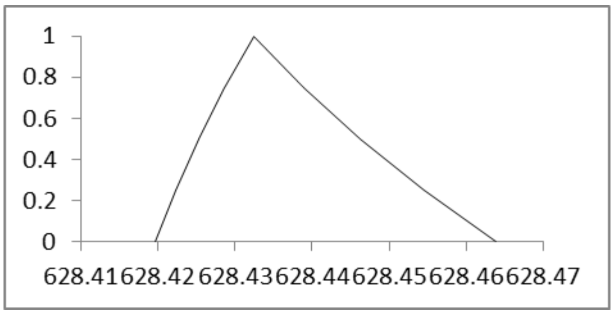


**Figure 4. Membership Curve of Horizontal Dispersion Co-efficient with Friction Velocity ( $u_*$ ) Fuzzy**

Finally, the membership function by Lagrange's approximation and the membership curve of horizontal dispersion co-efficient with the parameter wind speed ( $\bar{u}$ ) as fuzzy and all the other input parameters as non-fuzzy are as follows

$$\mu_{\mu_{\sigma_{xy(u)}}}(X)=\begin{cases}F(X), & 37.478096\le X\le 628.432536\\ G(X), & 628.432536\le X\le 8578.496582\\ 0, & otherwise.\end{cases}$$

where

$$F(X)=\frac{(X-37.478096)(X-130.483585)(X-259.734550)(X-628.432536)0.25}{-208693753.6}+\frac{(X-37.478096)(X-70.189432)(X-259.734550)(X-628.432536)0.5}{360912894}$$
$$+\frac{(X-37.478096)(X-70.189432)(X-130.483585)(X-628.432536)0.75}{-2007573921}+\frac{(X-37.478096)(X-70.189432)(X-130.483585)(X-259.734550)}{60566566150},$$
$$37.478096\le X\le 628.432536$$

and

$$G(X)=\frac{(X-8578.496582)(X-2379.233268)(X-1288.124617)(X-628.432536)0.25}{-9.61755\times10^{13}}+\frac{(X-8578.496582)(X-4364.761469)(X-1288.124617)(X-628.432536)0.5}{2.35137\times10^{13}}$$
$$+\frac{(X-8578.496582)(X-4364.761469)(X-2379.233268)(X-628.432536)0.75}{-1.61449\times10^{13}}+\frac{(X-8578.496582)(X-4364.761469)(X-2379.233268)(X-1288.124617)}{3.43079\times10^{13}},$$
$$628.432536\le X\le 8578.496582$$

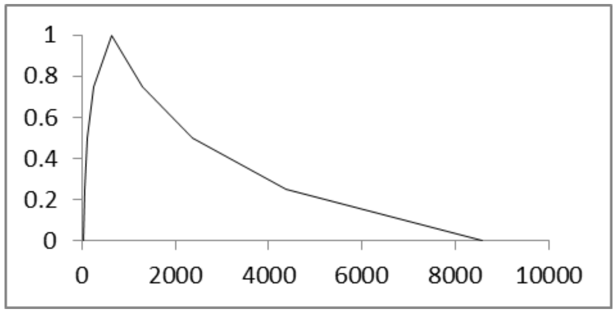


**Figure 5. Membership Curve of Horizontal Dispersion Co-efficient with Wind Speed ($\bar{u}$) Fuzzy**

Simulating 10,000 times the sample points for the left as well as for the right reference function to find correlation-coefficient with reference to horizontal dispersion coefficient are generated. The pair of correlation co-efficient is shown in Table 2.

**Table 2. Correlation Coefficients of Left and Right Distribution Functions**

| | Correlation Coefficients for the Left Distribution Function | Correlation Coefficients for the Right Distribution Function |
|---|---|---|
| Horizontal Dispersion Co-efficient with Fuzzy Boundary Layer Height | 0.851988 | 0.902736 |
| Horizontal Dispersion Co-efficient with Fuzzy Heat Flux | 0.913253 | 0.947145 |
| Horizontal Dispersion Co-efficient with Fuzzy Friction Velocity | 0.937331 | 0.964386 |
| Horizontal Dispersion Co-efficient with Fuzzy Wind Speed | 0.999995 | 0.999986 |

Table 2 reveals that the average wind speed ($\bar{u}$) is the most influential parameter followed by friction velocity ($u_*$), heat flux ($w_*$) and boundary layer height (h) with reference to the horizontal dispersion coefficient for the left as well as right distribution functions.

