Sensitivity Analysis with Reference to Emission Concentration of Gaussian Plume Model

Supahi Mahanta[1*], Rituparna Chutia[2], D. Datta[3] and Hemanta K. Baruah[1]

[1]*Department of Statistics, Gauhati University, Guwahati, Assam, India*
[2]*Department of Mathematics, Gauhati University, Guwahati, Assam, India*
[3]*Health Physics Division, BARC, Mumbai, India*
supahi_mahanta@rediffmail.com, Rituparnachutia7@rediffmail.com, ddatta@barc.gov.in, hemanta_bh@yahoo.com
** Corresponding Author (supahi_mahanta@rediffmail.com)*

Abstract

In this article, a randomness based approach of analyzing sensitivity of a fuzzy variable with reference to independent fuzzy variables based on the randomness-fuzziness consistency principle has been explained. The randomness-fuzziness consistency principle leads to defining a normal law of fuzziness using two different laws of randomness. For the two laws of randomness defined for every normal law of fuzziness, we can therefore have a pair of Correlation Coefficients. This leads to an analysis of sensitivity of the parameter of the Gaussian Plume Model with reference to various fuzzy parameters defining concentration.

Keywords: *Sensitivity Analysis, Randomness-Fuzziness Consistency Principle, Correlation Coefficient, Gaussian Plume Model*

1. Introduction

Air pollution models are routinely used in environmental impact assessments, risk analysis, emergency planning and source apportionment studies. Atmospheric dispersion is a phenomenon based on uncertainties, and in general, the concentration of pollutants observed at a given time and location downwind of a source cannot be predicted precisely [1]. Uncertainty here refers to lack of knowledge or information about an unknown quantity whose true value could be established if a perfect measurement device were available. Concentration would be a random variable, if the variables defining it are assumed to be random [2, 3]. In that case, statistical analysis of the data using the theory of probability would be enough. On the other hand, if the variables defining concentration are fuzzy in nature, then it would have to be studied using the mathematics of fuzziness.

It was long recognized that sensitivity and uncertainties in atmospheric dispersion models must be studied as part of any comprehensive model performance evaluation (See e.g., [4]). In applications of the mathematics of fuzziness in particular, the objective of sensitivity analysis is to identify the most important parameter, the variation of which results in the maximum of model output.

For carrying out sensitivity and uncertainty analysis of atmospheric dispersion the most commonly used model is the Standard Gaussian Plume Model. Gaussian Plume Model [5] is the most widely used method of estimating downwind concentration of airborne material released to the atmosphere. Sutton [6] derived an air pollutant plume dispersion equation which included the assumption of Gaussian distribution for the vertical and crosswind dispersion of the plume and also included the effect of ground reflection of the plume. Input parameters of the standard Gaussian Plume model considered are: wind speed, source height, horizontal and vertical standard deviations, quantity of the contaminant and stability category of weather.

The Gaussian plume model that provides the time integrated air concentration at any downwind distance is given by

$$C(x,y,z)=\frac{Q}{2\pi\sigma_y\sigma_z u}\exp\left(-\frac{y^2}{2\sigma_y{}^2}\right)\times\left\{\exp\left(-\frac{(z-h)^2}{2\sigma_z{}^2}\right)+\exp\left(-\frac{(z+h)^2}{2\sigma_z{}^2}\right)\right\} \quad - \quad (1)$$

where C(x, y, z) is the concentration of the emission (in micrograms per cubic metre) at any point x metres downwind of the source, y metres crosswind from the emission plume centreline and z metres above ground level, Q is the quantity or mass of the emission (in grams) per unit of time (seconds), u is the average wind speed (in metres per second), σ_z is vertical standard deviation of the emission distribution (in metre), σ_y is horizontal standard deviation of the emission distribution (in metre), h is the effective height of the source above ground level (in metres).

The values of horizontal and vertical dispersion co-efficient (σ_y and σ_z) here can be seen to be

$$\sigma_y = a_y x^{0.9071}\ ,\ \sigma_z = a_z x^{b_z} + c_z \quad - \quad (2)$$

where the co-efficient a_y, a_z, b_z and c_z can be obtained from the table of parameters for Pasquill-Gifford σ_y and σ_z [7]. Based on the temperature gradient, atmospheric conditions are categorized into six classes (A-F), so called the Pasquill stability classes.

