

Arithmetic of Triangular Fuzzy Variable from Credibility Theory

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Abstract

In this article, it has been shown that the credibility distribution of triangular fuzzy variable leads us to find a very simple alternative method of finding the membership function for functions of triangular fuzzy variables. This concept of credibility theory also leads us in finding an alternative method of computing basic arithmetic operations of triangular fuzzy variables and generalised membership function for the root of triangular fuzzy variable. The method has been demonstrated with the help of some examples.

Keywords: Credibility distribution, Credibility density function, Credibility measure, Membership function.

1. Introduction

The basic arithmetic operation of fuzzy number has been developed well through the years. Indeed Chou [4, 5] has actually developed a method of finding the membership function of the square root and cube root of a triangular fuzzy number. Although there are many arithmetical operation approaches, none of the approaches presents a generalised method for finding the n^{th} root of the fuzzy number.

Zadeh [7] proposed the concept of possibility measure and thereafter this has been widely used in solving fuzzy problems. The necessity measure is defined as a dual part of possibility measure. However, both possibility measure and necessity measure are not self-dual. In order to define a self-dual measure, Liu and Liu [1] present the concept of credibility measure.

In this paper the triangular fuzzy variable are considered and we try to develop a generalised method for finding the membership function for functions of triangular fuzzy variables, and very well applied in dealing the basic arithmetical operations of triangular fuzzy variables, based on the concept of credibility distribution.

2. Preliminaries

2.1. Triangular Fuzzy Variable

A fuzzy variable determined by the triplet $\beta = [a, b, c]$ of crisp number with $(a < b < c)$ and whose membership function is given by

$$\mu_{\beta}(x) = \begin{cases} \frac{x-a}{b-a}, & \text{if } a \leq x \leq b \\ \frac{c-x}{c-b}, & \text{if } b \leq x \leq c \\ 0, & \text{otherwise} \end{cases} \quad (2.1.1)$$

is called a triangular fuzzy variable.

2.2. Credibility Measure

Let Θ be a non-empty set, and P the power set of Θ and $A \in P$, Liu and Liu [1] defined the credibility measure as

$$Cr\{A\} = \frac{1}{2}(Pos\{A\} + Nec\{A\}) \quad (2.2.1)$$

Furthermore, for any $A \in P$ we have

$$Pos\{A\} = \min(2Cr\{A\}, 1) \quad (2.2.2)$$

Li and Liu [8] defined that a set function Cr is a credibility measure if it holds the following

1. $Cr\{\Theta\} = 1$.
2. $Cr\{A\} \leq Cr\{B\}$ whenever $A \subset B$
3. $Cr\{A\} + Cr\{A^c\} = 1$ for any event A .
4. $Cr\{\cup_i A_i\} = \sup_i Cr\{A_i\}$ for any events $\{A_i\}$ with $\sup_i Cr\{A_i\} < 0.5$.

Let ξ be a fuzzy variable defined on the credibility space (Θ, P, Cr) . Then its membership function is defined from the credibility measure by

$$\mu_{\xi}(x) = \min(2Cr\{\xi = x\}, 1). \quad (2.2.3)$$

2.3. Credibility Distribution

Liu [2] defined credibility distribution as $\Phi_{\xi} : R \rightarrow [0, 1]$ of any fuzzy variable ξ as

$$\Phi_{\xi}(x) = Cr\{\theta \in \Theta : \xi(\theta) \leq x\}.$$

That is the credibility that the fuzzy variable ξ takes a value less than or equal to x . If the fuzzy variable ξ is given by a membership function μ , then its credibility distribution is determined by

$$\Phi_{\xi}(x) = \frac{1}{2} \{ \sup_{y \leq x} \mu(y) + 1 - \sup_{y > x} \mu(y) \}, \forall x \in R.$$

