An Extended Approach to Generalized Fuzzy Soft Sets

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Abstract

The purpose of this paper is to restructure the notion of generalized fuzzy soft sets in the light of extended notion of fuzzy sets initiated by Baruah. We have verified the results on generalized fuzzy soft sets with examples and counter examples and some new results have been put forward in our work. Finally, we have put forward an extended notion regarding similarity between two fuzzy soft sets and similarity between two generalized fuzzy soft sets.

Keywords: *Soft set, Fuzzy soft set, Generalized fuzzy soft set, Similarity between two generalized fuzzy soft sets*

1. Introduction

Zadeh [5] initiated the concept of fuzzy sets in 1965 which is considered as generalization of classical or crisp sets. In the Zadehian definition, it has been accepted that the classical set theoretic axioms of exclusion and contradiction are not satisfied. In this regard, Baruah [2,3] proposed that two functions, namely fuzzy membership function and fuzzy reference function are necessary to represent a fuzzy set. Accordingly, Baruah [2, 3] reintroduced the notion of complement of a fuzzy set in a way that the set theoretic axioms of exclusion and contradiction can be seen valid for fuzzy sets also.

In 1999, Molodtsov [1] introduced the novel concept of soft sets, which is a new mathematical approach to vagueness. In recent years the researchers have contributed a lot towards fuzzification of soft set theory. Maji et al. [6] put forward the concept of fuzzy soft sets, which is a hybrid model of fuzzy sets and soft sets. Recently, Neog et al. [10] have studied the theory of fuzzy soft sets from a new perspective and put forward a new notion regarding complement of a fuzzy soft set. While doing so, fuzzy sets have been replaced by extended fuzzy sets initiated by Baruah [2, 3]. In 2010, Majumder and Samanta [7] gave a more generalized form of fuzzy soft sets, known as generalized fuzzy soft sets, by attaching a degree with the parameterization of fuzzy sets. These results were further studied by Yang [4] and some modifications were forwarded.

In this article, an attempt has been made to apply the extended definition of fuzzy set in the context of generalized fuzzy soft set.

2. Preliminaries

In this section, we first recall some concepts and definitions which would be needed in the sequel.

In [2], Baruah put forward an extended definition of fuzzy sets and with the help of this extended definition, he put forward the notion of union and intersection of two fuzzy sets in the following way -

2.1. Extended Definition of Union and Intersection of Fuzzy Sets

Let $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in U\}$ and $B(\mu_3, \mu_4) = \{x, \mu_3(x), \mu_4(x); x \in U\}$ be two fuzzy sets defined over the same universe *U*. Then the operations intersection and union are defined as

$$A(\mu_1, \mu_2) \cap B(\mu_3, \mu_4) = \{x, \min(\mu_1(x), \mu_3(x)), \max(\mu_2(x), \mu_4(x)); x \in U\}$$

and $A(\mu_1, \mu_2) \cup B(\mu_3, \mu_4) = \{x, \max(\mu_1(x), \mu_3(x)), \min(\mu_2(x), \mu_4(x)); x \in U\}$.

Neog et al. [9] showed by an example that this definition sometimes gives degenerate cases and revised the above definition as follows -

2.2. Extended Definition of Union and Intersection of Fuzzy Sets Revised

Let $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in U\}$ and $B(\mu_3, \mu_4) = \{x, \mu_3(x), \mu_4(x); x \in U\}$ be two fuzzy sets defined over the same universe *U*. To avoid degenerate cases we assume that $\min(\mu_1(x), \mu_3(x)) \geq \max(\mu_2(x), \mu_4(x)) \forall x \in U$. Then the operations intersection and union are defined as

$$A(\mu_1, \mu_2) \cap B(\mu_3, \mu_4) = \{x, \min(\mu_1(x), \mu_3(x)), \max(\mu_2(x), \mu_4(x)); x \in U\}$$

and $A(\mu_1, \mu_2) \cup B(\mu_3, \mu_4) = \{x, \max(\mu_1(x), \mu_3(x)), \min(\mu_2(x), \mu_4(x)); x \in U\}$.

In [2], Baruah put forward the notion of complement of usual fuzzy sets with fuzzy reference function 0 in the following way –

2.3. Complement of a Fuzzy Set Using Extended Definition

For usual fuzzy sets $A(\mu, 0) = \{x, \mu(x), 0; x \in U\}$ and $B(1, \mu) = \{x, 1, \mu(x); x \in U\}$ defined over the same universe *U*, we have

$$A(\mu, 0) \cap B(1, \mu) = \{x, \min(\mu(x), 1), \max(0, \mu(x)); x \in U\}$$

$= \{x, \mu(x), \mu(x); x \in U\}$, which is nothing but the null set φ.

and $A(\mu, 0) \cup B(1, \mu) = \{x, \max(\mu(x), 1), \min(0, \mu(x)); x \in U\}$

$= \{x, 1, 0; x \in U\}$, which is nothing but the universal set *U*.

This means if we define a fuzzy set $(A(\mu, 0))^c = \{x, 1, \mu(x); x \in U\}$, it is nothing but the complement of $A(\mu, 0) = \{x, \mu(x), 0; x \in U\}$.

Neog et al. [9] put forward the notion of fuzzy subset using the extended notion of fuzzy sets in the following manner –

2.4. Extended Definition of Fuzzy Subset

Let $A(\mu_1,\mu_2)=\{x,\mu_1(x),\mu_2(x);x\in U\}$ and $B(\mu_3,\mu_4)=\{x,\mu_3(x),\mu_4(x);x\in U\}$ be two fuzzy sets defined over the same universe *U*. The fuzzy set $A(\mu_1,\mu_2)$ is a subset of the fuzzy set $B(\mu_3,\mu_4)$ if $\forall x\in U$, $\mu_1(x)\le\mu_3(x)$ and $\mu_4(x)\le\mu_2(x)$.