## 5. Sensitivity Analysis of the Vertical Dispersion Co-efficient

The membership function approximated by Lagrange's polynomial of degree four and the membership curve of vertical dispersion co-efficient ($\sigma_z$) with all the input parameters as fuzzy are as follows

$$\mu_{\sigma_z}(X) = \begin{cases} F(X), 15.000187 \le X \le 309.331873 \\ G(X), 309.331873 \le X \le 5347.145399 \\ 0, \ otherwise. \end{cases}$$

Where

$$F(X)=\frac{(X-15.000187)(X-57.406369)(X-120.517389)(X-309.331873)0.25}{-10287729.23}+\frac{(X-15.000187)(X-29.406369)(X-120.517389)(X-309.331873)0.5}{18878335}$$
$$+\frac{(X-15.000187)(X-29.406369)(X-57.406319)(X-309.331873)0.75}{-114560533}+\frac{(X-15.000187)(X-29.406369)(X-57.406319)(X-120.517389)}{3919106057},$$
$$15.000187\le X\le 309.331873$$

and

$$G(X)=\frac{(X-5347.145399)(X-1302.180813)(X-667.128194)(X-309.331873)0.25}{-1.45011\times10^{13}}+\frac{(X-5347.145399)(X-2539.253900)(X-667.128194)(X-309.331873)0.5}{3.15503\times10^{12}}$$
$$+\frac{(X-5347.145399)(X-2539.253900)(X-1302.180813)(X-309.331873)0.75}{-1.9908\times10^{12}}+\frac{(X-5347.145399)(X-2539.253900)(X-1302.180813)(X-667.128194)}{3.99072\times10^{12}},$$
$$309.331873\le X\le 5347.145399$$

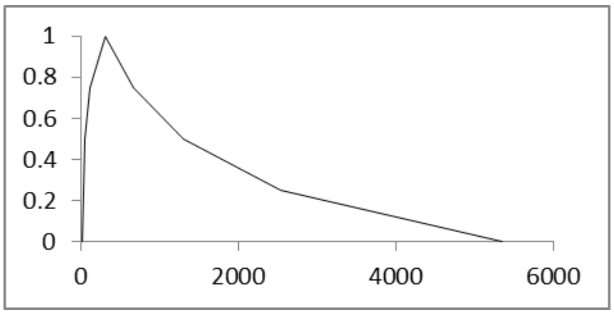


**Figure 6. Membership Curve of Vertical Dispersion Co-efficient Taking all Input Parameters as Fuzzy**

The membership function approximated by Lagrange's polynomial of degree four and the membership curve of vertical dispersion co-efficient ($\sigma_z$) with the parameter boundary layer height (h) fuzzy are as follows

$$\mu_{\sigma_{z(h)}}(X)=\begin{cases}F(X),\ 261.775180\le X\le 309.331873\\ G(X),\ 309.331873\le X\le 372.970979\\ 0,\ \ otherwise.\end{cases}$$

where

$$F(X)=\frac{(X-261.775180)(X-282.001588)(X-294.477391)(X-309.331873)0.25}{-90086.9}+\frac{(X-261.775180)(X-271.254083)(X-294.477391)(X-309.331873)0.5}{74120.62859}$$
$$+\frac{(X-261.775180)(X-271.254083)(X-282.001588)(X-309.331873)0.75}{-140743.1}+\frac{(X-261.775180)(X-271.254083)(X-282.001588)(X-294.477391)}{735165.3855},$$
$$261.775180\le X\le 309.331873$$

and

$$G(X)=\frac{(X-372.970979)(X-335.517231)(X-321.430956)(X-309.331873)0.25}{-460723.7831}+\frac{(X-372.970979)(X-352.318832)(X-321.430956)(X-309.331873)0.5}{232113.6298}$$
$$+\frac{(X-372.970979)(X-352.318832)(X-335.517231)(X-309.331873)0.75}{-271319.6743}+\frac{(X-372.970979)(X-352.318832)(X-335.517231)(X-321.430956)}{866705.879},$$
$$309.331873\le X\le 372.970979$$

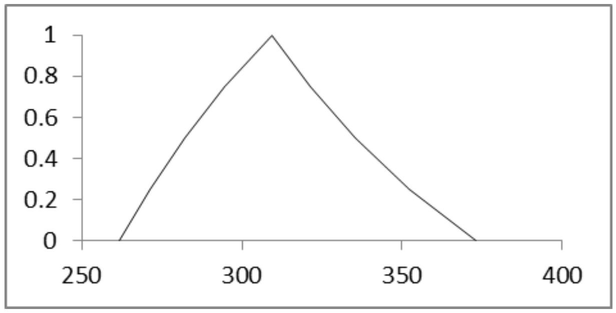


**Figure 7. Membership Curve of vertical Dispersion Co-efficient with Boundary Layer Height (h) Fuzzy**

Considering the parameter heat flux ($w_*$) as fuzzy and all the other input parameters as fixed the membership function by Lagrange's approximation and the membership curve of vertical dispersion co-efficient are as follows

$$\mu_{\mu_{\sigma_z(w_*)}}(X)=\begin{cases}F(X), & 297.230750 \le X \le 309.331585\\ G(X), & 309.331585 \le X \le 324.856101\\ 0, & otherwise.\end{cases}$$