2.4. Credibility Distribution of Triangular Fuzzy Variable

The credibility distribution of a triangular fuzzy variable $\beta = [a, b, c]$ is given by

$$\Phi_{\beta}(x) = Cr\{\beta \leq x\} = \begin{cases} 0, & \text{if } x < a \\ \frac{x-a}{2(b-a)}, & \text{if } a \leq x < b \\ \frac{x+c-2b}{2(c-b)}, & \text{if } b \leq x < c \\ 1, & \text{if } x \geq c \end{cases} \quad (2.4.1)$$

2.5. Credibility Density Function

The credibility density function defined by Liu [3] as $\phi_{\xi} : R \rightarrow [0, \infty)$ of any fuzzy variable ξ is a function such that

$$\phi_{\xi}(x) = \int_{-\infty}^x \phi(y) dy, \forall x \in R.$$

2.6. Credibility Density Function of Triangular Fuzzy Variable,

The credibility density function of a triangular fuzzy variable $\beta = [a, b, c]$ is given by

$$\phi_{\beta}(x) = \begin{cases} f(x) = \frac{1}{2(b-a)}, & \text{if } a \leq x \leq b \\ g(x) = \frac{1}{2(c-b)}, & \text{if } b \leq x \leq c \\ 0, & \text{otherwise} \end{cases} \quad (2.6.1)$$

3. The Membership Function for Functions of Triangular Fuzzy Variable

Let $X = [a, b, c], (a, b, c > 0)$, be a triangular fuzzy variable. Let $F(X) = [F(a), F(b), F(c)]$, be the fuzzy variable of the function $F(X)$. Suppose the membership function of X is given by (2.1.1). The credibility distribution function is as given in (2.4.1) and the credibility density function is as given in (2.6.1).

Let $y = F(x)$ or, $x = \phi(y)$ or, $\left| \frac{dx}{dy} \right| = \frac{d}{dy}(\phi(y)) = m(y)$, say. Replacing x by $\phi(y)$ in $f(x)$ and $g(x)$, we obtain $f(x) = \phi_1(y)$ and $g(x) = \phi_2(y)$, say. Then the credibility distribution function of $F(X)$ is

$$\Phi_x(x) = \begin{cases} 0, & \text{if } x < F(a) \\ \Phi_1(x) = \int_{F(a)}^x \phi_1(y)m(y)dy, & \text{if } F(a) \leq x < F(b) \\ \Phi_2(x) = 1 + \int_{F(c)}^x \phi_2(y)m(y)dy, & \text{if } F(b) \leq x < F(c) \\ 1, & \text{if } x \geq F(c) \end{cases}$$

Thus the membership function of the fuzzy variable $F(X)$ is given by

$$\mu_{F(X)}(x) = \begin{cases} 2\Phi_1(x), & \text{if } F(a) \leq x < F(b) \\ 2(1 - \Phi_2(x)), & \text{if } F(b) \leq x < F(c) \\ 0, & \text{otherwise} \end{cases}$$

3.1. Inverse of a Triangular Fuzzy Variable

Consider a triangular fuzzy variable $X = [a, b, c]$, $(a, b, c > 0)$, with membership function given in (2.1.1) and let $X^{-1} = [c^{-1}, b^{-1}, a^{-1}]$. The credibility distribution function and the credibility density function are as given in (2.4.1) and (2.6.1) respectively.

Let $y = \frac{1}{x}$ so that $\left| \frac{dx}{dy} \right| = \frac{1}{y^2}$. The credibility distribution of X^{-1} is

$$\Phi_{X^{-1}}(x) = \begin{cases} 0, & \text{if } x < c^{-1} \\ \int_{c^{-1}}^x \frac{1}{2(c-b)} \left(\frac{1}{y^2} \right) dy, & \text{if } c^{-1} \leq x < b^{-1} \\ 1 + \int_{a^{-1}}^x \frac{1}{2(b-a)} \left(\frac{1}{y^2} \right) dy, & \text{if } b^{-1} \leq x < a^{-1} \\ 1, & \text{if } x \geq a^{-1} \end{cases}$$

The membership function of X^{-1} is

$$\mu_{X^{-1}}(x) = \begin{cases} \frac{cx-1}{x(c-b)} & \text{if } c^{-1} \leq x < b^{-1} \\ \frac{1-ax}{x(b-a)}, & \text{if } b^{-1} \leq x < a^{-1} \\ 0, & \text{otherwise} \end{cases}$$

3.2. Negative of a Triangular Fuzzy Variable

Consider a triangular fuzzy variable $X = [a, b, c]$, $(a, b, c > 0)$, with membership function as given in (2.1.1) and let $-X = [-c, -b, -a]$. The credibility distribution function and the credibility density function are as given in (2.4.1) and (2.6.1) respectively.

