A New Combination Rule for Conflict Problem of Dempster-Shafer Evidence Theory

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Abstract

Dempster-Shafer theory of evidence is a tool of uncertainty modeling when information is provided by experts. Dempster has given a rule to combine evidences coming from different independent sources. However, Dempster's rule of combination has been criticized as some times it gives some illogical results. Many alternatives have been proposed by different researchers. In this paper we have also proposed a new rule of combination. Efficiency and validity of our approach have been demonstrated with numerical examples and comparing with other existing methods.

Keywords*: Evidence theory, combination rule, conflicts evidences*

1. Introduction

More often, it is seen that available information is interpreted in probabilistic sense because probability theory is a very strong and well established mathematical tool to deal with objective uncertainty (i.e., uncertainty arises from heterogeneity or the random character of natural processes). However, it is clear that not all available information, data or model parameters are affected by objective uncertainty (i.e., nature of the data, information or parameters are random) and can be handled by traditional probability theory. Imprecision may occur due to scarce or incomplete information or data, measurement error or data obtain from expert judgment or subjective interpretation of available data or information. Thus, model parameters, data may be affected by subjective uncertainty. Traditional probability theory is inappropriate to represent subjective uncertainty (i.e., uncertainty arises from the partial character of our knowledge of the natural world). To overcome the limitation of probabilistic method, Dempster put forward a theory and now it is known as evidence theory or Dempster- Shefer theory (1976). This theory is nowadays widely used for the objective and subjective uncertainty analysis. The use of Dempster-Shefer theory in risk analysis has many advantages over the conventional probabilistic approach. It provides convenient and comprehensive way to handle engineering problems including, imprecisely specified distributions, poorly known and unknown correlation between different variables, modeling uncertainty, small sample size, and measurement uncertainty. Unfortunately, the rule of combination of evidences as provided by Dempster sometimes fails to give realistic results. So it has been debated upon by different researchers like (Yager 1987), (Dubois et al. 1988), (Sun et al. 2000), (Xin et al. 2005), (Deng et al. 2004), (Deqiang et al. 2008).

2. Proposed Combination Rule

Dempster Shafer theory of evidence is widely used for modeling both epistemic and aleatory uncertainty. The most important parts of evidence theory are frame of discernment and basic probability assignment (bpa). A frame of discernment Θ is a set of mutually exclusive and exhaustive propositional hypotheses, one and only one of them is true and bpa is a function $m: 2^{\Theta} \rightarrow [0,1]$ satisfying the following two conditions:

$$\left.\begin{array}{l} m(\phi)=0 \\ \sum_{A \subseteq \Theta} m(A)=1 \end{array}\right\}$$

where ϕ is an empty set and A is any subset of Θ .

Given two mass functions m_1 and m_2, Dempster-Shafer theory also provides a combination rule for combining them, which is defined as follows:

$$m(C)=\frac{\sum_{A \cap B=C} m_1(A) m_2(B)}{1-\sum_{A \cap B=\phi} m_1(A) m_2(B)}$$

The above rule of combination produces unreasonable results as demonstrated in the following example (Deqiang et al. 2008). Zadeh had given a compelling example of erroneous result found by using D-S theory. Suppose two doctors examine a patient and agree that it suffers from either meningitis (*M*), contusion (*C*) or brain tumor (*T*). Thus frame of discernment Θ = {*M*, *C*, *T*}. Assume that the doctors agree in their low expectation of a tumor, but disagree in likely cause and provide following diagnosis:

$$m_1(M) = 0.99,\ \mathrm{m}_1(T) = 0.01$$
$$\text{and } m_2(C) = 0.99, m_2(T) = 0.01.$$

Based on Dempster rule of combination, we get the unexpected final conclusion $m(T)=1$. It means that the patient suffers with certainty from brain tumor. This unexpected result arises from the fact that the two bodies of evidence (doctors) agree that the patient most likely does not suffer from tumor but are in almost full contradiction for the other causes of the disease.

In this section we put forward a new rule for combining evidences coming from two or more different sources.

