On Fuzzy Soft Complement and Related Properties

Tridiv Jyoti Neog[1], Dusmanta Kumar Sut[2]

[1]Dept. of Mathematics, CMJ University, Shillong, India
[2]Dept. of Mathematics, N. N. Saikia College, Titabor, India
tridivjyoti@gmail.com, sutdk001@yahoo.com

Abstract

In this paper, we have reintroduced the notion of complement of a fuzzy soft set initiated by Maji. In our work, fuzzy sets have been replaced by the extended fuzzy sets with fuzzy reference function zero initiated by Baruah.

Keywords: *Fuzzy set, soft set, fuzzy soft set, complement of a fuzzy soft set.*

1. Introduction

In the fuzzy set theory initiated by Zadeh [5] in 1965, it has been accepted that the classical set theoretic axioms of exclusion and contradiction are not satisfied. In this regard, Baruah [3,4] proposed that two functions, namely fuzzy membership function and fuzzy reference function are necessary to represent a fuzzy set. Accordingly, Baruah [3,4] reintroduced the notion of complement of a fuzzy set in a different way and proved that indeed the set theoretic axioms of exclusion and contradiction are valid for fuzzy sets also.

In 1999, Molodtsov [2] introduced the theory of soft sets, which is a new mathematical approach to vagueness. In recent years the researchers have contributed a lot towards fuzzification of soft set theory. Maji et al. [6] initiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan’s Law etc. These results were further revised and improved by Ahmad and Kharal [1]. They defined arbitrary fuzzy soft union and intersection and proved De Morgan’s Inclusions and De Morgan’s Laws in Fuzzy Soft Set Theory.

It has been observed that till date we don’t have a proper definition of complement of a fuzzy soft set which satisfies the set theoretic axioms of exclusion and contradiction, *i.e.* union of a fuzzy soft set and its complement is absolute fuzzy soft set and intersection of a fuzzy soft set and its complement is null fuzzy soft set.

In this article, we shall apply the extended definition of fuzzy set in the context of fuzzy soft set and put forward a new definition of complement of a fuzzy soft set in a way that gives $(F,A)\cap(F,A)^c=\tilde{\phi}$, the null fuzzy soft set, and $(F,A)\cup(F,A)^c=\tilde{A}$, the absolute fuzzy soft set.

2. Preliminaries

In this section, we recall some concepts and definitions which will be needed in the sequel.

In [**3**], Baruah put forward an extended definition of fuzzy sets and with the help of this extended definition, he put forward the notion of union and intersection of two fuzzy sets in the following way -

2.1. Extended Definition of Union and Intersection of Fuzzy Sets

Let $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in U\}$ and $B(\mu_3, \mu_4) = \{x, \mu_3(x), \mu_4(x); x \in U\}$ be two fuzzy sets defined over the same universe *U*. Then the operations intersection and union are defined as

$$A(\mu_1, \mu_2) \cap B(\mu_3, \mu_4) = \{x, \min(\mu_1(x), \mu_3(x)), \max(\mu_2(x), \mu_4(x)); x \in U\}$$

and, $A(\mu_1, \mu_2) \cup B(\mu_3, \mu_4) = \{x, \max(\mu_1(x), \mu_3(x)), \min(\mu_2(x), \mu_4(x)); x \in U\}$.

Neog et al. [**7**] showed by an example that this definition sometimes gives degenerate cases and revised the above definition as follows -

2.2. Extended Definition of Union and Intersection of Fuzzy Sets Revised

Let $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in U\}$ and $B(\mu_3, \mu_4) = \{x, \mu_3(x), \mu_4(x); x \in U\}$ be two fuzzy sets defined over the same universe *U*. To avoid degenerate cases we assume that

$$\min(\mu_1(x), \mu_3(x)) \geq \max(\mu_2(x), \mu_4(x)) \forall x \in U .$$

Then the operations intersection and union are defined as

$$A(\mu_1, \mu_2) \cap B(\mu_3, \mu_4) = \{x, \min(\mu_1(x), \mu_3(x)), \max(\mu_2(x), \mu_4(x)); x \in U\}$$

and, $A(\mu_1, \mu_2) \cup B(\mu_3, \mu_4) = \{x, \max(\mu_1(x), \mu_3(x)), \min(\mu_2(x), \mu_4(x)); x \in U\}$.