Let $y = -x$, so that $\left| \frac{dx}{dy} \right| = 1$. The credibility distribution of $-X$ is

$$\Phi_{-X}(x) = \begin{cases} 0, & \text{if } x < -c \\ \frac{x+c}{2(c-b)}, & \text{if } -c \leq x < -b \\ \frac{x-a+2b}{2(b-a)}, & \text{if } -b \leq x < -a \\ 1, & \text{if } x \geq -a \end{cases}$$

Then the membership function of $-X$ is

$$\mu_{(-X)}(x) = \begin{cases} \frac{x+c}{c-b}, & \text{if } -c \leq x \leq -b \\ \frac{x+a}{a-b}, & \text{if } -b \leq x \leq -a \\ 0, & \text{otherwise} \end{cases}$$

3.3. Square Root of a Triangular Fuzzy Variable

Consider a triangular fuzzy variable $X = [a, b, c], (a, b, c > 0)$, with membership function is as given in (2.1.1), and let $\sqrt{X} = [\sqrt{a}, \sqrt{b}, \sqrt{c}]$. Accordingly the credibility distribution and the credibility density function are as given in (2.4.1) and (2.6.1) respectively.

Let $y = \sqrt{x}$ so that $\left| \frac{dx}{dy} \right| = 2y$. The credibility distribution function for \sqrt{X} , is

$$\Phi_{\sqrt{X}}(x) = \begin{cases} 0, & \text{if } x < \sqrt{a} \\ \frac{x^2 - a}{2(b-a)}, & \text{if } \sqrt{a} \leq x < \sqrt{b} \\ \frac{x^2 + c - 2b}{2(c-b)}, & \text{if } \sqrt{b} \leq x < \sqrt{c} \\ 1, & \text{if } x \geq \sqrt{c} \end{cases}$$

Then the membership function of the fuzzy variable \sqrt{X} is

$$\mu_{\sqrt{X}}(x) = \begin{cases} \frac{x^2 - a}{b - a}, & \text{if } \sqrt{a} \leq x \leq \sqrt{b} \\ \frac{c - x^2}{c - b}, & \text{if } \sqrt{b} \leq x \leq \sqrt{c} \\ 0, & \text{otherwise} \end{cases}$$

3.4. n^{th} Root of a Triangular Fuzzy Variable

Consider a triangular fuzzy variable $X = [a, b, c], (a, b, c > 0)$, with membership function as given in (2.1.1), and let $\sqrt[n]{X} = [\sqrt[n]{a}, \sqrt[n]{b}, \sqrt[n]{c}]$. Accordingly the credibility

distribution and the credibility density function are as given in (2.4.1) and (2.6.1) respectively.

Let $y = \sqrt[n]{x}$ so that $\left| \frac{dx}{dy} \right| = ny^{n-1}$. The credibility distribution for $\sqrt[n]{X}$ is

$$\Phi_{\sqrt[n]{X}}(x) = \begin{cases} 0, & \text{if } x < \sqrt[n]{a} \\ \frac{x^n - a}{2(b-a)}, & \text{if } \sqrt[n]{a} \leq x < \sqrt[n]{b} \\ \frac{x^n + c - 2b}{2(c-b)}, & \text{if } \sqrt[n]{b} \leq x < \sqrt[n]{c} \\ 1, & \text{if } x \geq \sqrt[n]{c} \end{cases}$$