The methodology used here is based on a result linking fuzziness with randomness. The existence of two laws of randomness is required to define a law of fuzziness [8, 9, 10, 11, 12, 13, 14]. The principle states that the left reference function of any normal fuzzy number is actually a distribution function, and that the right reference function is actually a complementary distribution function, for which however one needs to look into the matters through application of the Glivenko-Centelli theorem of Order Statistics on superimposed uniformly fuzzy intervals.

2. Sensitivity Analysis of the Model

Here we are going to find out using non-realistic data, which of the parameters among wind speed (u), effective stack height (h) and quantity of emission (Q) is the most effective one with reference to concentration of emission of the Gaussian Plume model at the ground level $C(x,y,0)$. When a random variable is a function of certain other random variables, we can use the correlation co-efficient between the dependent variable and the independent variables taken one by one to find the most highly correlated independent variable. This would reflect the variable to the changes of which the dependent variable is most sensitive.

The correlation co-efficient of emission concentration is computed by evaluating concentration of emission taking all the parameters fuzzy once and then evaluating the concentration of emission taking only that parameter as fuzzy for which we are going to find the coefficient of correlation by performing computer based simulation, repeating the experiments a large number of times. Simulation in this case is based on the classical assertion that for any random variable, the probability distribution function is again randomly distributed following the uniform law of randomness. This is done for all parameters and the parameter with the numerically highest correlation coefficient is taken to be the one for changes of which emission concentration is most sensitive.

We have obtained the sensitivity analysis to find the most sensitive one of the input parameters of the Gaussian plume model *C*(*x, y, 0*) for the extremely unstable atmospheric condition (category A), distance along the downwind direction for $x = 1600m$ and along the crosswind direction for $y = 2m$ under the super-adiabatic condition of plume rise due to Moses - Carson equation [15]. We have considered 5% uncertainty for fuzzifying the Eimuties and Konicek parameters [7]. Input data: Quantity of emission (Q) = TFN [*100, 500, 1000*] *gm/sec*, Average wind speed (u) = TFN [*2, 4, 6*] *m/sec*, Stack gas exit speed (Vs) = TFN (*1.2, 3.4, 6.3*) *m/sec*, Stack diameter (d) = *5m*, Stack heat emission rate (Q_h) = TFN (*100, 500, 1000*) C_i and Physical stack height (H) = *100m*.

3. Membership Functions with reference to Concentration of Emission

The membership function of $C(x,y,0)$ has been found by using Lagrangian polynomial on discretized values of the α-cuts of $C(x,y,0)$. We had to do this because here in this case the method of α-cuts as well as the alternative method ([16] and [17]) would fail to supply the results for the reason that we here have a non-invertible function. So the only alternative is to use a method of interpolation. This would give us the membership function approximated by a Lagrangian polynomial. To make the matters simple, we have considered Lagrangian polynomial of degree four.

The membership function and curve for Concentration of emission *C*(*x, y, 0*) taking all the input parameters as fuzzy has found to be

$$\mu_{C(x,y,0)}(X) = \begin{cases} F(X), \ 0.000005626 \le X \le 0.000109906 \\ G(X), \ 0.000109906 \le X \le 0.001063421 \\ 0, \ otherwise. \end{cases} \tag{3}$$

where

$$\begin{aligned} F(X) = & \frac{(X-0.000005626)(X-0.000031339)(X-0.000058788)(X-0.000109906)0.25}{-6.386283997\times10^{-19}} \\ & + \frac{(X-0.000005626)(X-0.000015358)(X-0.000058788)(X-0.000109906)0.50}{8.861829681\times10^{-19}} \\ & + \frac{(X-0.000005626)(X-0.000015358)(X-0.000031339)(X-0.000109906)0.75}{-3.239600977\times10^{-18}} \\ & + \frac{(X-0.000005626)(X-0.000015358)(X-0.000031339)(X-0.000058788)1}{3.959746586\times10^{-17}}, \\ & 0.000005626 \le X \le 0.000109906 \end{aligned}$$

and

$$\begin{aligned} G(X) = & \frac{(X-0.001063421)(X-0.000317637)(X-0.000186548)(X-0.000109906)0.25}{-2.051676691\times10^{-14}} \\ & + \frac{(X-0.001063421)(X-0.000560057)(X-0.000186548)(X-0.000109906)0.50}{4.923194348\times10^{-15}} \\ & + \frac{(X-0.001063421)(X-0.000560057)(X-0.000317637)(X-0.000109906)0.75}{-3.290524903\times10^{-15}} \\ & + \frac{(X-0.001063421)(X-0.000560057)(X-0.000317637)(X-0.00001865548)1}{6.833523815\times10^{-15}}, \\ & 0.000109906 \le X \le 0.0001063421 \end{aligned}$$