In [**3**], Baruah put forward the notion of complement of usual fuzzy sets with fuzzy reference function 0 in the following way -

2.3. Complement of a Fuzzy Set Using Extended Definition

For usual fuzzy sets $A(\mu,0) = \{x, \mu(x), 0; x \in U\}$ and $B(1, \mu) = \{x, 1, \mu(x); x \in U\}$ defined over the same universe *U*, we have

$$A(\mu,0) \cap B(1, \mu) = \{x, \min(\mu(x), 1), \max(0, \mu(x)); x \in U\}$$

$= \{x, \mu(x), \mu(x); x \in U\}$, which is nothing but the null set φ .

and, $A(\mu,0) \cup B(1, \mu) = \{x, \max(\mu(x), 1), \min(0, \mu(x)); x \in U\}$

$= \{x, 1, 0; x \in U\}$, which is nothing but the universal set *U*.

This means if we define a fuzzy set $(A(\mu,0))^c = \{x, 1, \mu(x); x \in U\}$, it is nothing but the complement of $A(\mu,0) = \{x, \mu(x), 0; x \in U\}$.

Neog et al. [**7**] put forward the notion of fuzzy subset and some results using the extended notion of fuzzy sets in the following manner -

2.4. Extended Definition of Fuzzy Subset

Let $A(\mu_1,\mu_2)=\{x,\mu_1(x),\mu_2(x);x\in U\}$ and $B(\mu_3,\mu_4)=\{x,\mu_3(x),\mu_4(x);x\in U\}$ be two fuzzy sets defined over the same universe *U*. The fuzzy set $A(\mu_1,\mu_2)$ is a subset of the fuzzy set $B(\mu_3,\mu_4)$ if $\forall x\in U$, $\mu_1(x)\le\mu_3(x)$ and $\mu_4(x)\le\mu_2(x)$.

Two fuzzy sets $C=\{x,\mu_C(x):x\in U\}$ and $D=\{x,\mu_D(x):x\in U\}$ in the usual definition would be expressed as $C(\mu_C,0)=\{x,\mu_C(x),0;x\in U\}$ and $D(\mu_D,0)=\{x,\mu_D(x),0;x\in U\}$.

Accordingly, we have $C(\mu_C,0)\subseteq D(\mu_D,0)$ if $\forall x\in U$, $\mu_C(x)\le\mu_D(x)$, Which can be obtained by putting $\mu_2(x)=\mu_4(x)=0$ in our new definition.

2.4.1. Proposition: For fuzzy sets $A(\mu_1,\mu_2),B(\mu_3,\mu_4),C(\mu_5,\mu_6)$ over the same universe *U*, the following results are valid.

1. $A(\mu_1,\mu_2)\subseteq B(\mu_3,\mu_4),B(\mu_3,\mu_4)\subseteq C(\mu_5,\mu_6)\Rightarrow A(\mu_1,\mu_2)\subseteq C(\mu_5,\mu_6)$
2. $A(\mu_1,\mu_2)\cap B(\mu_3,\mu_4)\subseteq A(\mu_1,\mu_2),A(\mu_1,\mu_2)\cap B(\mu_3,\mu_4)\subseteq B(\mu_3,\mu_4)$
3. $A(\mu_1,\mu_2)\subseteq A(\mu_1,\mu_2)\cup B(\mu_3,\mu_4),B(\mu_3,\mu_4)\subseteq A(\mu_1,\mu_2)\cup B(\mu_3,\mu_4)$
4. $A(\mu_1,\mu_2)\subseteq B(\mu_3,\mu_4)\Rightarrow A(\mu_1,\mu_2)\cap B(\mu_3,\mu_4)=A(\mu_1,\mu_2)$
5. $A(\mu_1,\mu_2)\subseteq B(\mu_3,\mu_4)\Rightarrow A(\mu_1,\mu_2)\cup B(\mu_3,\mu_4)=B(\mu_3,\mu_4)$

Taking $\mu_2(x)=\mu_4(x)=\mu_6(x)=0$, we obtain the same results for usual fuzzy sets.