Then the membership function of n^{th} root of the fuzzy variable X is

$$\mu_{\sqrt[n]{X}}(x) = \begin{cases} \frac{x^n - a}{b-a}, & \text{if } \sqrt[n]{a} \leq x < \sqrt[n]{b} \\ \frac{c - x^n}{c-b}, & \text{if } \sqrt[n]{a} \leq x < \sqrt[n]{b} \\ 0, & \text{otherwise} \end{cases}$$

4. Arithmetic of Triangular Fuzzy Variables

4.1. Addition of Fuzzy Variable

Consider the fuzzy variables $X = [a, b, c]$ and $Y = [p, q, r]$. Suppose $Z = X + Y = [a + p, b + q, c + r]$ be the fuzzy number of $X + Y$. Let the membership function of X and Y be $\mu_X(x)$ and $\mu_Y(y)$, where,

$$\mu_X(x) = \begin{cases} L(x), & \text{if } a \leq x \leq b \\ R(x), & \text{if } b \leq x \leq c \\ 0, & \text{otherwise} \end{cases} \quad (4.1.1)$$

$$\mu_Y(y) = \begin{cases} L(y), & \text{if } p \leq y \leq q \\ R(y), & \text{if } q \leq y \leq r \\ 0, & \text{otherwise} \end{cases} \quad (4.1.2)$$

Let the credibility distribution of the triangular fuzzy variables (4.1.1) and (4.1.2) are

$$\Phi_X(x) = \begin{cases} 0, & \text{if } x < a \\ \Phi_1(x), & \text{if } a \leq x < b \\ \Phi_2(x), & \text{if } b \leq x < c \\ 1, & \text{if } x \geq c \end{cases} \quad (4.1.3)$$

$$\Phi_Y(y) = \begin{cases} 0, & \text{if } y < p \\ \Phi_1(y), & \text{if } p \leq y < q \\ \Phi_2(y), & \text{if } q \leq y < r \\ 1, & \text{if } y \geq c \end{cases} \quad (4.1.4)$$

Let the credibility density function of the credibility distribution function (4.1.3) is

$$\phi_x(x) = \begin{cases} f(x), & \text{if } a \leq x < b \\ g(x), & \text{if } b \leq x < c \end{cases} \quad (4.1.5)$$

We start with equating $\Phi_1(x)$ with $\Phi_1(y)$ and $\Phi_2(x)$ with $\Phi_2(y)$. And so, we obtain $y = \psi_1(x)$ and $y = \psi_2(x)$ respectively. Let $z = x + y$, so we have $z = x + \psi_1(x)$ and $z = x + \psi_2(x)$, so that $x = \omega_1(x)$ and $x = \omega_2(x)$, say. The credibility density function (4.1.5) should be transformed in terms of the function of z by replacing x by $\omega_1(x)$ and $\omega_2(x)$ in $f(x)$ and $g(x)$, so that $f(x) = \chi_1(z)$ and $g(x) = \chi_2(z)$, say.

Now let, $\left| \frac{dx}{dz} \right| = \left| \frac{d}{dz}(\omega_1(z)) \right| = m_1(z)$, and $\left| \frac{dx}{dz} \right| = \left| \frac{d}{dz}(\omega_2(z)) \right| = m_2(z)$. The credibility distribution function of $X + Y$ is

$$\Phi_{X+Y}(x) = \begin{cases} 0, & \text{if } x < a + p \\ \int_{a+p}^x \chi_1(z)m_1(z)dz, & \text{if } a + p \leq x < b + q \\ 1 + \int_{c+r}^x \chi_2(z)m_2(z)dz, & \text{if } b + q \leq x < c + r \\ 1, & \text{if } x \geq c + r \end{cases}$$

Thus the membership function of addition of the fuzzy variables can be easily found from the credibility distribution function $\Phi_{X+Y}(x)$.

4.2. Subtraction of Fuzzy Variable

Consider the triangular fuzzy variables $X = [a, b, c]$ and $Y = [p, q, r]$. Suppose $Z = X - Y$, then the membership function of $Z = X - Y$ is given by $Z = X + (-Y)$.