Two fuzzy sets $C=\{x,\mu_C(x):x\in U\}$ and $D=\{x,\mu_D(x):x\in U\}$ in the usual definition would be expressed as $C(\mu_C,0)=\{x,\mu_C(x),0;x\in U\}$ and $D(\mu_D,0)=\{x,\mu_D(x),0;x\in U\}$.

Accordingly, we have $C(\mu_C,0)\subseteq D(\mu_D,0)$ if $\forall x\in U$, $\mu_C(x)\le\mu_D(x)$, Which can be obtained by putting $\mu_2(x)=\mu_4(x)=0$ in our new definition.

Molodtsov [1] defined soft set in the following way –

2.5. Soft Set

A pair (*F*, *E*) is called a soft set (over *U*) if and only if *F* is a mapping of *E* into the set of all subsets of the set *U*.

In other words, the soft set is a parameterized family of subsets of the set *U*. Every set $F(\varepsilon),\varepsilon\in E$, from this family may be considered as the set of ε - elements of the soft set (*F*, *E*), or as the set of ε - approximate elements of the soft set.

The following definition of fuzzy soft set is due to Maji et al. [6]

2.6. Fuzzy Soft Set

A pair (*F*, *A*) is called a fuzzy soft set over *U* where $F:A\to\tilde{P}(U)$ is a mapping from *A* into $\tilde{P}(U)$.

Mazumder and Samanta [7] put forward the notions related to generalized fuzzy soft sets as follows:

2.7. Generalized Fuzzy Soft Set

Let $U=\{x_1,x_2,x_3,............,x_n\}$ be the universal set of elements and $E=\{e_1,e_2,e_3,.....e_m\}$ be the universal set of parameters. The pair (*U*, *E*) will be called a soft universe. Let $F:E\to I^U$ and μ be a fuzzy subset of *E*, *i.e.* $\mu:E\to I=[0,1]$, where I^U is the collection of all fuzzy subsets of *U*. Let F_μ be the mapping $F_\mu:E\to I^U\times I$ defined as follows: $F_\mu(e)=(F(e),\mu(e))$, where $F(e)\in I^U$. Then F_μ is called generalized fuzzy soft sets over the soft universe (*U*, *E*). Here for each parameter e_i, $F_\mu(e_i)=(F(e_i),\mu(e_i))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$.

2.8. Generalized Fuzzy Soft Subset

Let F_μ and G_δ be two GFSS over (U, E). Now F_μ is said to be a generalized fuzzy soft subset of G_δ if

(i) μ is a fuzzy subset of δ (ii) $F(e)$ is also a fuzzy subset of $G(e)\ \forall e \in E$.

In this case, we write $F_\mu \tilde{\subseteq} G_\delta$

2.9. Complement of a Generalized Fuzzy Soft Set

Let F_μ be a GFSS over (U, E). Then the complement of F_μ, denoted by ${F_\mu}^c$ and is defined by ${F_\mu}^c = G_\delta$, where $\delta(e) = \mu^c(e)$ and $G(e) = F^c(e),\ \forall e \in E$.

2.10. Union of Generalized Fuzzy Soft Sets

The union of two GFSS F_λ and G_μ over (U, E) is denoted by $F_\lambda \tilde{\cup} G_\mu$ and defined by GFSS $H_\delta : E \to I^U \times I$ such that for each $e \in E$, $H_\delta(e) = (F(e) \lozenge G(e), \lambda(e) \lozenge \mu(e))$

2.11. Intersection of Generalized Fuzzy Soft Sets

The intersection of two GFSS F_λ and G_μ over (U, E) is denoted by $F_\lambda \tilde{\cap} G_\mu$ and defined by GFSS $H_\delta : E \to I^U \times I$ such that for each $e \in E$, $H_\delta(e) = (F(e) * G(e), \lambda(e) * \mu(e))$

2.12. Generalized Null Fuzzy Soft Set

A GFSS is said to be a generalized null fuzzy soft set, denoted by θ_ϕ, if $\theta_\phi : E \to I^U \times I$ such that
$\theta_\phi(e) = (F(e), \phi(e))$, where $F(e) = \bar{0}, \phi(e) = 0, \forall e \in E$

2.13. Generalized Absolute Fuzzy Soft Set

A GFSS is said to be a generalized absolute fuzzy soft set, denoted by $\tilde{A}_\lambda$, if $\tilde{A}_\lambda : E \to I^U \times I$ such that $\theta_\phi(e) = (F(e), \lambda(e))$, where $F(e) = \bar{1}, \lambda(e) = 1, \forall e \in E$.

3. Extended Notion of Generalized Fuzzy Soft Sets

3.1. Extended Definition of Generalized Fuzzy Soft Set

Let $U = \{x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_m\}$ be the universal set of elements and $E = \{e_1, e_2, e_3, \ldots\ldots e_n\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to I^U$ and μ be a fuzzy subset of E, *i.e.* $\mu = \{(e, \mu_1(e), \mu_2(e)) : e \in E\}$, where I^U is the collection of all fuzzy subsets of U. Let F_μ be the mapping $F_\mu : E \to I^U \times \mu$ defined as follows : $F_\mu(e)=\left(F(e),\mu(e)\right)$, where $F(e)\in I^U$. Then F_μ is called generalized fuzzy soft sets over the soft universe (*U*, *E*). Here for each parameter e_i, $F_\mu(e_i)=\left(F(e_i),\mu(e_i)\right)$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness.

