Theory of Fuzzy Sets: The Case of Subnormality

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Abstract

The theory of fuzzy sets with reference to subnormality needs to be properly explained. We shall in this article put forward the actual mathematical explanation of subnormal partial presence of an element in a set, better known as subnormal fuzziness. We shall first show that every law of subnormal fuzziness can be expressed in terms of two Lebesgue-Stieltjes measures. Thereafter we would define the complement of a subnormal fuzzy set in such a way that subnormal fuzzy numbers form a field.

Keywords**:** *Subnormal fuzzy numbers.*

1. Introduction

Using a set operation called superimposition and the Glivenko-Cantelli Theorem - a theorem on Order Statistics, we have recently explained in [1] that with reference to a normal fuzzy number the following theorem linking fuzziness and randomness holds:

Theorem-1: For a *normal* fuzzy number $N = [\alpha, \beta, \gamma]$ with membership function

$$\mu_N(x) = \Psi_1(x), \text{ if } \alpha \le x \le \beta,$$
$$= \Psi_2(x), \text{ if } \beta \le x \le \gamma, \text{ and}$$
$$= 0, \text{ otherwise},$$

such that $\Psi_1(\alpha) = \Psi_2(\gamma) = 0$, $\Psi_1(\beta) = \Psi_2(\beta) = 1$, where $\Psi_1(x)$ is the distribution function of a random variable defined in the interval $[\alpha, \beta]$, and $\Psi_2(x)$ is the complementary distribution function of another random variable defined in the interval $[\beta, \gamma]$, with randomness defined in the measure theoretic sense.

In other words, every normal law of fuzziness can be explained with the help of two laws of randomness. Hence to construct a normal fuzzy number, we need to have two laws of randomness, one defining the left reference function and the other defining the right reference function keeping in view the Dubois-Prade definition of a normal fuzzy number. In effect, two probability measures together can define a law of fuzziness.

It has been further illustrated in [1] that the complement of a normal fuzzy number has actually to be defined as follows. For a normal fuzzy number $N = [\alpha, \beta, \gamma]$ as defined above, the complement N^C will have the membership function $\mu_N{}^C(x)$, where $\mu_N{}^C(x) = 1$, $-\infty < x < \infty$, where the values of $\mu_N{}^C(x)$ are to be counted from $\Psi_1(x)$, if $\alpha \le x \le \beta$, from $\Psi_2(x)$, if $\beta \le x \le \gamma$, and from 0, otherwise, so that there happens to be a difference between a fuzzy membership *function* and the corresponding fuzzy membership *values*. This definition of the complement of a fuzzy set is based on the following axiom:

Axiom-1: The fuzzy membership function of the complement of a *normal* fuzzy number is equal to 1 for the entire real line, with the membership values counted not from zero but from the membership function of the fuzzy number concerned.

We shall now extend the theorem stated above to deal with subnormal fuzzy numbers. Further, based on an extension of the axiom stated above, we would define the complement of a subnormal fuzzy number so that subnormal fuzzy numbers qualify to define a field. Subnormality of fuzzy numbers can be of importance for the following two reasons. First, a normal fuzzy number is a special case of a subnormal fuzzy number in the sense that a subnormal fuzzy number is nothing but a generalized fuzzy number. Accordingly, unless the concept of subnormal fuzziness is clear, that of normal fuzziness itself would remain unclear. Secondly, in order to define a fuzzy vector, we would need to define a fuzzy membership surface, and to define a fuzzy membership surface we may take help of fuzzy membership surface sections, which in the two dimensions is nothing but the membership function of a subnormal fuzzy number.

We have shown in [1] that most of the basics of the mathematical theory of fuzzy sets have been improperly explained, and that the theory needs a complete overhauling. Based on the theorem and on the axiom stated above, in this article we shall put forward the mathematical explanation of *subnormal partial presence* of an element in a set, better known as *subnormal fuzziness*. There are misunderstandings regarding subnormal fuzzy sets too. We are, in what follows, going to explain the mathematics behind subnormality of fuzzy sets, as a special case of which one can study the theory of fuzzy sets.

It is obvious that if a principle is incorrect, logically and therefore mathematically, its generalizations would surely be incorrect in turn. We would now like to cite a particular example of how subnormality of fuzzy numbers has been wrongly defined by some workers, which is why we have chosen to put forward the proper explanation of exactly how a subnormal fuzzy number has to be constructed.

We would like to refer to a work done by Sheen [2] who suggested a method of *probabilistic conversion* of fuzzy numbers. The author has gone for conversion of the membership function $\mu(x)$ of a fuzzy number into an equivalent *probability density function* by using one of the following two transformations:

1) proportional probability density function : $p(x) = k_p\, \mu(x)$, and
2) uniform probability density function : $u(x) = \mu(x) + k_u$

where k_p and k_u are values of the conversion constants which ensure that the area under the continuous probability function is equal to 1.