where

$$F(X)=\frac{(X-297.230750)(X-303.281125)(X-306.306344)(X-309.331585)0.25}{-502.537}+\frac{(X-297.230750)(X-300.255928)(X-306.306344)(X-309.331585)0.5}{335.028}$$
$$+\frac{(X-297.230750)(X-300.255928)(X-303.281125)(X-309.331585)0.75}{-502.5473}+\frac{(X-297.230750)(X-300.255928)(X-303.281125)(X-306.306344)}{2010.211273},$$
$$297.230750 \le X \le 309.331585$$

and

$$G(X)=\frac{(X-324.856101)(X-317.093805)(X-313.212686)(X-309.331585)0.25}{-1361.403107}+\frac{(X-324.856101)(X-320.974943)(X-313.212686)(X-309.331585)0.5}{907.595355}$$
$$+\frac{(X-324.856101)(X-320.974943)(X-317.093805)(X-309.331585)0.75}{-1361.383146}+\frac{(X-324.856101)(X-320.974943)(X-317.093805)(X-313.212686)}{5445.493767},$$
$$309.331585 \le X \le 324.856101$$

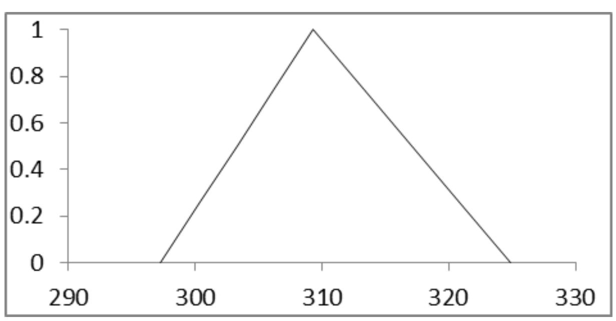


**Figure 8. Membership Curve of Vertical Dispersion Co-efficient with Heat Flux ($w_*$) Fuzzy**

The membership function and the membership curve of vertical dispersion coefficient with the input parameter friction velocity ($u_*$) fuzzy and all the other parameters as fixed are as follows

$$\mu_{\sigma_z(w_*)}(X)=\begin{cases}\dfrac{\sqrt{24.537529-7.322564(95620.00947-X^2)}-4.953537}{3.661282}, & 309.320618\le X\le 309.331584\\ \dfrac{20.587173-\sqrt{423.831692-15.836288(95643.42178-X^2)}}{7.918144}, & 309.331584\le X\le 309.358461\\ 0,\ otherwise. & \end{cases}$$

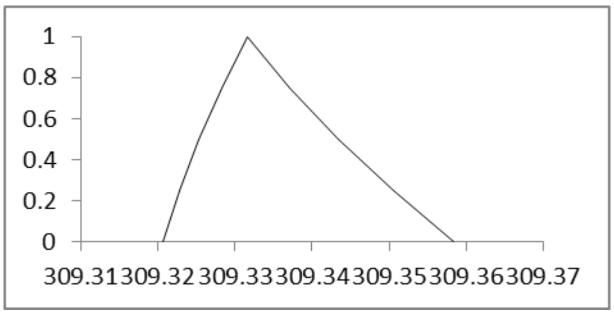


**Figure 9. Membership Curve of Vertical Dispersion Co-efficient with Friction Velocity ($u_*$) Fuzzy**

Finally, the membership function by Lagrange's approximation and the membership curve of vertical dispersion co-efficient with the parameter wind speed ($\bar{u}$) fuzzy and all the other input parameters as non-fuzzy are as follows

$$\mu_{\mu_{\sigma_z(\bar{u})}}(X)=\begin{cases}F(X), 18.447738\le X\le 309.331583\\ G(X), 309.331583\le X\le 4222.56929\\ 0,\ otherwise.\end{cases}$$

where

$$F(X)=\frac{(X-18.447738)(X-64.227569)(X-127.848409)(X-309.331583)0.25}{-12251009}+\frac{(X-18.447738)(X-34.549147)(X-127.848409)(X-309.331583)0.5}{21186772.78}$$
$$+\frac{(X-18.447738)(X-34.549147)(X-64.227569)(X-309.331583)0.75}{-117851186}+\frac{(X-18.447738)(X-34.549147)(X-64.227569)(X-127.848409)}{3555456442},$$
$$18.447738\le X\le 309.331583$$

and

$$G(X)=\frac{(X-4222.56929)(X-1171.12331)(X-634.049965)(X-309.331583)0.25}{-5.64582\times 10^{12}}+\frac{(X-4222.56929)(X-2148.45429)(X-634.049965)(X-309.331583)0.5}{1.38033\times 10^{12}}$$
$$+\frac{(X-4222.56929)(X-2148.45429)(X-1171.12331)(X-309.331583)0.75}{-9.47758\times 10^{11}}+\frac{(X-4222.56929)(X-2148.45429)(X-1171.12331)(X-634.049965)}{2.01398\times 10^{12}},$$
$$309.331583\le X\le 4222.56929$$