3.2. Generalized Fuzzy Soft Sub Set

Let F_μ and G_δ be two GFSS over (*U*, *E*), where
$\mu=\{(e,\mu_1(e),\mu_2(e)):e\in E\},\delta=\{(e,\delta_1(e),\delta_2(e)):e\in E\}$ and

$$\begin{aligned}F_\mu(e) &=\left(F(e),\mu(e)\right)\\ &=\left(\{(x,\xi_1(x),\xi_2(x)):x\in U\},(e,\mu_1(e),\mu_2(e))\right),\\ G_\delta(e) &=\left(G(e),\delta(e)\right)\\ &=\left(\{(x,\psi_1(x),\psi_2(x)):x\in U\},(e,\delta_1(e),\delta_2(e))\right)\end{aligned}$$

, where $F(e),G(e)\in I^U$ and μ,δ are fuzzy sets over *E*. Then F_μ is called generalized fuzzy soft sub set of G_δ , denoted by $F_\mu\,\tilde{\subseteq}\,G_\delta$, if the following conditions hold:

(*i*) μ is a fuzzy subset of δ *i.e.* $\forall e\in E,\mu_1(e)\le\delta_1(e),\delta_2(e)\le\mu_2(e)$

(*ii*) $F(e)$ is also a fuzzy subset of $G(e)\ \forall e\in E$ *i.e.*
$\forall e\in E$ and $\forall x\in U,\xi_1(x)\le\psi_1(x),\psi_2(x)\le\xi_2(x)$

3.3. Complement of Generalized Fuzzy Soft Set

Let F_μ be a GFSS over (*U*, *E*), where $\mu=\{(e,\mu_1(e),\mu_2(e)):e\in E\}$ and

$$\begin{aligned}F_\mu(e) &=\left(F(e),\mu(e)\right)\\ &=\left(\{(x,\xi_1(x),\xi_2(x)):x\in U\},(e,\mu_1(e),\mu_2(e))\right)\end{aligned}$$

, where $F(e)\in I^U$ and μ is a fuzzy set over *E*. For usual fuzzy sets, we would take $\xi_2(x)=0\ \forall x\in U$ and $\mu_2(e)=0\ \forall e\in E$. Then complement of

$F_\mu(e)\ =\left(\{(x,\xi_1(x),0):x\in U\},(e,\mu_1(e),0)\right)$ is denoted by $F^c{}_\mu$ and is defined by $F^c{}_\mu=G_\delta$, where

$$G_\delta(e)\ =\left(F^c(e),\mu^c(e)\right)=\left(\{(x,1,\xi_1(x)):x\in U\},(e,1,\mu_1(e))\right)\ \forall x\in U \text{ and } \forall e\in E.$$

3.4. Union of Two Generalized Fuzzy Soft Sets

Let F_μ and G_δ be two GFSS over (*U*, *E*), where
$\mu=\{(e,\mu_1(e),\mu_2(e)):e\in E\},\delta=\{(e,\delta_1(e),\delta_2(e)):e\in E\}$ and

$$\begin{aligned}F_\mu(e) &=\left(F(e),\mu(e)\right)\\ &=\left(\{(x,\xi_1(x),\xi_2(x)):x\in U\},(e,\mu_1(e),\mu_2(e))\right),\\ G_\delta(e) &=\left(G(e),\delta(e)\right)\\ &=\left(\{(x,\psi_1(x),\psi_2(x)):x\in U\},(e,\delta_1(e),\delta_2(e))\right)\end{aligned}$$

, where $F(e),G(e)\in I^U$ and μ,δ are fuzzy sets over *E*. Then the union of F_μ and G_δ is denoted by $F_\mu\,\tilde{\cup}\,G_\delta=H_\nu$ and is defined as

$$H_\nu(e)\ =\left(H(e),\nu(e)\right)$$

$$=\left(\left\{\left(x,\max\left(\xi_1(x),\psi_1(x)\right),\min\left(\xi_2(x),\psi_2(x)\right)\right):x\in U\right\},\left(e,\max\left(\mu_1(e),\delta_1(e)\right),\min\left(\mu_2(e),\delta_2(e)\right)\right)\right)$$

3.5. Intersection of Two Generalized Fuzzy Soft Sets

Let F_μ and G_δ be two GFSS over (*U*, *E*), where
$\mu=\{(e,\mu_1(e),\mu_2(e)):e\in E\},\delta=\{(e,\delta_1(e),\delta_2(e)):e\in E\}$ and

$$F_\mu(e)=\left(F(e),\mu(e)\right)=\left(\left\{\left(x,\xi_1(x),\xi_2(x)\right):x\in U\right\},\left(e,\mu_1(e),\mu_2(e)\right)\right),$$

$$G_\delta(e)=\left(G(e),\delta(e)\right)=\left(\left\{\left(x,\psi_1(x),\psi_2(x)\right):x\in U\right\},\left(e,\delta_1(e),\delta_2(e)\right)\right),$$

where $F(e),G(e)\in I^U$ and μ,δ are fuzzy sets over *E*. Then the intersection of F_μ and G_δ is denoted by $F_\mu\,\tilde{\cap}\,G_\delta=H_\nu$ and is defined as

$$H_\nu(e)=\left(H(e),\nu(e)\right)=\left(\left\{\left(x,\min\left(\xi_1(x),\psi_1(x)\right),\max\left(\xi_2(x),\psi_2(x)\right)\right):x\in U\right\},\left(e,\min\left(\mu_1(e),\delta_1(e)\right),\max\left(\mu_2(e),\delta_2(e)\right)\right)\right)$$

3.6. Generalized Null Fuzzy Soft Set

Let F_θ be a GFSS over (*U*, *E*), where $\theta=\{(e,\theta(e),\theta(e)):e\in E\}$ and

$$F_\theta(e)=\left(F(e),\theta(e)\right)=\left(\left\{\left(x,\xi(x),\xi(x)\right):x\in U\right\},\left(e,\theta(e),\theta(e)\right)\right),$$

where $\forall e\in E,F(e)\in I^U$ is a null fuzzy set over *U* and θ is a null fuzzy set over *E*. Then F_θ is said to be generalized null fuzzy soft set and is denoted by $\tilde{\varphi}_\theta$.