As we can see, Sheen's first method of conversion is similar to Lemaire's conversion [3] of a subnormal fuzzy number to a normal fuzzy number. Lemaire had suggested that a subnormal fuzzy set can be made normal by using the conversion

$$\mu_A(x) = \upsilon_A(x) / \sup(\upsilon_A(x))$$

where $\upsilon_A(x)$ is the membership function of a subnormal fuzzy number defined for any x in the interval A.

Depending on the value of k_p in $p(x) = k_p\, \mu(x)$

one might end up getting a *subnormal fuzzy number* also. If Sheen is correct, the subnormal fuzzy number defined by Lemaire could actually be a probability density function too! Obviously, not both Sheen and Lemaire could be correct at the same time. While Lemaire stayed in fuzziness after his conversion from subnormality to normality, Sheen has ended up defining the very concept of probability density function in his own way. In [1], we have proved that two laws of randomness can define one law of fuzziness, and therefore conversion of a fuzzy membership function to a probability density function is meaningless. We are not interested to raise that point again. What we are interested now is to discuss the misconceptions regarding construction of subnormal fuzzy numbers. After all, had Sheen been correct, and therefore had Lemaire been wrong, the theory of probability and the theory of subnormal fuzzy sets would in fact have been the same, which is simply not true anyway. We would like to emphasize that Lemaire's conversion is indeed correct, and therefore Sheen's conversion must necessarily be wrong.

As far as the *uniform probability density* referred by Sheen in the second conversion is concerned, the very nomenclature used here is improper. It is well known that in the theory of probability, the uniform probability density function is defined as one that takes a constant value in the interval of reference. In this case, the usage of the words *uniform probability density* has been made in an altogether different sense anyway. Further, here too, the author went for a conversion of a membership function to a probability density function. Hence, our objection remains the same with reference to this case also. We can not get a law of randomness from a law of fuzziness, and therefore such conversions are against the philosophy of mathematics.

2. Subnormal Fuzzy Numbers

Let $A = [a, b, c]$ be a fuzzy number with a continuous non-decreasing left reference function $L(x)$ and a continuous right reference function $R(x)$, such that the membership function is given by $\mu_A(x)$ is $L(x)$, $a \le x \le b$, $R(x)$, $b \le x \le c$, and is 0, otherwise, with $L(a) = R(c) = 0$, and $L(b) = R(b) = T$, $0 \le T < 1$. A here is subnormal. For $T = 1$, A will be a normal fuzzy number. For example, for $T = ½$, $\mu_A(x)$ is $(x - 2)/4$, $2 \le x \le 4$, is $½ - (x - 4)/4$, $4 \le x \le 6$, and is 0 otherwise, A will be a subnormal triangular fuzzy number.

We now proceed to study the structure of such a subnormal fuzzy number from our perspective ([4], pages 46-51). We have shown in [1] that a *normal* fuzzy number can be expressed with the help of two probability laws. We are now going to discuss how to express a *subnormal* fuzzy number with the help of two measures. So that the present article is readable independently of our earlier work, we would first discuss in short our standpoint of defining fuzzy numbers.

It can be seen that for two intervals $A = [a_1, b_1]$ and $B = [a_2, b_2]$, superimposition of the intervals would give us

$$[a_1, b_1]\ (S)\ [a_2, b_2] = [a_{(1)}, a_{(2)}] \cup [a_{(2)}, b_{(1)}]^{(2)} \cup [b_{(1)}, b_{(2)}]$$

where $a_{(1)} = \min(a_1, a_2)$, $a_{(2)} = \max(a_1, a_2)$, $b_{(1)} = \min(b_1, b_2)$, $b_{(2)} = \max(b_1, b_2)$, with $[a_{(2)}, b_{(1)}]^{(2)}$ representing the interval $[a_{(2)}, b_{(1)}]$ all elements being *doubly* present.

Now let $[a_1, b_1]^{(T/2)}$ and $[a_2, b_2]^{(T/2)}$ represent two *uniformly* fuzzy intervals both with membership value equal to $T/2$ everywhere. Then superimposition of the two sets $[a_1, b_1]^{(T/2)}$ and $[a_2, b_2]^{(T/2)}$ would give rise to $[a_1, b_1]^{(T/2)}\ (S)\ [a_2, b_2]^{(T/2)}$

$$= [a_{(1)}, a_{(2)}]^{(T/2))} \cup [a_{(2)}, b_{(1)}]^{(T)} \cup [b_{(1)}, b_{(2)}]^{(T/2)}.$$

This is like placing one translucent paper over another of equal opacity $T/2$ to get the opacity doubled as a result in the common portion.