Molodtsov [2] defined soft set in the following way -

2.5. Soft Set

A pair (*F*, *E*) is called a soft set (over *U*) if and only if *F* is a mapping of *E* into the set of all subsets of the set *U*.

In other words, the soft set is a parameterized family of subsets of the set *U*. Every set $F(\varepsilon),\varepsilon\in E$, from this family may be considered as the set of ε - elements of the soft set (*F*, *E*), or as the set of ε - approximate elements of the soft set.

The following definition of fuzzy soft set is due to Maji et al. [**6**]

2.6. Fuzzy Soft Set

A pair (*F*, *A*) is called a fuzzy soft set over *U* where $F:A\to\tilde{P}(U)$ is a mapping from *A* into $\tilde{P}(U)$.

Ahmad and Kharal [**1**] defined fuzzy soft class in the following manner -

2.7. Fuzzy Soft Class

Let *U* be a universe and *E* a set of attributes. Then the pair (*U*, *E*) denotes the collection of all fuzzy soft sets on *U* with attributes from *E* and is called a fuzzy soft class.

Maji et al. [**6**] gave the definition of null fuzzy soft set, absolute fuzzy soft set, union and intersection of fuzzy soft sets in the following way.

2.8. Null Fuzzy Soft Set

A soft set (*F*, *A*) over *U* is said to be null fuzzy soft set denoted by φ if $\forall \varepsilon \in A, F(\varepsilon)$ is the null fuzzy set $\overline{0}$ of *U* where $\overline{0}(x) = 0 \forall x \in U$.

2.9. Absolute Fuzzy Soft Set

A soft set (*F*, *A*) over *U* is said to be absolute fuzzy soft set denoted by $\tilde{A}$ if $\forall \varepsilon \in A, F(\varepsilon)$ is the absolute fuzzy set $\overline{1}$ of *U* where $\overline{1}(x) = 1 \forall x \in U$.

2.10. Union of Fuzzy Soft Sets

Union of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

and is written as $(F, A) \tilde{\cup} (G, B) = (H, C)$.

2.11. Intersection of Fuzzy Soft Sets

Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon)$ or $G(\varepsilon)$ (as both are same fuzzy set) and is written as $(F, A) \tilde{\cap} (G, B) = (H, C)$.

Ahmad and Kharal [**1**] pointed out that generally $F(\varepsilon)$ or $G(\varepsilon)$ may not be identical. Moreover in order to avoid the degenerate case, he proposed that $A \cap B$ must be non-empty and thus revised the above definition as follows -

2.12. Intersection of Fuzzy Soft Sets Revised

Let (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets in a soft class (*U*, *E*) with $A \cap B \neq \phi$ .Then Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*,*C*) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$. We write $(F, A) \tilde{\cap} (G, B) = (H, C)$.

The following definition of complement of a fuzzy soft set is due to Maji et al. [**6**]

2.13. Complement of a Fuzzy Soft Set

The complement of a fuzzy soft set (*F*, *A*) is denoted by $(F, A)^c$ and is defined by $(F, A)^c$ = (F^c, $\rceil A$), where F^c: $\rceil A \to \tilde{P}(U)$ is a mapping given by $F^c(\sigma) = (F(\neg\sigma))^c$ for all $\sigma \in \rceil A$.

3. Complement of a Fuzzy Soft Set Redefined

From our stand point, we first give the definition of Null Fuzzy Soft Set and Absolute Fuzzy Soft Set as given below -

3.1. Null Fuzzy Soft Set

A fuzzy soft set (*F*, *A*) over *U* is said to be null fuzzy soft set (with respect to the parameter set *A*), denoted by $\tilde{\varphi}$ if $\forall \varepsilon \in A, F(\varepsilon)$ is the null fuzzy set φ. In other words, for a null fuzzy soft set (*F*, *A*), $\forall \varepsilon \in A, F(\varepsilon) = \{x, \mu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x); x \in U\}$. In the case of usual fuzzy sets, it is obvious that for a null fuzzy soft set (*F*, *A*), $\forall \varepsilon \in A, F(\varepsilon) = \{x, 0, 0; x \in U\}$.