3.7. Generalized Absolute Fuzzy Soft Set

Let $F_{\bar{1}}$ be a GFSS over (*U*, *E*), where $\bar{1}=\{(e,1,0):e\in E\}$ and

$$F_{\bar{1}}(e)=\left(F(e),\bar{1}(e)\right)=\left(\left\{\left(x,1,0\right):x\in U\right\},\left(e,1,0\right)\right),$$

where $\forall e\in E,F(e)\in I^U$ is the absolute fuzzy set over *U* and $\bar{1}$ is the absolute fuzzy set over *E*. Then $F_{\bar{1}}$ is said to be generalized absolute fuzzy soft set and is denoted by $\tilde{\Lambda}_{\bar{1}}$.

3.8. Proposition

Let F_μ be a GFSS over (*U*, *E*), where $\mu=\{(e,\mu_1(e),\mu_2(e)):e\in E\}$ and

$$F_\mu(e)=\left(F(e),\mu(e)\right)=\left(\left\{\left(x,\xi_1(x),\xi_2(x)\right):x\in U\right\},\left(e,\mu_1(e),\mu_2(e)\right)\right),$$

where $F(e)\in I^U$ and μ is a fuzzy set over *E*. Then the following results are valid.

(*i*) $F_\mu=F_\mu\,\tilde{\cup}\,F_\mu$

(*ii*) $F_\mu=F_\mu\,\tilde{\cap}\,F_\mu$

(*iii*) $F_\mu \tilde{\cup} \tilde{\varphi}_\theta = F_\mu$

(*iv*) $F_\mu \tilde{\cap} \tilde{\varphi}_\theta = \tilde{\varphi}_\theta$

(*v*) $F_\mu \tilde{\cup} \tilde{\Lambda}_{\bar{1}} = \tilde{\Lambda}_{\bar{1}}$

(*vi*) $F_\mu \tilde{\cap} \tilde{\Lambda}_{\bar{1}} = F_\mu$

Proof:

(*i*) We have,

$$F_\mu(e) = (F(e), \mu(e))$$
$$= (\{(x, \xi_1(x), \xi_2(x)) : x \in U\}, (e, \mu_1(e), \mu_2(e)))$$

Let $F_\mu \tilde{\cup} F_\mu = H_\nu$

$$H_\nu(e) = (H(e), \nu(e))$$
$$= (\{(x, \max(\xi_1(x), \xi_1(x)), \min(\xi_2(x), \xi_2(x))) : x \in U\}, (e, \max(\mu_1(e), \mu_1(e)), \min(\mu_2(e), \mu_2(e))))$$
$$= (\{(x, \xi_1(x), \xi_2(x)) : x \in U\}, (e, \mu_1(e), \mu_2(e)))$$

Thus $F_\mu = F_\mu \tilde{\cup} F_\mu$

(*ii*) We have

$$F_\mu(e) = (F(e), \mu(e))$$
$$= (\{(x, \xi_1(x), \xi_2(x)) : x \in U\}, (e, \mu_1(e), \mu_2(e)))$$

Let $F_\mu \tilde{\cap} F_\mu = H_\nu$

$$H_\nu(e) = (H(e), \nu(e))$$
$$= (\{(x, \min(\xi_1(x), \xi_1(x)), \max(\xi_2(x), \xi_2(x))) : x \in U\}, (e, \min(\mu_1(e), \mu_1(e)), \max(\mu_2(e), \mu_2(e))))$$
$$= (\{(x, \xi_1(x), \xi_2(x)) : x \in U\}, (e, \mu_1(e), \mu_2(e)))$$

Thus $F_\mu = F_\mu \tilde{\cap} F_\mu$

(*iii*) We have,

$$F_\mu(e) = (F(e), \mu(e))$$
$$= (\{(x, \xi_1(x), \xi_2(x)) : x \in U\}, (e, \mu_1(e), \mu_2(e)))$$

Also,

$$\tilde{\varphi}_\theta = (\{(x, \xi_2(x), \xi_2(x)) : x \in U\}, (e, \mu_2(e), \mu_2(e)))$$

Let $F_\mu \tilde{\cup} \tilde{\varphi}_\theta = H_\nu$

$$H_\nu(e) = (H(e), \nu(e))$$
$$= (\{(x, \max(\xi_1(x), \xi_2(x)), \min(\xi_2(x), \xi_2(x))) : x \in U\}, (e, \max(\mu_1(e), \mu_2(e)), \min(\mu_2(e), \mu_2(e))))$$
$$= (\{(x, \xi_1(x), \xi_2(x)) : x \in U\}, (e, \mu_1(e), \mu_2(e)))$$

Thus $F_\mu \tilde{\cup} \tilde{\varphi}_\theta = F_\mu$

(*iv*) We have,

$$F_\mu(e) = (F(e), \mu(e))$$
$$= (\{(x, \xi_1(x), \xi_2(x)) : x \in U\}, (e, \mu_1(e), \mu_2(e)))$$

Also,

$\tilde{\varphi}_\theta \quad = (\{(x, \xi_2(x), \xi_2(x)) : x \in U\}, (e, \mu_2(e), \mu_2(e)))$

Let $\quad F_\mu \tilde{\cap} \tilde{\varphi}_\theta \quad = H_\nu$

$$
\begin{aligned}
H_\nu(e) &= (H(e), \nu(e)) \\
&= (\{(x, \min(\xi_1(x), \xi_2(x)), \max(\xi_2(x), \xi_2(x))) : x \in U\}, (e, \min(\mu_1(e), \mu_2(e)), \max(\mu_2(e), \mu_2(e)))) \\
&= (\{(x, \xi_2(x), \xi_2(x)) : x \in U\}, (e, \mu_2(e), \mu_2(e)))
\end{aligned}
$$

Thus $F_\mu \tilde{\cap} \tilde{\varphi}_\theta = \tilde{\varphi}_\theta$

(*v*)We have

$$
\begin{aligned}
F_\mu(e) &= (F(e), \mu(e)) \\
&= (\{(x, \xi_1(x), \xi_2(x)) : x \in U\}, (e, \mu_1(e), \mu_2(e)))
\end{aligned}
$$