Let (F, m_1) and (F, m_2) be two body of evidences given by two independent experts.
Suppose $A_1, A_2, \ldots, A_n$ be the focal elements. We will construct a combine body of evidence by assigning new bpa to each of the focal elements $A_1, A_2, \ldots, A_n$. This is done as follows:

For each A_i we associate a value $m'(A_i)$ as

$$m'(A_i)=m_1(A_i)+m_2(A_i)-m_1(A_i)m_2(A_i)$$

This value can be interpreted as the combined evidence in support of the focal elements A_i. The motivation behind this formulation is the probability rule for union of two events. It also resembles that algebraic sum of fuzzy sets. Then the quantity $m'(A_i)/\{1-m'(A_i)\}$ can be interpreted as the odds ratio in favor of A_i, which can also be considered as evidence in support of A_i. Since we will deal with conflict situation in evidence theory the denominator of the above expression may be 0. So, we replace the denominator by $1+\{1-m'(A_i))$, which gives $m'(A_i)/[1+\{1-m'(A_i)\}]$ which can also be written as

$$m^{'}(A_i)=\frac{1-(1-m_1(A_i)\times(1-m_2(A_i)}{1+(1-m_1(A_i)\times(1-m_2(A_i)} \qquad ...(1)$$

Finally we assign bpa to A_i as

$$m\ (A_i)=\frac{m^{'}(A_i)}{\sum_n m^{'}(A_i)} \qquad ...(2)$$

Using our proposed combined rule we solved the compelling example given by Zadeh in which the frame of discernment is $\Theta=\{M,C,\ T\}$ and corresponding bpas are $m_1\ (M)$ = 0.99, $m_1\ (T)$ = 0.01 and $m_2\ (C)$ = 0.99, $m_2\ (T)$ = 0.01. Our method gives the bpa as $m(M)=0.494745, m(C)=0.49745$ and $m(T)=0.0051$ which is more reliable than Dempster's rule of combination.

3. Numerical Example

Example 1: Let Θ = {*A*, *B*, *C*} be the frame of discernment and the corresponding basic probability assignments (bpa) from four different independent sources are:

Table 1. BPAs Obtained from Four Different Bodies of Evidences

$m_1(A)$ = 0.98	$m_1(B)$ = 0.01	$m_1(C)$ = 0.01
$m_2(A)$ = 0.00`	$m_2(B)$ = 0.01	$m_2(C)$ = 0.99
$m_3(A)$ = 0.90	$m_3(B)$ = 0.01	$m_3(C)$ = 0.09
$m_4(A)$ = 0.90	$m_4(B)$ = 0.01	$m_4(C)$ = 0.09

Here we will compare different types of combination rule with our proposed combination rule (Table 2).

Table 2. Results of Different Combination Rules of Evidence

Combination rules	m_{12}	m_{123}	m_{1234}
Dempster Rule	$m(A)$ = 0, $m(B)$ = 0.01 $m(C)$ = 0.99, $m(\Theta)$ = 0	$m(A)$ = 0, $m(B)$ = 0.0011 $m(C)$ = 0.9989, $m(\Theta)$ =0	$m(A)$ = 0, $m(B)$ = 0.0001247 $m(C)$ = 0.998753, $m(\Theta)$ =0
Yager Rule	$m(A)$ = 0, $m(B)$ = 0.0001 $m(C)$ = 0.0099, $m(\Theta)$ = 0.99	$m(A)$ = 0, $m(B)$ =0.000001, $m(C)$ = 0.000891, $m(\Theta)$ = 0.999108	$m(A)$ = 0, $m(B)$ = 0.00000001, $m(C)$ = 0.00008019, $m(\Theta)$= 0.9999198
Dubois & Prade Rule	$m(A)$ = 0, $m(B)$ = 0.0001 $m(C)$ = 0.0099, $m(A \cup B)$ = 0.0098 $m(A \cup C)$ = 0.9702 $m(B \cup C)$ = 0.01 $m(\Theta)$ = 0	$m(A)$ = 0, $m(B)$ = 0.000001 $m(C)$ = 0.000891, $m(A \cup B)$ = 0.009008 $m(A \cup C)$ = 0.969408 $m(B \cup C)$ = 0.00108 $m(\Theta)$ = 0.019584	$m(A)$ = 0, $m(B)$ = 0.00000001 $m(C)$ = 0.00008019, $m(A \cup B)$ = 0.00819818 $m(A \cup C)$ = 0.96051582 $m(B \cup C)$ = 0.0001198 $m(\Theta)$ = 0.031086
Sun rule	$m(A)$=0.18, $m(B)$=0.0004 $m(C)$=0.194, $m(\Theta)$=0.622	$m(A)$=0.321, $m(B)$=0.003 $m(C)$=0.188, $m(\Theta)$=0.488	$m(A)$=0.42, $m(B)$=0.003 $m(C)$=0.181, $m(\Theta)$=0.396
Deng rule	$m(A)$=0.4898, $m(B)$=0.0002 $m(C)$=0.51, $m(\Theta)$=0	$m(A)$=0.998299, $m(B)$=0.000001, $m(C)$=0.017, $m(\Theta)$=0	$m(A)$=0.9985426, $m(B)$=0.00000001, $m(C)$=0.00014573, $m(\Theta)$=0
Deqiang rule	$m(A)$=0.4703, $m(B)$=0.01 $m(C)$=0.5197, $m(\Theta)$=0	$m(A)$=0.7431, $m(B)$=0.01 $m(C)$=0.2469, $m(\Theta)$=0	$m(A)$=0.8358, $m(B)$=0.01 $m(C)$=0.1542, $m(\Theta)$=0
Our Proposed combined Rule	$m(A)$=0.4924, $m(B)$=0.0051, m(C)=0.5025, $m(\Theta)$=0	$m(A)$=0.7016, $m(B)$=0.0059, $m(C)$=0.2925, $m(\Theta)$=0	$m(A)$=0.8075, $m(B)$=0.0068 $m(C)$=0.1857, $m(\Theta)$=0