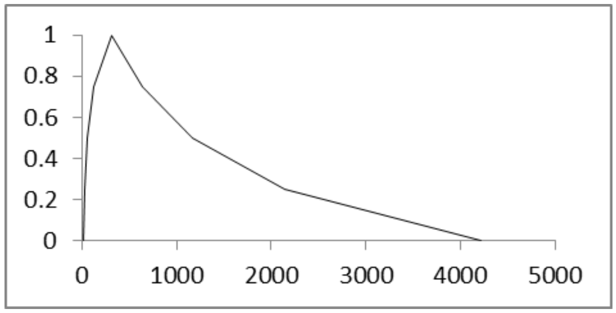


**Figure 10. Membership Curve of Vertical Dispersion Co-efficient with Wind Speed ($\bar{u}$) Fuzzy**

10,000 sample points for the left as well as for the right reference function are generated to find correlation-coefficient with reference to vertical dispersion coefficient. The pair of correlation co-efficient is shown in Table 3.

**Table 3. Correlation Coefficient of Left and Right Distribution Function**

| | Correlation Coefficients for the Left Distribution Function | Correlation Coefficients for the Right Distribution Function |
|---|---|---|
| Vertical Dispersion Co-efficient with Fuzzy Boundary Layer Height | 0.934898 | 0.966317 |
| Vertical Dispersion Co-efficient with Fuzzy Heat Flux | 0.906064 | 0.938903 |
| Vertical Dispersion Co-efficient with Fuzzy Friction Velocity | 0.931048 | 0.957408 |
| Vertical Dispersion Co-efficient with Fuzzy Wind Speed | 0.999781 | 0.999511 |

Table 3 reveals that the average wind speed ($\bar{u}$) is the most influential parameter followed by boundary layer height (h), friction velocity ($u_*$) and heat flux ($w_*$) with reference to the vertical dispersion coefficient.

## 6. Sensitivity Analysis of the Pollutant Concentration

The membership function by Lagrange's approximation and the membership curve of pollutant concentration $\chi(x_g, y_g, z_g)$ of the RIMPUFF model taking all the input parameters as fuzzy are as follows

$$\mu_{\mu_{\chi(x_g,y_g,z_g)}}(X) = \begin{cases} F(X), 5.15707\times10^{-15} \leq X \leq 5.14169\times10^{-10} \\ G(X), 5.14169\times10^{-10} \leq X \leq 0.75593901 \\ 0, \ otherwise. \end{cases}$$

where

$$F(X)=\frac{(X-5.15707\times10^{-15})(X-6.30348\times10^{-12})(X-5.24422\times10^{-11})(X-5.14169\times10^{-10})0.25}{-7.39188\times10^{-44}}$$
$$+\frac{(X-5.15707\times10^{-15})(X-4.80764\times10^{-13})(X-5.24422\times10^{-11})(X-5.14169\times10^{-10})0.5}{8.59341\times10^{-43}}$$
$$+\frac{(X-5.15707\times10^{-15})(X-4.80764\times10^{-13})(X-6.30348\times10^{-12})(X-5.14169\times10^{-10})0.75}{-5.80458\times10^{-41}}$$
$$+\frac{(X-5.15707\times10^{-15})(X-4.80764\times10^{-13})(X-6.30348\times10^{-12})(X-5.24422\times10^{-11})}{6.19349\times10^{-38}},$$
$$5.15707\times10^{-15}\le X\le5.14169\times10^{-10}$$

and

$$G(X)=\frac{(X-0.75593901)(X-0.008934117)(X-2.35586\times10^{-8})(X-5.14169\times10^{-10})0.25}{-0.010938142}$$
$$+\frac{(X-0.75593901)(X-0.28917314)(X-2.35586\times10^{-8})(X-5.14169\times10^{-10})0.5}{1.67091\times10^{-05}}$$
$$+\frac{(X-0.75593901)(X-0.28917314)(X-0.008934117)(X-5.14169\times10^{-10})0.75}{-4.50051\times10^{-11}}$$
$$+\frac{(X-0.75593901)(X-0.28917314)(X-0.008934117)(X-2.35586\times10^{-8})}{4.50052\times10^{-11}},$$
$$5.14169\times10^{-10}\le X\le0.75593901$$

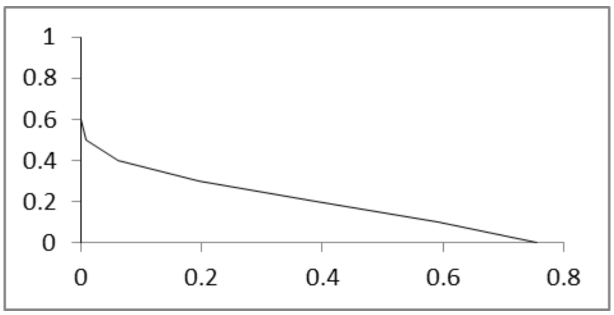


**Figure 11. Membership Curve of Pollutant Concentration Taking all Input Parameter as Fuzzy**

The approximated membership function by Lagrange's interpolation and the membership curve of pollutant concentration under the parameter boundary layer height ($h$) fuzzy are as follows