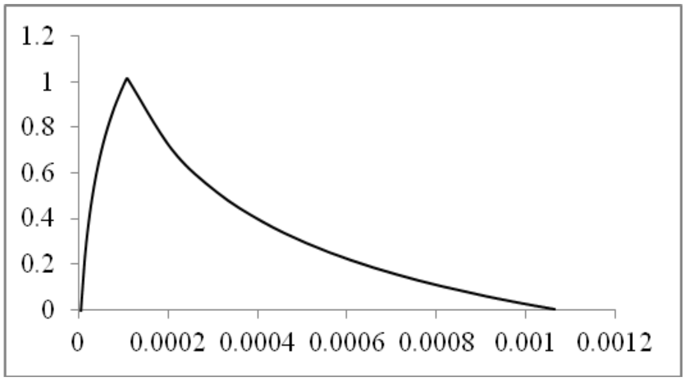

Figure 1. The Membership Curve of Concentration under all the Input Parameters Fuzzy

By Lagrange's inverse interpolation we can determine the functions in terms of the distribution functions. Simulating 10,000 times the sample points for the left as well as for the right reference functions, were found out.

The membership function and curve for the concentration of emission of the Gaussian plume model *C*(*x*,*y*,*0*) taking only the wind speed parameter (*u*) as fuzzy and keeping the other parameters as constant are shown as below

$$\mu_{C(x,y,0)}(X) = \begin{cases} \frac{6X-0.00043963}{2X}, 0.00007327 \leq X \leq 0.00010991 \\ \frac{0.00043963-2X}{2X}, 0.00010991 \leq X \leq 0.00021982 \\ 0, \quad otherwise \end{cases} \quad -$$

(4)

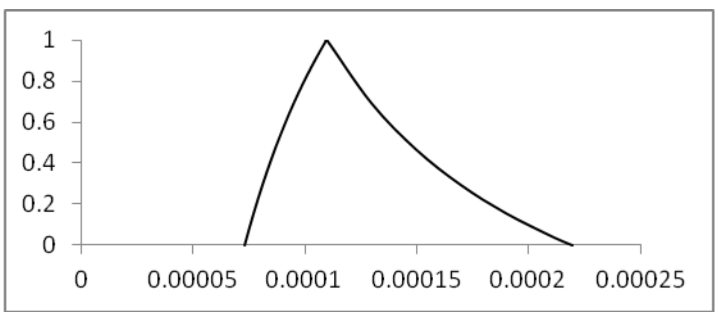

Figure 2. Membership Curve of Concentration under the Parameter Wind Speed (*u*) Fuzzy

By simulating 10,000 times the sample points for the left as well as for the right reference functions were found out.

The membership function and curve for the concentration of emission of the Gaussian Plume Model *C*(*x*,*y*,*0*) taking only the parameter quantity of emission (*Q*) as fuzzy and keeping all the other input parameters as constant are as shown below

$$\mu_{C(x,y,0)}(X) = \begin{cases} \dfrac{\frac{X}{2.1982\times10^{-7}}-100}{400}, 0.00002198 \le X \le 0.00010991 \\ \dfrac{1000-\frac{X}{2.1982\times10^{-7}}}{500}, 0.00010991 \le X \le 0.00021982 \\ 0, \quad otherwise \end{cases} \quad - (5)$$

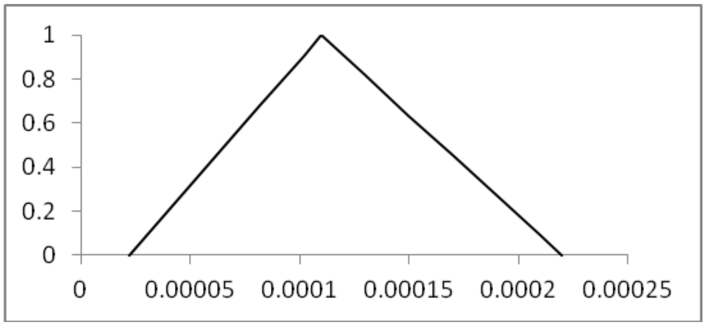

Figure 3. Membership Curve of Concentration under the Parameter Quantity of Emission (*Q*) Fuzzy

We have then evaluated the sample points by simulating 10,000 times for the left as well as for the right reference functions.