3.2. Absolute Fuzzy Soft Set

A fuzzy soft set (*F*, *A*) over *U* is said to be absolute fuzzy soft set (with respect to the parameter set *A*), denoted by $\tilde{A}$ if $\forall \varepsilon \in A, F(\varepsilon)$ is the absolute fuzzy set *U*. In other words, for an absolute fuzzy soft set (*F*, *A*), $\forall \varepsilon \in A, F(\varepsilon) = \{x, 1, 0; x \in U\}$.

In the context of complement of a fuzzy soft set initiated by Maji et al. [**6**], we see that F^c is a mapping from the set $\rceil A$ of not parameters to $\tilde{P}(U)$. This definition of complement of a fuzzy soft set does not satisfy the set theoretic axioms of contradiction and exclusion. As such, we shall endeavor to make the definition of complement of a fuzzy soft set more rational. Accordingly, our definition of complement of a fuzzy soft set is as follows -

3.3. Complement of a Fuzzy Soft Set

The complement of a fuzzy soft set (*F*, *A*) is denoted by $(F, A)^c$ and is defined by $(F, A)^c$ = (F^c, A) where $F^c : A \rightarrow \tilde{P}(U)$ is a mapping given by $F^c(\alpha) = [F(\alpha)]^c$, $\forall \alpha \in A$. In other words, $\forall \varepsilon \in A$, if $F(\varepsilon) = \{x, \mu_{F(\varepsilon)}(x), 0; x \in U\}$, then $F^c(\varepsilon) = \{x, 1, \mu_{F(\varepsilon)}(x); x \in U\}$.

3.3.1. Proposition: For a fuzzy soft set (*F*, *A*) over *U*, we have,

1. $(F, A) \tilde{\cup} (F, A)^c = \tilde{A}$ (Exclusion)
2. $(F, A) \tilde{\cap} (F, A)^c = \tilde{\varphi}$ (Contradiction)

Proof

1. Let $(F, A) \tilde{\cup} (F, A)^c = (F, A) \tilde{\cup} (F^c, A) = (H, A)$, where $\forall \varepsilon \in A, H(\varepsilon) = F(\varepsilon) \cup F^c(\varepsilon)$

$$
\begin{aligned}
&= F(\varepsilon) \cup (F(\varepsilon))^c \\
&= \{x, \mu_{F(\varepsilon)}(x), 0; x \in U\} \cup \{x, 1, \mu_{F(\varepsilon)}(x); x \in U\} \\
&= \{x, \max(\mu_{F(\varepsilon)}(x), 1), \min(0, \mu_{F(\varepsilon)}(x)); x \in U\} \\
&= \{x, 1, 0; x \in U\} \\
&= U
\end{aligned}
$$

Thus $(F, A) \tilde{\cup} (F, A)^c = \tilde{A}$

2. Let $(F,A)\tilde{\cap}(F,A)^c = (F,A)\tilde{\cap}(F^c,A) = (H,A)$, where $\forall \varepsilon \in A, H(\varepsilon) = F(\varepsilon) \cap F^c(\varepsilon)$

$$= F(\varepsilon) \cup (F(\varepsilon))^c$$
$$= \{x, \mu_{F(\varepsilon)}(x), 0; x \in U\} \cap \{x, 1, \mu_{F(\varepsilon)}(x); x \in U\}$$
$$= \{x, \min(\mu_{F(\varepsilon)}(x), 1), \max(0, \mu_{F(\varepsilon)}(x)); x \in U\}$$
$$= \{x, \mu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x); x \in U\}$$
$$= \varphi$$

Thus $(F,A)\tilde{\cap}(F,A)^c = \tilde{\varphi}$

It can be verified that in our definition, the following properties are valid for fuzzy soft sets (*F*, *A*), (*G*, *B*) and (*H*, *C*).