$$\mu_{\chi(h)}(X)=\begin{cases}F(X), & 4.26349\times10^{-10}\le X\le5.14169\times10^{-10}\\ G(X), & 5.14169\times10^{-10}\le X\le6.07678\times10^{-10}\\ 0, & otherwise.\end{cases}$$

where

$$F(X)=\frac{(X-4.26349\times10^{-10})(X-4.7399\times10^{-10})(X-4.94788\times10^{-10})(X-5.14169\times10^{-10})0.25}{-1.54365\times10^{-42}}$$
$$+\frac{(X-4.26349\times10^{-10})(X-4.51363\times10^{-10})(X-4.94788\times10^{-10})(X-5.14169\times10^{-10})0.5}{9.00785\times10^{-43}}$$
$$+\frac{(X-4.26349\times10^{-10})(X-4.51363\times10^{-10})(X-4.7399\times10^{-10})(X-5.14169\times10^{-10})0.75}{-1.19793\times10^{-42}}$$
$$+\frac{(X-4.26349\times10^{-10})(X-4.51363\times10^{-10})(X-4.7399\times10^{-10})(X-4.94788\times10^{-10})}{4.29494\times10^{-42}},$$
$$4.26349\times10^{-10}\le X\le5.14169\times10^{-10}$$

and

$$G(X)=\frac{\left(X-6.07678\times10^{-10}\right)\left(X-5.64044\times10^{-10}\right)\left(X-5.40126\times10^{-10}\right)\left(X-5.14169\times10^{-10}\right)0.25}{-1.59078\times10^{-42}}$$
$$+\frac{\left(X-6.07678\times10^{-10}\right)\left(X-5.86417\times10^{-10}\right)\left(X-5.40126\times10^{-10}\right)\left(X-5.14169\times10^{-10}\right)0.5}{1.16451\times10^{-42}}$$
$$+\frac{\left(X-6.07678\times10^{-10}\right)\left(X-5.86417\times10^{-10}\right)\left(X-5.64044\times10^{-10}\right)\left(X-5.14169\times10^{-10}\right)0.75}{-1.94136\times10^{-42}}$$
$$+\frac{\left(X-6.07678\times10^{-10}\right)\left(X-5.86417\times10^{-10}\right)\left(X-5.64044\times10^{-10}\right)\left(X-5.40126\times10^{-10}\right)}{8.74626\times10^{-42}},$$
$$5.14169\times10^{-10}\le X\le 6.07678\times10^{-10}$$

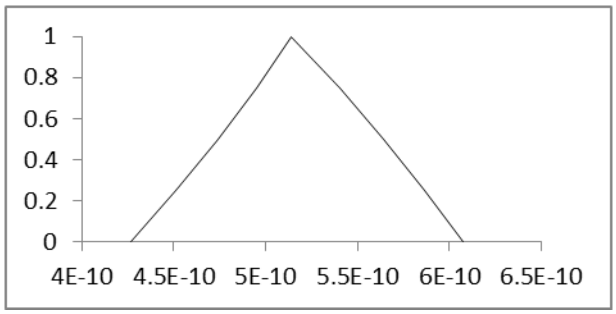


**Figure 12. Membership Curve of Pollutant Concentration with Boundary Layer Height (*h*) Fuzzy**

Considering the parameter heat flux ($w_*$) as fuzzy and all the other input parameters as fixed the membership function by Lagrange's approximation and the membership curve of pollutant concentration are obtained as follows

$$\mu_{\chi(w_*)}(X)=\begin{cases}F(X), & 4.43531\times10^{-10}\le X\le 5.14169\times10^{-10}\\ G(X), & 5.14169\times10^{-10}\le X\le 5.80134\times10^{-10}\\ 0, & otherwise.\end{cases}$$

where

$$F(X)=\frac{\left(X-4.43531\times10^{-10}\right)\left(X-4.77123\times10^{-10}\right)\left(X-4.95188\times10^{-10}\right)\left(X-5.14169\times10^{-10}\right)0.25}{-5.39415\times10^{-43}}$$
$$+\frac{\left(X-4.43531\times10^{-10}\right)\left(X-4.59921\times10^{-10}\right)\left(X-4.95188\times10^{-10}\right)\left(X-5.14169\times10^{-10}\right)0.5}{3.8672\times10^{-43}}$$
$$+\frac{\left(X-4.43531\times10^{-10}\right)\left(X-4.59921\times10^{-10}\right)\left(X-4.77123\times10^{-10}\right)\left(X-5.14169\times10^{-10}\right)0.75}{-6.24685\times10^{-43}}$$
$$+\frac{\left(X-4.43531\times10^{-10}\right)\left(X-4.59921\times10^{-10}\right)\left(X-4.77123\times10^{-10}\right)\left(X-4.95188\times10^{-10}\right)}{2.69476\times10^{-42}},$$
$$4.43531\times10^{-10}\le X\le 5.14169\times10^{-10}$$