4.3. Multiplication of Fuzzy Variables

Consider the triangular fuzzy variables $X = [a, b, c], (a, b, c > 0)$ and $Y = [p, q, r], (p, q, r > 0)$. Suppose $Z = X.Y = [a.p, b.q, c.r]$ be the fuzzy variable of $X.Y$. Let the membership function of X and Y be $\mu_x(x)$ and $\mu_y(y)$ as mentioned in (4.1.1) and (4.1.2) respectively. Let the credibility distribution of (4.1.1) and (4.1.2) are as mentioned in (4.1.3) and (4.1.4) respectively. Let the credibility density function of the credibility distribution function (4.1.3) is (4.1.5).

We start with equating $\Phi_1(x)$ with $\Phi_1(y)$ and $\Phi_2(x)$ with $\Phi_2(y)$. And so, we obtain $y = \psi_1(x)$ and $y = \psi_2(x)$ respectively. Let $z = x.y$, so we have $z = x.\psi_1(x)$ and $z = x.\psi_2(x)$, so that $x = \omega_1(x)$ and $x = \omega_2(x)$, say. The credibility density function (4.1.5) should be transformed in terms of the function of z by replacing x by $\omega_1(x)$ and $\omega_2(x)$ in $f(x)$ and $g(x)$, so that $f(x) = \chi_1(z)$ and $g(x) = \chi_2(z)$, say.

Now let, $\left| \frac{dx}{dz} \right| = \left| \frac{d}{dz}(\omega_1(z)) \right| = m_1(z)$ and $\left| \frac{dx}{dz} \right| = \left| \frac{d}{dz}(\omega_2(z)) \right| = m_2(z)$. The credibility distribution function of $X.Y$, is given by

$$\Phi_{X.Y}(x) = \begin{cases} 0, & \text{if } x < a.p \\ \int_{a.p}^x \chi_1(z)m_1(z)dz, & \text{if } a.p \leq x < b.q \\ 1 + \int_{c.r}^x \chi_2(z)m_2(z)dz & \text{if } b.q \leq x < c.r \\ 1, & \text{if } x \geq c.r \end{cases}$$

Thus the membership function of multiplication of the fuzzy variables can be easily found from the credibility distribution function $\Phi_{X.Y}(x)$.

4.4. Division of Fuzzy Variables

Consider the triangular fuzzy variables $X = [a, b, c], (a, b, c > 0)$ and $Y = [p, q, r], (p, q, r > 0)$. Suppose $Z = \frac{X}{Y}$ then membership function of $Z = \frac{X}{Y}$ is given by $Z = X.Y^{-1}$.

5. Numerical Examples

Example

Consider two triangular fuzzy variables $X = [12, 16, 28]$ and $Y = [5, 6, 8]$ with membership function as given below, and let $Z = \frac{X}{Y} = \left[\frac{3}{2}, \frac{8}{3}, \frac{28}{5} \right]$.

$$\mu_X(x) = \begin{cases} \frac{x-12}{4}, & \text{if } 12 \leq x \leq 16 \\ \frac{84-3x}{36}, & \text{if } 16 \leq x \leq 28 \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_Y(y) = \begin{cases} y-5, & \text{if } 5 \leq y \leq 6 \\ \frac{8-y}{2}, & \text{if } 6 \leq y \leq 8 \\ 0, & \text{otherwise} \end{cases}$$

The credibility distribution function and credibility density function of $\mu_X(x)$ are as below respectively.

$$\Phi_X(x) = \begin{cases} 0, & \text{if } x < 12 \\ \frac{x-12}{8}, & \text{if } 12 \leq x < 16 \\ \frac{x-4}{24}, & \text{if } 16 \leq x < 28 \\ 1, & \text{if } x \geq 28 \end{cases}$$

$$\phi_X(z) = \begin{cases} \chi_1(z) = \frac{1}{8}, & \text{if } 12 \leq z < 16 \\ \chi_2(z) = \frac{1}{24}, & \text{if } 16 \leq z < 28 \end{cases}$$