1.(*i*) $(F,A)\tilde{\cup}(G,B) = (G,B)\tilde{\cup}(F,A)$
(*ii*) $(F,A)\tilde{\cap}(G,B) = (G,B)\tilde{\cap}(F,A)$
2.(*i*) $(F,A)\tilde{\cup}((G,B)\tilde{\cup}(H,C)) = ((F,A)\tilde{\cup}(G,B))\tilde{\cup}(H,C)$
(*ii*) $(F,A)\tilde{\cap}((G,B)\tilde{\cap}(H,C)) = ((F,A)\tilde{\cap}(G,B))\tilde{\cap}(H,C)$
3.(*i*) $(F,A)\tilde{\cup}((G,B)\tilde{\cap}(H,C)) = ((F,A)\tilde{\cup}(G,B))\tilde{\cap}((F,A)\tilde{\cup}(H,C))$
(*ii*) $(F,A)\tilde{\cap}((G,B)\tilde{\cup}(H,C)) = ((F,A)\tilde{\cap}(G,B))\tilde{\cup}((F,A)\tilde{\cap}(H,C))$
4.(*i*) $(F,A)\tilde{\cup}(F,A) = (F,A)$
(*ii*) $(F,A)\tilde{\cap}(F,A) = (F,A)$
5. $((F,A)^c)^c = (F,A)$

Let us now come to the context of De Morgan Inclusions as well as De Morgan Laws. Maji et al. [6] proved the following propositions.

3.3.2. Proposition:

1. $((F,A)\tilde{\cup}(G,B))^c = (F,A)^c\,\tilde{\cup}(G,B)^c$
2. $((F,A)\tilde{\cap}(G,B))^c = (F,A)^c\,\tilde{\cap}(G,B)^c$

Ahmad and Kharal [1] showed by a counter example that these two propositions are not valid. However they partially established the following results.

3.3.3. Proposition: For fuzzy soft sets (*F*, *A*) and (*G*, *B*) in (*U*, *E*), one has the following:

1. $((F,A)\tilde{\cup}(G,B))^c \tilde{\subseteq} (F,A)^c\,\tilde{\cup}(G,B)^c$
2. $(F,A)^c\,\tilde{\cap}(G,B)^c \tilde{\subseteq} ((F,A)\tilde{\cap}(G,B))^c$

We shall show that these two results are valid for fuzzy soft sets (*F*, *A*) and (*G*, *B*) in (*U*, *E*) under our new notion of complement.

3.3.4. Proposition: For fuzzy soft sets (*F*, *A*) and (*G*, *B*) in (*U*, *E*), one has the following.

1. $((F,A)\tilde{\cup}(G,B))^c \tilde{\subseteq} (F,A)^c\,\tilde{\cup}(G,B)^c$

2. $(F,A)^c \tilde{\cap} (G,B)^c \tilde{\subseteq} ((F,A)\tilde{\cap}(G,B))^c$

Proof

1. Let $(F,A)\tilde{\cup}(G,B) = (H,C)$, where $C = A \cup B$ and

$$\forall \varepsilon \in C, H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B \\ G(\varepsilon) & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon) & \text{if } \varepsilon \in A \cap B \end{cases}$$

Thus

$$((F,A)\tilde{\cup}(G,B))^c = (H,C)^c = (H^c, C), \text{ where } C = A \cup B \text{ and}$$
$$= (H(\varepsilon))^c$$

$$\forall \varepsilon \in C, H^c(\varepsilon)$$

$$= \begin{cases} (F(\varepsilon))^c & \text{if } \varepsilon \in A\text{-}B \\ (G(\varepsilon))^c & \text{if } \varepsilon \in B\text{-}A \\ (F(\varepsilon) \cup G(\varepsilon))^c & \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \in A\text{-}B \\ G^c(\varepsilon) & \text{if } \varepsilon \in B\text{-}A \\ (F(\varepsilon))^c \cap (G(\varepsilon))^c & \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \in A\text{-}B \\ G^c(\varepsilon) & \text{if } \varepsilon \in B\text{-}A \\ F^c(\varepsilon) \cap G^c(\varepsilon) & \text{if } \varepsilon \in A \cap B \end{cases}$$