The membership function and curve for the concentration of emission of the Gaussian Plume Model $C(x,y,0)$ taking only the parameter effective stack height (h) as fuzzy and keeping all the other input parameters as non-fuzzy are as given below

$$\mu_{C(x,y,0)}(x) = \begin{cases} F(X), \ 0.00010862 \le X \le 0.00010991 \\ G(X), \ 0.00010991 \le X \le 0.00011021 \\ 0, \ otherwise. \end{cases} \quad - (6)$$

where

$$F(X) = \frac{(X-0.00010862)(X-0.00010954)(X-0.00010975)(X-0.00010991)0.25}{-7.70066\times10^{-26}} + \frac{(X-0.00010862)(X-0.00010920)(X-0.00010975)(X-0.00010991)0.50}{2.43046\times10^{-26}} + \frac{(X-0.00010862)(X-0.00010920)(X-0.00010954)(X-0.00010991)0.75}{-2.08824\times10^{-26}} + \frac{(X-0.00010862)(X-0.00010920)(X-0.00010954)(X-0.00010975)1}{5.42213\times10^{-26}},$$
$$0.00010862 \le X \le 0.00010991$$

and

$$G(X) = \frac{(X-0.00011021)(X-0.00011008)(X-0.00011001)(X-0.00010991)0.25}{-1.4112\times10^{-28}} + \frac{(X-0.00011021)(X-0.00011015)(X-0.00011001)(X-0.00010991)0.50}{1.0829\times10^{-28}} + \frac{(X-0.00011021)(X-0.00011015)(X-0.00011008)(X-0.00010991)0.75}{-1.96\times10^{-28}} + \frac{(X-0.00011021)(X-0.00011015)(X-0.00011008)(X-0.00011001)1}{1.224\times10^{-27}},$$
$$0.00010991 \le X \le 0.00011021$$

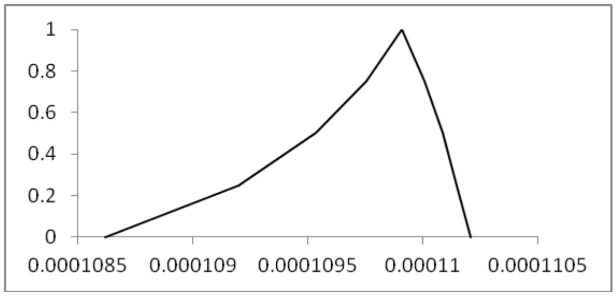

Figure 4. Membership Curve of Concentration under the Parameter Effective Stack Height (*h*) Fuzzy

We have found out the function in terms of the distribution function by the method of Lagrange's Inverse Interpolation and then simulating 10,000 times we generated the sample points for the left as well as for the right reference function to find correlation-coefficient with reference to emission concentration.

4. Correlation Co-efficient with reference to Concentration of Emission

In the literature of fuzziness, fuzzy correlation has been defined in a number of ways [18]. Indeed, correlation coefficient is defined with reference to two random variables. However, the term *fuzzy correlation* as defined in the literature on fuzziness considers the membership functions of fuzzy numbers. In fact, taking the membership function as equivalent to a density function leads to such definitions of fuzzy correlation.

Baruah ([9, 10 and 11]) has established that two laws of randomness can together define one normal law of fuzziness, and that the left reference function of a fuzzy number is a distribution function and the right reference function is a complementary distribution function. Accordingly, we can define fuzzy correlation *probabilistically*, taking the left reference functions of two fuzzy numbers to find a correlation coefficient, and the right reference functions to find another correlation coefficient. This would give a pair of correlation coefficients, which would lead to measure randomness based fuzzy correlation. This however would need simulation of data based on the probability laws concerned. From the sample points for the left as well as for the right reference function obtained by simulation, the pair correlation coefficients were evaluated.