and

$$G(X) = \frac{(X-5.80134\times10^{-10})(X-5.45841\times10^{-10})(X-5.29694\times10^{-10})(X-5.14169\times10^{-10})0.25}{-4.69375\times10^{-43}}$$
$$+\frac{(X-5.80134\times10^{-10})(X-5.62643\times10^{-10})(X-5.29694\times10^{-10})(X-5.14169\times10^{-10})0.5}{2.94665\times10^{-43}}$$
$$+\frac{(X-5.80134\times10^{-10})(X-5.62643\times10^{-10})(X-5.45841\times10^{-10})(X-5.14169\times10^{-10})0.75}{-4.16605\times10^{-43}}$$
$$+\frac{(X-5.80134\times10^{-10})(X-5.62643\times10^{-10})(X-5.45841\times10^{-10})(X-5.29694\times10^{-10})0.75}{-4.16605\times10^{-43}},$$
$$5.14169\times10^{-10} \le X \le 5.80134\times10^{-10}$$

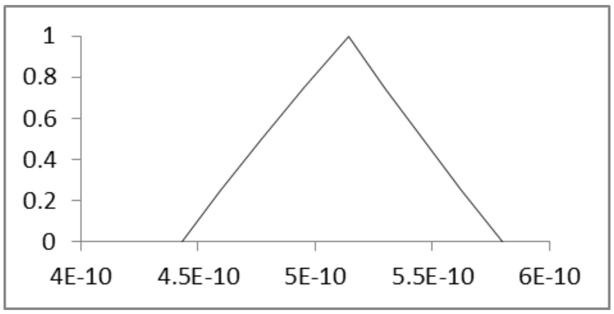


**Figure 13. Membership Curve of Pollutant Concentration with Heat Flux ($w_*$) Fuzzy**

The membership function by Lagrange's polynomial approximation and the membership curve of pollutant concentration under the input parameter friction velocity ($u_*$) fuzzy and all the other parameters as fixed are as follows

$$\mu_{\chi(u_*)}(X) = \begin{cases} F(X), & 5.14074\times10^{-10} \le X \le 5.14169\times10^{-10} \\ G(X), & 5.14169\times10^{-10} \le X \le 5.14209\times10^{-10} \\ 0, & otherwise. \end{cases}$$

where

$$F(X) = \frac{(X-5.14074\times10^{-10})(X-5.14127\times10^{-10})(X-5.1415\times10^{-10})(X-5.14169\times10^{-10})0.25}{-2.34893\times10^{-54}}$$
$$+\frac{(X-5.14074\times10^{-10})(X-5.14102\times10^{-10})(X-5.1415\times10^{-10})(X-5.14169\times10^{-10})0.5}{1.31125\times10^{-54}}$$
$$+\frac{(X-5.14074\times10^{-10})(X-5.14102\times10^{-10})(X-5.14127\times10^{-10})(X-5.14169\times10^{-10})0.75}{-1.64066\times10^{-54}}$$
$$+\frac{(X-5.14074\times10^{-10})(X-5.14102\times10^{-10})(X-5.14127\times10^{-10})(X-5.1415\times10^{-10})}{5.46251\times10^{-54}},$$
$$5.14074\times10^{-10} \le X \le 5.14169\times10^{-10}$$

and

$$G(X)=\frac{(X-5.14209\times10^{-10})(X-5.14192\times10^{-10})(X-5.14182\times10^{-10})(X-5.14169\times10^{-10})0.25}{-4.65976\times10^{-56}}$$
$$+\frac{(X-5.14209\times10^{-10})(X-5.14201\times10^{-10})(X-5.14182\times10^{-10})(X-5.14169\times10^{-10})0.5}{3.80129\times10^{-56}}$$
$$+\frac{(X-5.14209\times10^{-10})(X-5.14201\times10^{-10})(X-5.14192\times10^{-10})(X-5.14169\times10^{-10})0.75}{-6.9439\times10^{-56}}$$
$$+\frac{(X-5.14209\times10^{-10})(X-5.14201\times10^{-10})(X-5.14192\times10^{-10})(X-5.14182\times10^{-10})}{3.36266\times10^{-55}},$$
$$5.14169\times10^{-10}\le X\le5.14209\times10^{-10}$$