Again,

$$(F,A)^c \tilde{\cup} (G,B)^c = (F^c, A)\tilde{\cup}(G^c, B) = (I, J), \text{ say}$$

Where $J = A \cup B$ and

$$\forall \varepsilon \in J, I(\varepsilon) = \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \in A\text{-}B \\ G^c(\varepsilon) & \text{if } \varepsilon \in B\text{-}A \\ F^c(\varepsilon) \cup G^c(\varepsilon) & \text{if } \varepsilon \in A \cap B \end{cases}$$

We see that $C = J$ and $\forall \varepsilon \in C, H^c(\varepsilon) \subseteq I(\varepsilon)$

Thus $((F,A)\tilde{\cup}(G,B))^c \tilde{\subseteq} (F,A)^c \tilde{\cup} (G,B)^c$

2. Let $(F,A)\tilde{\cap}(G,B) = (H,C)$, where $C = A \cap B$ and

$$\forall \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$$

Thus $((F,A)\tilde{\cap}(G,B))^c = (H,C)^c = (H^c, C)$, where $C = A \cap B$ and

$$\forall \varepsilon \in C, H^c(\varepsilon) = (F(\varepsilon) \cap G(\varepsilon))^c$$
$$= (F(\varepsilon))^c \cup (G(\varepsilon))^c$$

$$= F^c(\varepsilon) \cup G^c(\varepsilon)$$

Again, $(F,A)^c \,\tilde{\cap}\, (G,B)^c = (F^c, A) \tilde{\cap} (G^c, B) = (I, J)$, say, where $J = A \cap B$ and

$$\forall \varepsilon \in J, I(\varepsilon) \quad = F^c(\varepsilon) \cap G^c(\varepsilon)$$

We see that $C = J$ and $\forall \varepsilon \in C, I(\varepsilon) \subseteq H^c(\varepsilon)$

Thus $(F,A)^c \,\tilde{\cap}\, (G,B)^c \,\tilde{\subseteq}\, ((F,A) \tilde{\cap} (G,B))^c$

It is well known that De Morgan Law interrelate union and intersection via complements. Ahmad and Kharal [**1**] proved the following De Morgan Inclusions.

3.3.5. Proposition: For fuzzy soft sets (*F*, *A*) and (*G*, *B*) of a soft class (*U*, *E*), one has the following.

1. $(F,A)^c \,\tilde{\cap}\, (G,B)^c \,\tilde{\subseteq}\, ((F,A) \tilde{\cup} (G,B))^c$
2. $((F,A) \tilde{\cap} (G,B))^c \,\tilde{\subseteq}\, (F,A)^c \,\tilde{\cup}\, (G,B)^c$

The above De Morgan Inclusions are, in general, irreversible, which is shown by Ahmad and Kharal [**1**] with the help of a counter example.

We shall show that these two results are true for fuzzy soft sets (*F*, *A*) and (*G*, *B*) in (*U*, *E*) according to our new notion.

3.3.6. Proposition (De Morgan Inclusions): For fuzzy soft sets (*F*, *A*) and (*G*, *B*) of a soft class (*U*, *E*), one has the following:

1. $(F,A)^c \,\tilde{\cap}\, (G,B)^c \,\tilde{\subseteq}\, ((F,A) \tilde{\cup} (G,B))^c$
2. $((F,A) \tilde{\cap} (G,B))^c \,\tilde{\subseteq}\, (F,A)^c \,\tilde{\cup}\, (G,B)^c$

Proof

1. Let $(F,A) \tilde{\cup} (G,B) = (H, C)$, where $C = A \cup B$ and

$$\forall \varepsilon \in C, H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A\text{-}B \\ G(\varepsilon) & \text{if } \varepsilon \in B\text{-}A \\ F(\varepsilon) \cup G(\varepsilon) & \text{if } \varepsilon \in A \cap B \end{cases}$$