Also,

$\tilde{\Lambda}_{\bar{1}} \quad = (\{(x, 1, 0) : x \in U\}, (e, 1, 0))$

Let $\quad F_\mu \tilde{\cup} \tilde{\Lambda}_{\bar{1}} \quad = H_\nu$

$$
\begin{aligned}
H_\nu(e) &= (H(e), \nu(e)) \\
&= (\{(x, \max(\xi_1(x), 1), \min(\xi_2(x), 0)) : x \in U\}, (e, \max(\mu_1(e), 1), \min(\mu_2(e), 0))) \\
&= (\{(x, 1, 0) : x \in U\}, (e, 1, 0))
\end{aligned}
$$

Thus $F_\mu \tilde{\cup} \tilde{\Lambda}_{\bar{1}} = \tilde{\Lambda}_{\bar{1}}$

(*vi*) We have

$$
\begin{aligned}
F_\mu(e) &= (F(e), \mu(e)) \\
&= (\{(x, \xi_1(x), \xi_2(x)) : x \in U\}, (e, \mu_1(e), \mu_2(e)))
\end{aligned}
$$

Also,

$\tilde{\Lambda}_{\bar{1}} \quad = (\{(x, 1, 0) : x \in U\}, (e, 1, 0))$

Let $\quad F_\mu \tilde{\cap} \tilde{\Lambda}_{\bar{1}} \quad = H_\nu$

$$
\begin{aligned}
H_\nu(e) &= (H(e), \nu(e)) \\
&= (\{(x, \min(\xi_1(x), 1), \max(\xi_2(x), 0)) : x \in U\}, (e, \min(\mu_1(e), 1), \max(\mu_2(e), 0))) \\
&= (\{(x, \xi_1(x), \xi_2(x)) : x \in U\}, (e, \mu_1(e), \mu_2(e)))
\end{aligned}
$$

Thus $F_\mu \tilde{\cap} \tilde{\Lambda}_{\bar{1}} = F_\mu$

Majumder and Samanta [7] put forward that the law of excluded middle and the law of contradiction hold in case of generalized fuzzy soft sets, whereas Yang [4] established with the help of a counter example that these laws are not valid for generalized fuzzy soft sets. However, in our way, we have the following proposition.

3.9. Proposition

Let F_μ be a GFSS over (*U*, *E*), where $\mu = \{(e, \mu_1(e), 0) : e \in E\}$ and

$F_\mu(e) \quad = (F(e), \mu(e))$

$=\left(\{(x,\xi_1(x),0):x\in U\},(e,\mu_1(e),0)\right)$, where $F(e)\in I^U$ and μ is a fuzzy set over *E*. Then the following results are valid.

(*i*) $F_\mu \tilde{\cup} F^c{}_\mu = \tilde{\Lambda}_{\bar{1}}$

(*ii*) $F_\mu \tilde{\cap} F^c{}_\mu = \tilde{\varphi}_\theta$

Proof:

(*i*) $F^c{}_\mu = \left(\{(x,1,\xi_1(x)):x\in U\},(e,1,\mu_1(e))\right)$ $\forall x\in U$ and $\forall e\in E$.

Let $F_\mu \tilde{\cup} F^c{}_\mu = H_\nu$

$$\begin{aligned} H_\nu(e) &= (H(e),\nu(e)) \\ &= \left(\{(x,\max(\xi_1(x),1),\min(0,\xi_1(x))):x\in U\},(e,\max(\mu_1(e),1),\min(0,\mu_1(e)))\right) \\ &= \left(\{(x,1,0):x\in U\},(e,1,0)\right) \end{aligned}$$

Thus $F_\mu \tilde{\cup} F^c{}_\mu = \tilde{\Lambda}_{\bar{1}}$

(*ii*) $F^c{}_\mu = \left(\{(x,1,\xi_1(x)):x\in U\},(e,1,\mu_1(e))\right)$ $\forall x\in U$ and $\forall e\in E$.

Let $F_\mu \tilde{\cap} F^c{}_\mu = H_\nu$

$$\begin{aligned} H_\nu(e) &= (H(e),\nu(e)) \\ &= \left(\{(x,\min(\xi_1(x),1),\max(0,\xi_1(x))):x\in U\},(e,\min(\mu_1(e),1),\max(0,\mu_1(e)))\right) \\ &= \left(\{(x,\xi_1(x),\xi_1(x)):x\in U\},(e,\mu_1(e),\mu_1(e))\right) \end{aligned}$$

Thus $F_\mu \tilde{\cap} F^c{}_\mu = \tilde{\varphi}_\theta$

3.10. Proposition

Let F_μ, G_δ and H_λ be three GFSS over (*U*, *E*), where $\mu=\{(e,\mu_1(e),\mu_2(e)):e\in E\}$, $\delta=\{(e,\delta_1(e),\delta_2(e)):e\in E\}$ and $\lambda=\{(e,\lambda_1(e),\lambda_2(e)):e\in E\}$ with

$$\begin{aligned} F_\mu(e) &= (F(e),\mu(e)) \\ &= \left(\{(x,\xi_1(x),\xi_2(x)):x\in U\},(e,\mu_1(e),\mu_2(e))\right), \\ G_\delta(e) &= (G(e),\delta(e)) \\ &= \left(\{(x,\psi_1(x),\psi_2(x)):x\in U\},(e,\delta_1(e),\delta_2(e))\right), \\ H_\lambda(e) &= (H(e),\lambda(e)) \\ &= \left(\{(x,\zeta_1(x),\zeta_2(x)):x\in U\},(e,\lambda_1(e),\lambda_2(e))\right) \end{aligned}$$

, where $F(e),G(e),H(e)\in I^U$ and μ,δ,λ are fuzzy sets over *E*. Then it can be verified that the following results are valid.