Example 2: Let $\Theta = \{A, B, C\}$ be the frame of discernment and the corresponding basic probability assignments (bpa) from five different independent sources are:

Table 3. BPAs Obtained from Five Different Bodies of Evidences

$m_1(A)$ = 0.50	$m_1(B)$ = 0.20	$m_1(C)$ = 0.30
$m_2(A)$ = 0.00	$m_2(B)$ = 0.90	$m_2(C)$ = 0.10
$m_3(A)$ = 0.55	$m_3(B)$ = 0.1	$m_3(C)$ = 0.35
$m_4(A)$ = 0.55	$m_4(B)$ = 0. 1	$m_4(C)$ = 035
$m_5(A)$ = 0.60	$m_5(B)$ = 0. 1	$m_5(C)$ = 030

Here also we will compare different types of combination rule with our proposed combination rule (Table 4).

Table 4. Results of Different Combination Rules of Evidence

Combination rules	m_{12}	m_{123}	m_{1234}	m_{12345}
Dempster rule	$m(A) = 0$, $m(B) = 0.8571$ $m(C) = 0.1429$	$m(A) = 0$, $m(B) = 0.6316$ $m(C) = 0.3684$,	$m(A) = 0$, $m(B) = 0.3288$ $m(C) =.6712$	$m(A) = 0$, $m(B) = 0.1228$ $m(C) =.8772$
Murphy's average combination rule	$m(A) = 0.1543$, $m(B) = 0.7469$ $m(C) = 0.0988$,	$m(A) = 0.3500$ $m(B) = 0.5224$ $m(C) = 0.1276$	$m(A) = 0.6027$ $m(B) = 0.2627$ $m(C) = 0.1346$	$m(A) = 0.7958$ $m(B) = 0.0932$ $m(C) = 0.1110$
Deng rule	$m(A) = 0.1543$ $m(B) = 0.7469$ $m(C) = 0.0988$	$m(A) = 0.4861$ $m(B) = 0.3481$ $m(C) = 0.1657$	$m(A) = 0.7773$ $m(B) = 0.0628$ $m(C) = 0.1600$	$m(A) = 0.8909$ $m(B) = 0.0086$ $m(C) = 0.1005$
Our Proposed combined rule	$m(A) = 0.2360$ $m(B) = 0.6032$ $m(C) = 0.1608$	$m(A) = 0.3888$, $m(B) = 0.3771$, $m(C) = 0.2341$,	$m(A) = 0.4797$ $m(B) = 0.2375$ $m(C) = 0.2828$	$m(A) = 0.5588$ $m(B) = 0.1586$ $m(C) = 0.2816$

4. Conclusion

It has been observed in different situations by many researchers that Dempster's combination rule is a poor solution for decision making in conflict situations. It sometimes exhibit counter intuitive behavior and produces illogical conclusions. Several alternative methods have been proposed to remedy the drawback of Dempster's rule. None of the methods can be stated to be the best because utility of a particular method depends upon the problem under consideration. In this paper we have proposed a simple alternative rule of combination. From the two numerical examples we have given it appears that our proposed method produces result which are more realistic than that given by the other methods.

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