Thus $((F,A) \tilde{\cup} (G,B))^c = (H, C)^c \quad = (H^c, C)$, where $C = A \cup B$ and

$$= (H(\varepsilon))^c$$

$$\forall \varepsilon \in C, H^c(\varepsilon)$$

$$= \begin{cases} (F(\varepsilon))^c & \text{if } \varepsilon \in A\text{-}B \\ (G(\varepsilon))^c & \text{if } \varepsilon \in B\text{-}A \\ (F(\varepsilon) \cup G(\varepsilon))^c & \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \in A\text{-}B \\ G^c(\varepsilon) & \text{if } \varepsilon \in B\text{-}A \\ (F(\varepsilon))^c \cap (G(\varepsilon))^c & \text{if } \varepsilon \in A \cap B \end{cases}$$

Again, $(F,A)^c \tilde{\cap} (G,B)^c = (F^c,A) \tilde{\cap} (G^c,B) \quad = (I,J)$, say, where $J = A \cap B$ and

$\forall \varepsilon \in J, I(\varepsilon) \quad = F^c(\varepsilon) \cap G^c(\varepsilon)$

We see that $J \subseteq C$ and $\forall \varepsilon \in J, I(\varepsilon) \subseteq H^c(\varepsilon)$

Thus $(F,A)^c \tilde{\cap} (G,B)^c \tilde{\subseteq} ((F,A) \tilde{\cup} (G,B))^c$

It follows immediately that $(F,A)^c \tilde{\cap} (G,B)^c \tilde{\subseteq} ((F,A) \tilde{\cup} (G,B))^c$

2. Let $(F,A) \tilde{\cap} (G,B) \quad = (H,C)$, where $C = A \cap B$ and $\forall \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$

Thus $((F,A) \tilde{\cap} (G,B))^c = (H,C)^c \quad = (H^c,C)$, where $C = A \cap B$ and

$$\begin{aligned} \forall \varepsilon \in C, H^c(\varepsilon) &= (F(\varepsilon) \cap G(\varepsilon))^c \\ &= (F(\varepsilon))^c \cup (G(\varepsilon))^c \\ &= F^c(\varepsilon) \cup G^c(\varepsilon) \end{aligned}$$

Again, $(F,A)^c \tilde{\cup} (G,B)^c = (F^c,A) \tilde{\cup} (G^c,B) \; = (I,J)$, say, where $J = A \cup B$ and

$$\forall \varepsilon \in J, I(\varepsilon) = \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \in A\text{-}B \\ G^c(\varepsilon) & \text{if } \varepsilon \in B\text{-}A \\ F^c(\varepsilon) \cup G^c(\varepsilon) & \text{if } \varepsilon \in A \cap B \end{cases}$$

We see that $C \subseteq J$ and $\forall \varepsilon \in C, H^c(\varepsilon) \subseteq I(\varepsilon)$

It follows that $((F,A) \tilde{\cap} (G,B))^c \tilde{\subseteq} (F,A)^c \tilde{\cup} (G,B)^c$

Finally, regarding De Morgan Laws, we are applying our new notion of complement of a fuzzy soft set in the findings of Ahmad and Kharal [**1**] as follows.

3.3.7. Proposition (De Morgan Laws): For fuzzy soft sets (*F*, *A*) and (*G*, *A*) in a fuzzy soft class (*U*, *E*), one has the following:

1. $((F,A) \tilde{\cup} (G,A))^c = (F,A)^c \tilde{\cap} (G,A)^c$
2. $((F,A) \tilde{\cap} (G,A))^c = (F,A)^c \tilde{\cup} (G,A)^c$

Proof

1. Let $(F,A) \tilde{\cup} (G,A) \quad = (H,A)$, where $\forall \varepsilon \in A, H(\varepsilon) = F(\varepsilon) \cup G(\varepsilon)$

Thus $((F,A) \tilde{\cup} (G,A))^c = (H,A)^c \quad = (H^c,A)$,

Where
$$\begin{aligned} \forall \varepsilon \in A, H^c(\varepsilon) &= (H(\varepsilon))^c \\ &= (F(\varepsilon) \cup G(\varepsilon))^c \\ &= (F(\varepsilon))^c \cap (G(\varepsilon))^c \end{aligned}$$