(*i*) $F_\mu \tilde{\cup} G_\delta = G_\delta \tilde{\cup} F_\mu$

(*ii*) $F_\mu \tilde{\cap} G_\delta = G_\delta \tilde{\cap} F_\mu$

(*iii*) $F_\mu \tilde{\cup} (G_\delta \tilde{\cup} H_\lambda) = (F_\mu \tilde{\cup} G_\delta) \tilde{\cup} H_\lambda$

(*iv*) $F_\mu \tilde{\cap} (G_\delta \tilde{\cap} H_\lambda) = (F_\mu \tilde{\cap} G_\delta) \tilde{\cap} H_\lambda$

(*v*) $F_\mu \tilde{\cup} (G_\delta \tilde{\cap} H_\lambda) = (F_\mu \tilde{\cup} G_\delta) \tilde{\cap} (F_\mu \tilde{\cup} H_\lambda)$

(vi) $F_\mu \tilde{\cap} (G_\delta \tilde{\cup} H_\lambda) = (F_\mu \tilde{\cap} G_\delta) \tilde{\cup} (F_\mu \tilde{\cap} H_\lambda)$

3.11. Proposition

Let F_μ and G_δ be two GFSS over (U, E), where
$\mu = \{(e, \mu_1(e), \mu_2(e)) : e \in E\}, \delta = \{(e, \delta_1(e), \delta_2(e)) : e \in E\}$ and

$$\begin{aligned} F_\mu(e) &= (F(e), \mu(e)) \\ &= (\{(x, \xi_1(x), \xi_2(x)) : x \in U\}, (e, \mu_1(e), \mu_2(e))), \\ G_\delta(e) &= (G(e), \delta(e)) \\ &= (\{(x, \psi_1(x), \psi_2(x)) : x \in U\}, (e, \delta_1(e), \delta_2(e))) \end{aligned}$$

, where $F(e), G(e) \in I^U$ and μ, δ are fuzzy sets over E. Then the following De Morgan laws are valid.

(i) $(F_\mu \tilde{\cup} G_\delta)^c = F_\mu{}^c \tilde{\cap} G_\delta{}^c$

(ii) $(F_\mu \tilde{\cap} G_\delta)^c = F_\mu{}^c \tilde{\cup} G_\delta{}^c$

Proof:

(i) Let $F_\mu \tilde{\cup} G_\delta = H_\nu$, where $\forall e \in E$,

$$\begin{aligned} H_\nu(e) &= (H(e), \nu(e)) \\ &= (F(e) \cup G(e), \mu(e) \cup \delta(e)) \end{aligned}$$

Thus $(F_\mu \tilde{\cup} G_\delta)^c = H_\nu{}^c$, where $\forall e \in E$,

$$\begin{aligned} H_\nu{}^c(e) &= (H^c(e), \nu^c(e)) \\ &= ((F(e) \cup G(e))^c, (\mu(e) \cup \delta(e))^c) \\ &= (F^c(e) \cap G^c(e), \mu^c(e) \cap \delta^c(e)) \end{aligned}$$

Also, if $F_\mu{}^c \tilde{\cap} G_\delta{}^c = I_\lambda$, then

$$I_\lambda(e) = (F^c(e) \cap G^c(e), \mu^c(e) \cap \delta^c(e)) \;\; \forall e \in E.$$

It follows that $(F_\mu \tilde{\cup} G_\delta)^c = F_\mu{}^c \tilde{\cap} G_\delta{}^c$

(ii) Let $F_\mu \tilde{\cap} G_\delta = H_\nu$, where $\forall e \in E$,

$$\begin{aligned} H_\nu(e) &= (H(e), \nu(e)) \\ &= (F(e) \cap G(e), \mu(e) \cap \delta(e)) \end{aligned}$$

Thus $(F_\mu \tilde{\cup} G_\delta)^c = H_\nu{}^c$, where $\forall e \in E$,

$$\begin{aligned} H_\nu{}^c(e) &= (H^c(e), \nu^c(e)) \\ &= ((F(e) \cap G(e))^c, (\mu(e) \cap \delta(e))^c) \\ &= (F^c(e) \cup G^c(e), \mu^c(e) \cup \delta^c(e)) \end{aligned}$$

Also, if $F_\mu{}^c \tilde{\cap} G_\delta{}^c = I_\lambda$, then

$$I_\lambda(e) = (F^c(e) \cup G^c(e), \mu^c(e) \cup \delta^c(e)) \;\; \forall e \in E.$$

It follows that $\left(F_{\mu}\tilde{\cap}G_{\delta}\right)^{c}=F_{\mu}^{\ c}\tilde{\cup}G_{\delta}^{\ c}$

4. Extended Notion of Similarity Between Two Generalized Fuzzy Soft Sets

Similarity measures have extensive application in pattern recognition, region extraction, coding theory, image processing and in many other areas. Application of similarity measurement of two fuzzy soft sets has been studied by Majumder and Samanta in [8]. Here without going to the application part, we give an extended notion of similarity between two fuzzy soft sets and we would use this notion to find similarity between two generalized fuzzy soft sets. In this work, we are using the matrix representation of a fuzzy soft set discussed in our earlier works [11, 12].

4.1. Similarity Between Two Fuzzy Soft Sets

Let (*F*, *E*) and (*G*, *E*) be two fuzzy soft sets over (*U*, *E*), where $U=\{c_1,c_2,c_3,\ldots\ldots\ldots c_m\}$ and $E=\{e_1,e_2,e_3,\ldots\ldots\ldots e_n\}$. Let $(F,E)\tilde{\cup}(G,E)=(P,E)$ and $(F,E)\tilde{\cap}(G,E)=(Q,E)$. We assume that $A=[a_{ij}]$ and $B=[b_{ij}]$ are the fuzzy soft matrices corresponding to the fuzzy soft sets (*P*,*E*) and (*Q*,*E*) respectively, where $a_{ij}=\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)$ and $b_{ij}=\left(\chi_{j1}(c_i),\chi_{j2}(c_i)\right)$. The membership value matrices corresponding to the fuzzy soft matrices *A* and *B* are respectively, $MV(A)=[\delta_{(A)ij}]$, where $\delta_{(A)ij}=\mu_{j1}(c_i)-\mu_{j2}(c_i)$ and $MV(B)=[\delta_{(B)ij}]$, where $\delta_{(B)ij}=\chi_{j1}(c_i)-\chi_{j2}(c_i)$. Let $M((F,E),(G,E))$ denote the similarity between the fuzzy soft sets (*F*, *E*) and (*G*, *E*). Let $M_j((F,E),(G,E))$ represent the similarity between the e_j approximations $F(e_j)$ and $G(e_j)$ $\forall e_j$. Then we define

$$M_j((F,E),(G,E))=\frac{\sum_{i=1}^{m}\left(\delta_{(A)ij}\wedge\delta_{(B)ij}\right)}{\sum_{i=1}^{m}\left(\delta_{(A)ij}\vee\delta_{(B)ij}\right)}$$

and $M((F,E),(G,E))=\max_{j}\left\{M_j((F,E),(G,E))\right\},\ j=1,2,3\ldots\ldots n$