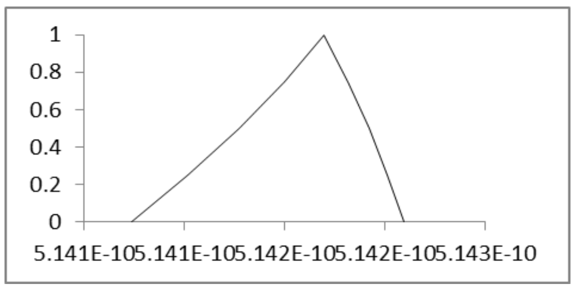


**Figure 14. Membership Curve of Pollutant Concentration with Friction Velocity ($u_*$) Fuzzy**

Finally, the membership function by Lagrange's approximation and the membership curve of pollutant concentration taking the parameter wind speed ($\bar{u}$) as fuzzy and all the other input parameters as non-fuzzy are found to be as follows

$$\mu_{\chi(\bar{u})}(X)=\begin{cases}F(X), & 9.86943\times10^{-15}\le X\le5.14169\times10^{-10}\\ G(X), & 5.14169\times10^{-10}\le X\le0.638473021\\ 0, & otherwise.\end{cases}$$

where

$$F(X)=\frac{(X-9.86943\times10^{-15})(X-7.45902\times10^{-12})(X-5.66616\times10^{-11})(X-5.14169\times10^{-10})0.25}{-1.26038\times10^{-43}}$$
$$+\frac{(X-9.86943\times10^{-15})(X-6.53833\times10^{-13})(X-5.66616\times10^{-11})(X-5.14169\times10^{-10})0.5}{1.26385\times10^{-42}}$$
$$+\frac{(X-9.86943\times10^{-15})(X-6.53833\times10^{-13})(X-7.45902\times10^{-12})(X-5.14169\times10^{-10})0.75}{-7.14245\times10^{-41}}$$
$$+\frac{(X-9.86943\times10^{-15})(X-6.53833\times10^{-13})(X-7.45902\times10^{-12})(X-5.66616\times10^{-11})}{6.12084\times10^{-38}},$$
$$9.86943\times10^{-15}\le X\le5.14169\times10^{-10}$$

and

$$G(X)=\frac{(X-0.638473021)(X-0.002929619)(X-9.62175\times10^{-9})(X-5.14169\times10^{-10})0.25}{-0.002506376}$$
$$+\frac{(X-0.638473021)(X-0.176726087)(X-9.62175\times10^{-9})(X-5.14169\times10^{-10})0.5}{9.47997\times10^{-7}}$$
$$+\frac{(X-0.638473021)(X-0.176726087)(X-0.002929619)(X-5.14169\times10^{-10})0.75}{-3.01062\times10^{-12}}$$
$$+\frac{(X-0.638473021)(X-0.176726087)(X-0.002929619)(X-9.62175\times10^{-9})}{3.01063\times10^{-12}},$$
$$5.14169\times10^{-15}\le X\le 0.638473021$$

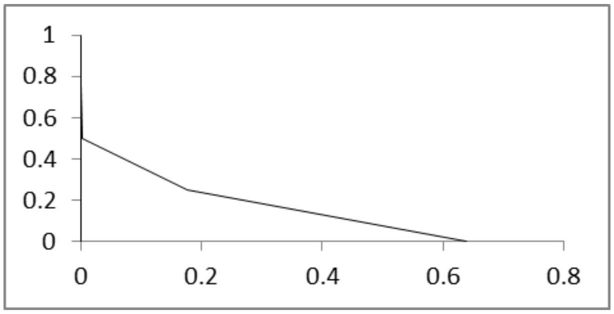


**Figure 15. Membership Curve of Pollutant Concentration with Wind Speed ($\bar{u}$) Fuzzy**

10,000 sample points for the left as well as for the right reference function are generated to find correlation-coefficient with reference to pollutant concentration. The pair of correlation co-efficient is shown in Table 4.

**Table 4. Correlation Coefficient of Left and Right Distribution Function**

| | Correlation Coefficients for the Left Distribution Function | Correlation Coefficients for the Right Distribution Function |
|---|---|---|
| Pollutant Concentration under Fuzzy Boundary Layer Height | 0.706507 | 0.822375 |
| Pollutant Concentration under Fuzzy Heat Flux | 0.744785 | 0.851729 |
| Pollutant Concentration under Fuzzy Friction Velocity | 0.700074 | 0.801028 |
| Pollutant Concentration under Fuzzy Wind Speed | 0.999917 | 0.993305 |

Table 4 reveals that the average wind speed ($\bar{u}$) is the most influential parameter followed by heat flux ($w_*$), boundary layer height (h) and friction velocity ($u_*$) with reference to the pollutant concentration defined by the RIMPUFF model for both left as well as right reference functions.

## 7. Conclusion

Here the approach in which a normal law of fuzziness is defined in terms of two independent laws of randomness has been utilised to analyze the effect of boundary layer height, heat flux, friction velocity and wind speed on pollutant concentration defined by the RIMPUFF model. For the horizontal dispersion coefficient, average wind speed is most highly correlated with reference to concentration of pollutant whereas boundary layer height has less influence on the downwind concentration. In the case of vertical dispersion also, wind speed is the most sensitive parameter while heat flux is the least sensitive parameter towards pollutant concentration. It has been found that among all the input parameters, average wind speed is the most sensitive parameter which influences the pollutant concentration defined by the RIMPUFF model along the downwind direction. Boundary layer height and heat flux are less sensitive towards downwind concentration of the model. Friction velocity has been found to have much less influence on the downwind concentration of the model.