$= F^c(\varepsilon) \cap G^c(\varepsilon)$

Again, $(F,A)^c \tilde{\cap} (G,A)^c = (F^c, A) \tilde{\cap} (G^c, A) = (I, A)$, say, where

$\forall \varepsilon \in A, I(\varepsilon) = F^c(\varepsilon) \cap G^c(\varepsilon)$

Thus $((F,A) \tilde{\cup} (G,A))^c = (F,A)^c \tilde{\cap} (G,A)^c$

2. Let $(F,A) \tilde{\cap} (G,A) = (H,A)$, where $\forall \varepsilon \in A, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$

Thus $((F,A) \tilde{\cap} (G,A))^c = (H,A)^c = (H^c, A)$, where

$$
\begin{aligned}
\forall \varepsilon \in A, H^c(\varepsilon) &= (H(\varepsilon))^c \\
&= (F(\varepsilon) \cap G(\varepsilon))^c \\
&= (F(\varepsilon))^c \cup (G(\varepsilon))^c \\
&= F^c(\varepsilon) \cup G^c(\varepsilon)
\end{aligned}
$$

Again, $(F,A)^c \tilde{\cup} (G,A)^c = (F^c, A) \tilde{\cup} (G^c, A) = (I, A)$, say, where

$\forall \varepsilon \in A, I(\varepsilon) = F^c(\varepsilon) \cup G^c(\varepsilon)$

Thus $((F,A) \tilde{\cap} (G,A))^c = (F,A)^c \tilde{\cup} (G,A)^c$

4. Conclusion

We have seen that if we use the extended definition of fuzzy set and newly defined complement of a fuzzy soft set, we arrive at the conclusion that the fuzzy soft sets follow the set theoretic axioms of exclusion and contradiction. We hope that our findings would help enhancing this study in fuzzy soft sets.

Acknowledgement

The authors are thankful to the reviewers and editor of this journal for their valuable suggestions and comments which have improved this work.

References

[1] B. Ahmad and A Kharal, "On Fuzzy Soft Sets", Advances in Fuzzy Systems, volume 2009, **(2009)**, pp. 1-6.

[2] D. A. Molodtsov, "Soft Set Theory - First Result", Computers and Mathematics with Applications, vol. 37, **(1999)**, pp. 19-31.

[3] Hemanta K. Baruah, "Towards Forming A Field of Fuzzy Sets", International Journal of Energy, Information and Communications, vol. 2, Issue 1, **(2011)** February, pp. 16-20.

[4] Hemanta K. Baruah, "The Theory of Fuzzy Sets: Beliefs and Realities", International Journal of Energy, Information and Communications, vol. 2, Issue 2, **(2011)** May, pp. 1-22.

[5] L. A. Zadeh, "Fuzzy Sets", Information and Control, 8, **(1965)**, pp. 338-353.

[6] P. K. Maji, R. Biswas and A. R. Roy, "Fuzzy Soft Sets", Journal of Fuzzy Mathematics, vol. 9, no. 3, **(2001)**, pp. 589-602.

[7] T. J. Neog, D. K. Sut, "Complement of an Extended Fuzzy Set" International Journal of Computer Applications, vol. 29, no. 3, **(2011)** September, pp. 39-45.

Authors

Tridiv Jyoti Neog received his M.Sc. degree in Mathematics from Dibrugarh University, India, in 2004. He is a research scholar in the department of Mathematics, Faculty of Science, CMJ University, Shillong, Meghalaya, India. He was awarded “BINANDI MEDHI MEMORIAL AWARD” for being the best graduate in B.Sc. examination, 2001, under Dibrugarh University.

Dusmanta Kumar Sut received his M.Sc. degree in Mathematics from Dibrugarh University, Dibrugarh, India, in 2002 and his Ph.d in Mathematics from Dibrugarh University, India, in 2007. He is an assistant professor in the department of Mathematics, N. N. Saikia College, Titabor, India. His research interests are in Fuzzy Mathematics, Fluid dynamics and Graph Theory.