The credibility distribution function of Y^{-1} is found to be

$$\Phi_{Y^{-1}}(y) = \begin{cases} 0, & \text{if } x < 8^{-1} \\ \frac{8y-1}{4y}, & \text{if } 8^{-1} \leq y < 6^{-1} \\ \frac{7y-1}{2y}, & \text{if } 6^{-1} \leq y < 5^{-1} \\ 1, & \text{if } y \geq 5^{-1} \end{cases}$$

Now equating the credibility distribution functions of X and Y^{-1} , we have

$$\left| \frac{dx}{dz} \right| = \frac{56}{(z+2)^2} = m_1(z) \quad \text{and similarly} \quad \left| \frac{dx}{dz} \right| = \frac{1056}{(z+12)^2} = m_2(z) .$$

Thus the credibility distribution function of $Z = X.Y^{-1}$ is of the form

$$\Phi_{X.Y^{-1}}(x) = \begin{cases} 0, & \text{if } x < 3/2 \\ \frac{2x-3}{x+2}, & \text{if } 3/2 \leq x < 8/3 \\ \frac{7x-4}{2(x+12)}, & \text{if } 8/3 \leq x < 28/5 \\ 1, & \text{if } x \geq 28/5 \end{cases}$$

Thus the membership function of the fuzzy variable $X.Y^{-1}$ is,

$$\mu_{X.Y^{-1}}(x) = \begin{cases} \frac{4x-6}{x+2}, & \text{if } 3/2 \leq x \leq 8/3 \\ \frac{28-5x}{x+12}, & \text{if } 8/3 \leq x \leq 28/5 \\ 0, & \text{otherwise} \end{cases}$$

Example

Consider two triangular fuzzy variables $X = [7,14,19]$ and $Y = [3,5,10]$ with membership function as given below, and let $Z = X - Y = [-3,9,16]$.

$$\mu_X(x) = \begin{cases} \frac{x-7}{7}, & \text{if } 7 \leq x \leq 14 \\ \frac{19-x}{5}, & \text{if } 14 \leq x \leq 19 \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_Y(y) = \begin{cases} \frac{y-3}{2}, & \text{if } 3 \leq y \leq 5 \\ \frac{10-y}{2}, & \text{if } 5 \leq y \leq 10 \\ 0, & \text{otherwise} \end{cases}$$

The credibility distribution function and credibility density function of $\mu_x(x)$ are as below respectively.

$$\Phi_X(x) = \begin{cases} 0, & \text{if } x < 7 \\ \frac{x-7}{8}, & \text{if } 7 \leq x < 14 \\ \frac{x-9}{10}, & \text{if } 14 \leq x < 19 \\ 1, & \text{if } x \geq 19 \end{cases}$$

$$\phi_X(z) = \begin{cases} \chi_1(z) = \frac{1}{8}, & \text{if } 7 \leq z < 14 \\ \chi_2(z) = \frac{1}{10}, & \text{if } 14 \leq z < 19 \end{cases}$$

The credibility distribution function of $-Y$ is found to be

$$\Phi_{-Y}(y) = \begin{cases} 0, & \text{if } y < -10 \\ \frac{y-10}{10}, & \text{if } -10 \leq y < -5 \\ \frac{y+7}{4}, & \text{if } -5 \leq y < -3 \\ 1, & \text{if } y \geq -3 \end{cases}$$

Now equating the credibility distribution functions of X and $-Y$, we have $\left|\frac{dx}{dz}\right| = \frac{7}{12} = m_1(z)$ and similarly $\left|\frac{dx}{dz}\right| = \frac{5}{7} = m_2(z)$. Thus the credibility distribution function of $Z = X + (-Y)$ is of the form

$$\Phi_{X+(-Y)}(x) = \begin{cases} 0, & \text{if } x < -3 \\ \frac{x+3}{24}, & \text{if } -3 \leq x < 9 \\ \frac{x-2}{14}, & \text{if } 9 \leq x < 16 \\ 1, & \text{if } x \geq 16 \end{cases}$$