## Acknowledgements

This work was carried out under a research project approved by the Board of Research in Nuclear Sciences, Bhabha Atomic Research Centre, Mumbai, sponsored by the Department of Atomic Energy, Government of India.

## References

[1] P. C. Chatwin, "The Use of Statistics in Describing and Predicting the Effects of Dispersing Gas Clouds", J. Hazard Mater, vol. 6, **(1982)**, pp. 213-230**.**
[2] A. A. A. Rahman, "On the Atmospheric Dispersion and Gaussian Plume Model", 2nd International Conference on Waste Management, Water Pollution, Air Pollution, Indoor Climate (WWAI'08), Corfu, Greece, **(2008)** October 26-28.
[3] D. G. Fox, "Uncertainty in Air Quality Modeling", Bull. Amer. Meteor Soc., vol. 65, **(1984)**, pp. 27-36**.**
[4] S. Mahanta, R. Chutia, D. Datta and H. Baruah, "Sensitivity Analysis with reference to Emission Concentration of Gaussian Plume Model", International Journal of Energy, Information and Communications, vol. 3, no. 2, **(2012)**, pp. 45-52**.**
[5] H. K. Baruah, "The Randomness – Fuzziness Consistency Principle", International Journal of Energy, Information and Communications, vol. 1, Issue 1, **(2010)**, pp. 37-48**.**
[6] H. K. Baruah, "The Theory of Fuzzy Sets: Beliefs and Realities", International Journal of Energy Information and Communications, vol. 2, no. 2, **(2011).**
[7] H. K. Baruah, "Construction of the Membership Function of a Fuzzy Number", ICIC Express Letters, vol. 5, Issue 2, **(2011)**, pp. 545-549**.**
[8] H. K. Baruah, "An Introduction to the Theory of Imprecise Sets: The Mathematics of Partial Presence", Journal of Mathematics and Computer Science, vol. 2, no. 2, **(2012)**, pp. 110-124**.**
[9] T. Mikkelsen, S. E. Larsen and S. Thykier-Nielsen, "Description of the Riso Puff Diffusion Model", Nuclear Technology, vol. 67, **(1984)**, pp. 56-65.
[10] S. Thykier-Nielsen, S. Deme and T. Mikkelsen, "Description of the Atmospheric Dispersion Module RIMPUFF", Report RODOS (WG2)-TN (98)-02, **(1999)**.
[11] T. Mikkelsen, "Riso-M-2361, Description of the Riso Puff Diffusion Model", Riso National Laboratory, DK 4000 Roskilde, Denmark, **(1982).**
[12] R. Meyers, "Encyclopaedia of Physical Science and Technology", Eighteen-Volume Set, Third Edition, Academic Press, California, U.S.A., **(2001)**.
[13] D. J. Carruthers, "Atmospheric Dispersion Modelling System (UK-ADMS) in Air Pollution Modelling and its Application IX", eds. H.vanDop and G. kallos, Plenum Press, New York, **(1992).**
[14] V. V. Shirvaikar and V. J. Daoo, "Air Pollution Meteorology", Bhabha Atomic Research Centre, Mumbai, India, **(2002).**

# Authors

**Supahi Mahanta**, is a Senior Research Fellow in the Department of Statistics, Gauhati University, Guwahati, India. She has completed her M.Sc in Statistics from Dibrugarh University, Dibrugarh, India in the year 2009. She is currently working on a project under Board of Research in Nuclear Sciences, Bhabha Atomic Research Centre, Mumbai, sponsored by the Department of Atomic Energy, Government of India.

**Rituparna Chutia**, is a Senior Research Fellow in the Department of Mathematics, Gauhati University, Guwahati, India. He has completed his M.Sc in Mathematics from the said university in the year 2009. He is currently working on a project under Board of Research in Nuclear Sciences, Bhabha Atomic Research Centre, Mumbai, sponsored by the Department of Atomic Energy, Government of India.

**D. Datta,** is scientist of Bhabha Atomic Research Centre, the Department of Atomic Energy, India. He is the recipient of "Millennium Plaques of Honour", award of Eminent Scientist. He is the author of more than 100 science papers and developer of 15 software for use in radiological safety science and engineering. His interests and competence field is soft computing, statistical and mathematical modeling, reliability, sensitivity and uncertainty analysis of systems. He is a life member of the Indian Science Congress, Indian Association of Radiation Protection, National Society of Radiation Physics, Indian Nuclear Society.

**Hemanta K. Baruah**, a Reviewer in the Mathematical Reviews, USA, received his M.Sc. Degree in Statistics from Gauhati University, Guwahati, India, in 1975, and his Ph.D. degree in Mathematics from Indian Institute of Technology, Kharagpur, India, in 1980. He is a Professor in the Department of Statistics, Gauhati University, since 1995. The Randomness – Fuzziness Consistency Principle used in this article has been theorized by him. His research interests are in mathematics of fuzziness, graph theory, data mining and mathematical modeling.