4.1.1. Proposition: Let (*F*, *E*), (*G*, *E*) and (*H*, *E*) be three fuzzy soft sets over (*U*, *E*). Then the following results are valid.

(*i*) $M((F,E),(F,E)^c)=0$

(*ii*) $M((F,E),(G,E))=M((G,E),(F,E))$

(*iii*) $(F,E)=(G,E)\Rightarrow M((F,E),(G,E))=1$

(*iv*) $(F,E)\tilde{\cap}(G,E)=\tilde{\varphi}\Leftrightarrow M((F,E),(G,E))=0$

(*v*) $(F,E)\tilde{\subseteq}(H,E)\tilde{\subseteq}(G,E)\Rightarrow M((F,E),(G,E))\leq M((H,E),(G,E))$

Proof: We only prove (*i*) and the others follow the similar lines.

We have $(F,E)\tilde{\cup}(F,E)^c=\tilde{E}$ and $(F,E)\tilde{\cap}(F,E)^c=\tilde{\varphi}$. The fuzzy soft matrices corresponding to $\tilde{E}$ and $\tilde{\varphi}$ are respectively $A=[a_{ij}]$ and $B=[b_{ij}]$, where a_{ij} = (1,0) and b_{ij} =

(0,0) $\forall i,j$. It follows that $MV(A) = [\delta_{(A)ij}]$, where $\delta_{(A)ij} = 1$ and $MV(B) = [\delta_{(B)ij}]$, where $\delta_{(B)ij} = 0 \ \forall\, i,j$.

Thus $M_j((F,E),(F,E)^c) = \dfrac{\sum_{i=1}^{m}\left(\delta_{(A)ij} \wedge \delta_{(B)ij}\right)}{\sum_{i=1}^{m}\left(\delta_{(A)ij} \vee \delta_{(B)ij}\right)} = 0 \ \forall\, j = 1,2,3.....................n$.

and $M((F,E),(G,E)) = \max_j \{M_j((F,E),(G,E))\} = 0$

4.2. Similarity Between Two Generalized Fuzzy Soft Sets

Let F_μ and G_ν be two GFSS over (*U*, *E*). Using the method discussed in *Section 4.1*, we first find out the similarity between the e_j approximations $F(e_j)$ and $G(e_j)\ \forall\, j$, which is given by

$$M_j((F,E),(G,E)) = \frac{\sum_{i=1}^{m}\left(\delta_{(A)ij} \wedge \delta_{(B)ij}\right)}{\sum_{i=1}^{m}\left(\delta_{(A)ij} \vee \delta_{(B)ij}\right)},$$

Where (*F*, *E*) and (*G*, *E*) are the constituent fuzzy soft sets in F_μ and G_ν . Let $M((F,E),(G,E))$ denote the similarity between the fuzzy soft sets (*F*, *E*) and (*G*, *E*). Then according to our definition,

$$M((F,E),(G,E)) = \max_j \{M_j((F,E),(G,E))\}$$

Let $m(\mu,\nu)$ be the similarity between μ and ν. We find the fuzzy membership values (*f.m.v.*) for each e_j in μ and ν respectively. Now, *f.m.v.* of e_j in μ , $f.m.v(\mu(e_j)) = \mu_{j1}(e_j) - \mu_{j2}(e_j)$ and *f.m.v.* of e_j in ν , $f.m.v(\nu(e_j)) = \nu_{j1}(e_j) - \nu_{j2}(e_j)$. We find the similarity between the fuzzy sets μ and ν in the following manner:

$$m(\mu,\nu) = \frac{\sum_j f.m.v.(\mu(e_j)) \wedge f.m.v.(\nu(e_j))}{\sum_j f.m.v.(\mu(e_j)) \vee f.m.v.(\nu(e_j))}$$

Let $M(F_\mu, G_\nu)$ denote the similarity between the generalized fuzzy soft sets F_μ and G_ν . The similarity between the generalized fuzzy soft sets F_μ and G_ν is defined as, $M(F_\mu, G_\nu) = M((F,E),(G,E)) \times m(\mu,\nu)$

4.2.1. Example: Let $U = \{c_1, c_2, c_3\}$ be the set of three cars under consideration and *E* = $\{e_1$ (in good condition), e_2 (luxurious), e_3 (new technology) } be a set of parameters.