The pair of correlation coefficients of the emission concentration $C(x, y, 0)$ under all the input parameters as fuzzy and emission concentration $C(x, y, 0)$ considering only the *wind speed* parameter, u as fuzzy for the left reference function and the right reference function are found to be 0.9841 and 0.9909 respectively.

The correlation coefficient of emission concentration $C(x, y, 0)$ under all the input parameters fuzzy and emission concentration $C(x, y, 0)$ considering only the *quantity of emission*, Q as fuzzy for the left reference function and the right reference function are found to be 0.9604 and 0.9513 respectively.

Finally, the correlation coefficient of emission concentration $C(x, y, 0)$ under all the input parameters fuzzy and emission concentration $C(x, y, 0)$ considering only the *stack height*, h as fuzzy for the left reference function and the right reference function are found to be 0.8742 and 0.9317 respectively.

It is evident that the correlation coefficients are highest for emission concentration when all the parameters are fuzzy and for emission concentration when only the parameter *wind speed* is fuzzy for both the left and the right reference functions. Hence we conclude that wind speed is the most sensitive of the three parameters effecting concentration. We have found that quantity of emission is the second most effective parameter with reference to emission concentration. Finally, we can conclude that stack height is the least sensitive among the three parameters - wind speed, quantity of emission and stack height effecting concentration of emission.

In fact, an empirical analysis is enough to see that as far as concentration is concerned, the shapes of the membership curve of concentration taking all the input parameters as fuzzy and the membership curve of concentration taking only the parameter wind speed as fuzzy are very nearly the same. The membership curve of concentration taking only the parameter effective stack height as fuzzy is very different from the membership curve of concentration taking all the input parameter as fuzzy. Concentration of emission of the Gaussian Plume model is inversely proportional to wind speed, directly proportional to quantity of emission, and it is negative exponential function of the square of effective stack height. Accordingly, the figures above give an idea that wind speed is the most effective parameter, and that effective stack height is the least effective parameter.

5. Conclusions

It could be seen that concentration of emission is most sensitive towards changes in the parameter wind speed. The next important parameter in this regard has been found to be the quantity of emission. Effective stack height is the least important of the three parameters. Even for realistic data this pattern would not change, because the shapes of the membership functions concerned would not actually be any different if realistic data would be used in place of the non-realistic data.

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Authors

Supahi Mahanta, is a Senior Research Fellow in the Department of Statistics, Gauhati University, Guwahati, India. She has completed her M.Sc in Statistics from Dibrugarh University, Dibrugarh, India in the year 2009. She is currently working on a project under Board of Research in Nuclear Sciences, Bhabha Atomic Research Centre, Mumbai, sponsored by the Department of Atomic Energy, Government of India.

Rituparna Chutia, is a Senior Research Fellow in the Department of Mathematics, Gauhati University, Guwahati, India. He has completed his M.Sc in Mathematics from the said university in the year 2009. He is currently working on a project under Board of Research in Nuclear Sciences, Bhabha Atomic Research Centre, Mumbai, sponsored by the Department of Atomic Energy, Government of India.

D. Datta, is scientist of Bhabha Atomic Research Centre, the Department of Atomic Energy, India. He is the recipient of "Millennium Plaques of Honour", award of Eminent Scientist. He is the author of more than 100 science papers and developer of 15 software for use in radiological safety science and engineering. His interests and competence field is soft computing, statistical and mathematical modeling, reliability, sensitivity and uncertainty analysis of systems. He is a life member of the Indian Science Congress, Indian Association of Radiation Protection, National Society of Radiation Physics, Indian Nuclear Society.

Hemanta K. Baruah, a Reviewer in the Mathematical Reviews, USA, received his M.Sc. Degree in Statistics from Gauhati University, Guwahati, India, in 1975, and his Ph.D. degree in Mathematics from Indian Institute of Technology, Kharagpur, India, in 1980. He is a Professor in the Department of Statistics, Gauhati University, since 1995. The Randomness – Fuzziness Consistency Principle used in this article has been theorized by him. His research interests are in mathematics of fuzziness, graph theory, data mining and mathematical modeling.