Thus the membership function of the fuzzy variable $X + (-Y)$ is

$$\mu_{X+(-Y)}(x) = \begin{cases} \frac{x+3}{12}, & \text{if } -3 \leq x \leq 9 \\ \frac{16-x}{7}, & \text{if } 9 \leq x \leq 16 \\ 0, & \text{otherwise} \end{cases}$$

Example

Consider two triangular fuzzy variables $X = [2,3,5]$ and $Y = [3,5,6]$ with membership function as given below, and let $Z = X.Y = [6,15,30]$

$$\mu_X(x) = \begin{cases} x-2, & \text{if } 2 \leq x \leq 3 \\ \frac{5-x}{2}, & \text{if } 3 \leq x \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_Y(y) = \begin{cases} \frac{y-3}{2}, & \text{if } 3 \leq y \leq 5 \\ 6-y, & \text{if } 5 \leq y \leq 6 \\ 0, & \text{otherwise} \end{cases}$$

The credibility distribution functions of $\mu_X(x)$ and $\mu_Y(y)$ are as below respectively.

$$\Phi_X(x) = \begin{cases} 0, & \text{if } x < 2 \\ \frac{x-2}{2}, & \text{if } 2 \leq x < 3 \\ \frac{x-1}{4}, & \text{if } 3 \leq x < 5 \\ 1, & \text{if } x \geq 5 \end{cases}$$

$$\Phi_Y(y) = \begin{cases} 0, & \text{if } y < 3 \\ \frac{y-3}{4}, & \text{if } 3 \leq y < 5 \\ \frac{y-4}{2}, & \text{if } 5 \leq y < 6 \\ 1, & \text{if } y \geq 6 \end{cases}$$

The credibility density function of the credibility distribution function $\Phi_X(x)$

$$\phi_X(z) = \begin{cases} \chi_1(z) = \frac{1}{2}, & \text{if } 2 \leq z < 3 \\ \chi_2(z) = \frac{1}{4}, & \text{if } 3 \leq z < 5 \end{cases}$$

Now equating the credibility distribution functions of X and Y , we have

$$\left| \frac{dx}{dz} \right| = \frac{1}{\sqrt{1+8z}} = m_1(z) \quad \text{and similarly} \quad \left| \frac{dx}{dz} \right| = \frac{2}{\sqrt{49+8z}} = m_2(z).$$

Thus the credibility distribution function of $Z = X.Y$ is of the form

$$\Phi_{X.Y}(x) = \begin{cases} 0, & \text{if } x < 6 \\ \frac{-7 + \sqrt{1+8x}}{4}, & \text{if } 6 \leq x < 15 \\ \frac{\sqrt{49+8x} - 9}{8}, & \text{if } 15 \leq x < 30 \\ 1, & \text{if } x \geq 30 \end{cases}$$

Thus the membership function of the fuzzy variable $X.Y$ is

$$\mu_{X.Y}(x) = \begin{cases} \frac{-7 + \sqrt{1+8x}}{4}, & \text{if } 6 \leq x \leq 15 \\ \frac{\sqrt{49+8x} - 17}{4}, & \text{if } 15 \leq x \leq 30 \\ 0, & \text{otherwise} \end{cases}$$

These examples have been quoted from the book by Bojadziev and Bojadziev [6], the result tallies with it, where it has been solved by the method of α -cuts.

6. Conclusion

Here we have tried to develop an alternative method of finding the membership function for functions of triangular fuzzy variable from the concept of credibility theory. A new method for computation of basic arithmetical operations of fuzzy variable is forwarded. This method validity has been tested evaluating some examples which were quoted from [6], where they were solved by the method of alpha-cuts and are compared with the results obtained here. The square root of triangular fuzzy variable found by our proposed method tallies with the one defined by Chou [2]. A generalised membership function for the n^{th} root of triangular fuzzy variable has been forwarded. The method can be applied in solving equations with fuzzy coefficients. Further the proposed method can be applied to the uncertainty analysis of dispersion models and engineering problems which can be taken for further research.

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