We consider two GFSS F_μ and G_ν as

$$F_\mu = \{F_\mu(e_1) = (\{(c_1,0.2,0),(c_2,0.3,0),(c_3,0.4,0)\},(e_1,0.4,0)),$$

$$F_\mu(e_2) = (\{(c_1,0.1,0),(c_2,0.4,0),(c_3,0.7,0)\},(e_2,0.6,0)),$$

$F_{\mu}(e_3) = (\{(c_1,0.7,0),(c_2,0.4,0),(c_3,0.3,0)\},(e_1,0.3,0))\}$

$G_{\nu} \quad = \{G_{\nu}(e_1) = (\{(c_1,0.4,0),(c_2,0.1,0),(c_3,0.6,0)\},(e_1,0.2,0)),$

$G_{\nu}(e_2) = (\{(c_1,0.1,0),(c_2,0.8,0),(c_3,0.6,0)\},(e_2,0.8,0)),$

$G_{\nu}(e_3) = (\{(c_1,0.5,0),(c_2,0.1,0),(c_3,0.3,0)\},(e_1,0.4,0))\}$

The constituent fuzzy soft sets (F, E) and (G, E) in F_{μ} and G_{ν} are

$(F,E) \quad = \{F(e_1) = \{(c_1,0.2,0),(c_2,0.3,0),(c_3,0.4,0)\},\ F(e_2) = \{(c_1,0.1,0),(c_2,0.4,0),(c_3,0.7,0)\},$

$F(e_3) = \{(c_1,0.7,0),(c_2,0.4,0),(c_3,0.3,0)\}\}$

$(G,E) \quad = \{G(e_1) = \{(c_1,0.4,0),(c_2,0.1,0),(c_3,0.6,0)\},\ G(e_2) = \{(c_1,0.1,0),(c_2,0.8,0),(c_3,0.6,0)\},$

$G(e_3) = \{(c_1,0.5,0),(c_2,0.1,0),(c_3,0.3,0)\}\}$

Let $(F,E)\tilde{\cup}(G,E)=(P,E)$ and $(F,E)\tilde{\cap}(G,E)=(Q,E)$. Then

$(P,E) \quad = \{P(e_1) = \{(c_1,0.4,0),(c_2,0.3,0),(c_3,0.6,0)\},\ P(e_2) = \{(c_1,0.1,0),(c_2,0.8,0),(c_3,0.7,0)\},$

$P(e_3) = \{(c_1,0.7,0),(c_2,0.4,0),(c_3,0.3,0)\}\}$

$(Q,E) \quad = \{Q(e_1) = \{(c_1,0.2,0),(c_2,0.1,0),(c_3,0.4,0)\},\ Q(e_2) = \{(c_1,0.1,0),(c_2,0.4,0),(c_3,0.6,0)\},$

$Q(e_3) = \{(c_1,0.5,0),(c_2,0.1,0),(c_3,0.3,0)\}\}$

The fuzzy soft matrices corresponding to (P, E) and (Q, E) are given by,

$$A = \begin{bmatrix} (0.4,0.0) & (0.1,0.0) & (0.7,0.0) \\ (0.3,0.0) & (0.8,0.0) & (0.4,0.0) \\ (0.6,0.0) & (0.7,0.0) & (0.3,0.0) \end{bmatrix} \text{ and } B = \begin{bmatrix} (0.2,0.0) & (0.1,0.0) & (0.5,0.0) \\ (0.1,0.0) & (0.4,0.0) & (0.1,0.0) \\ (0.4,0.0) & (0.6,0.0) & (0.3,0.0) \end{bmatrix}$$

The membership value matrices corresponding to the fuzzy soft matrices A and B are respectively,

$$MV(A) = \begin{bmatrix} 0.4 & 0.1 & 0.7 \\ 0.3 & 0.8 & 0.4 \\ 0.6 & 0.7 & 0.3 \end{bmatrix} \text{ and } MV(B) = \begin{bmatrix} 0.2 & 0.1 & 0.5 \\ 0.1 & 0.4 & 0.1 \\ 0.4 & 0.6 & 0.3 \end{bmatrix}$$

We have,

$M_1((F,E),(G,E)) = 0.538$, $M_2((F,E),(G,E)) = 0.688$, $M_3((F,E),(G,E)) = 0.643$

Thus $M((F,E),(G,E)) = \max_j \{M_j((F,E),(G,E))\},\ j = 1,2,3 \quad = 0.688$

Also, $\mu = \{(e_1,0.4,0),(e_2,0.6,0),(e_3,0.3,0)\}$ and $\nu = \{(e_1,0.2,0),(e_2,0.8,0),(e_3,0.4,0)\}$

$f.m.v(\mu(e_1)) = 0.4,\ f.m.v(\mu(e_2)) = 0.6,\ f.m.v(\mu(e_3)) = 0.3$

$f.m.v(\nu(e_1)) = 0.2,\ f.m.v(\nu(e_2)) = 0.8,\ f.m.v(\nu(e_3)) = 0.4$

$$m(\mu,\nu) = \frac{\sum_j f.m.v.(\mu(e_j)) \wedge f.m.v.(\nu(e_j))}{\sum_j f.m.v.(\mu(e_j)) \vee f.m.v.(\nu(e_j))} = \frac{0.2+0.6+0.3}{0.4+0.8+0.4} = \frac{1.1}{1.6} = 0.688$$

Thus the similarity between the generalized fuzzy soft sets F_{μ} and G_{ν} is given by,

$M(F_\mu, G_\nu) = M((F,E),(G,E)) \times m(\mu,\nu) \quad = 0.688 \times 0.688 \quad = 0.473$

4.2.2. Proposition: Let F_μ , G_ν and H_ρ be three GFSS over (U, E). Then the following results are valid.

(i) $M(F_\mu, F_\mu{}^c) = 0$

(ii) $M(F_\mu, G_\nu) = M(G_\nu, F_\mu)$

(iii) $F_\mu = G_\nu \Rightarrow M(F_\mu, G_\nu) = 1$

(iv) $F_\mu \tilde{\cap} G_\nu = \tilde{\varphi} \Leftrightarrow M(F_\mu, G_\nu) = 0$

(v) $F_\mu \tilde{\subseteq} H_\rho \tilde{\subseteq} G_\nu \Rightarrow M(F_\mu, G_\nu) \le M(H_\rho, G_\nu)$

Proof: The proof immediately follows from definition.

5. Conclusion

We have given a restructure of generalized fuzzy soft sets initiated by Majumder and Samanta [7] in the light of extended notion of fuzzy sets initiated by Baruah [2]. Finally, we have put forward an extended notion regarding similarity between two fuzzy soft sets and similarity between two generalized fuzzy soft sets. It is hoped that our findings would help enhancing this study in generalized fuzzy soft sets.

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