So for n fuzzy intervals $[a_1, b_1]^{(T/n)}$, $[a_2, b_2]^{(T/n)}$, …, $[a_n, b_n]^{(T/n)}$

all with membership value equal to T/n everywhere, we shall have

$$[a_1, b_1]^{(T/n)} \text{ (S) } [a_2, b_2]^{(T/n)} \text{ (S) } \ldots\ldots\ldots \text{ (S) } [a_n, b_n]^{(T/n)}$$

$$= [a_{(1)}, a_{(2)}]^{(T/n)} \cup [a_{(2)}, a_{(3)}]^{(2T/n)} \cup \ldots\ldots\ldots \cup [a_{(n-1)}, a_{(n)}]^{((n-1)\,T/n)}$$

$$\cup [a_{(n)}, b_{(1)}]^{(T)} \cup [b_{(1)}, b_{(2)}]^{((n-1)\,T/n)} \cup \ldots\ldots\ldots \cup [b_{(n-2)}, b_{(n-1)}]^{(2T/n)} \cup [b_{(n-1)}, b_{(n)}]^{(T/n)},$$

where for example $[b_{(1)}, b_{(2)}]^{((n-1)\,T/n)}$ represents the uniformly fuzzy interval $[b_{(1)}, b_{(2)}]$ with membership $((n\text{-}1)\ T/n)$ in the entire interval, $a_{(1)}, a_{(2)}, \ldots\ldots\ldots, a_{(n)}$ being values of $a_1, a_2, \ldots\ldots\ldots, a_n$ arranged in increasing order of magnitude, and $b_{(1)}, b_{(2)}, \ldots\ldots\ldots, b_{(n)}$ being values of $b_1, b_2, \ldots\ldots\ldots, b_n$ arranged in increasing order of magnitude.

Following the Dubois – Prade definition of a normal fuzzy number, let us now define a *subnormal* fuzzy number $N = [\alpha, \beta, \gamma]$ with membership function

$$\begin{aligned} \mu_N(x) &= \Psi_1(x), \text{ if } \alpha \le x \le \beta, \\ &= \Psi_2(x), \text{ if } \beta \le x \le \gamma, \\ &= 0, \text{ otherwise,} \end{aligned}$$

$\Psi_1(x)$ being a continuous *nondecreasing* function in the interval $[\alpha, \beta]$, and $\Psi_2(x)$ being a continuous *nonincreasing* function in the interval $[\beta, \gamma]$, where $\Psi_1(\alpha) = \Psi_2(\gamma) = 0$, and $\Psi_1(\beta) = \Psi_2(\beta) = T$, $0 \le T \le 1$.

Here $\Psi_1(x)$ and $(T - \Psi_2(x))$ are distribution functions and we would now study the membership function of a subnormal fuzzy number from this standpoint. We would first look into how such distribution functions can be constructed so that every fuzzy number can be explained with the help of *two* Lebesgue-Stieltjes measures.

Let us now define a vector $X = (X_1, X_2, \ldots\ldots, X_n)$

as a family of X_k, $k = 1, 2, \ldots\ldots, n$, with every X_k inducing a sub-σ field so that X is measurable. Let $(x_1, x_2 \ldots x_n)$ be a particular realization of X, and let $X_{(k)}$ realize the value $x_{(k)}$ where $x_{(1)}, x_{(2)}, \ldots\ldots, x_{(n)}$ are *ordered* values of $x_1, x_2, \ldots\ldots, x_n$ in increasing order of magnitude. Further let the sub-σ fields induced by X_k be independent and identical.

Define $\Phi_n(x)$ $= 0$, if $x < x_{(1)}$,

$= (r\text{-}1)\ T/n$, if $x_{(r-1)} \le x \le x_{(r)}$, r= 2, 3, …, n,

$= T$, if $x \ge x_{(n)}$;

$\Phi_n(x)$ here is an empirical distribution function of a theoretical distribution function $\Phi(x)$ with reference to a Lebesgue-Stieltjes measure.

Observe that $(r\text{-}1)\ T/n$ above, if $x_{(r-1)} \le x \le x_{(r)}$, are memberships of $[x_{(r-1)}, x_{(r)}]^{((r-1)\,T/n)}$ for $r = 2, 3, \ldots, n$. As there is a one to one correspondence between a Lebesgue-Stieltjes measure and the distribution function, we would have

$$\Pi(\alpha, \beta) = \Phi(\beta) - \Phi(\alpha)$$

where Π is a measure defining the space (Ω, A, Π), A being the σ- field common to every x_k.

Now the Glivenko-Cantelli theorem states that $\Phi_n(x)$ converges to $\Phi(x)$ uniformly in x. This means, Sup $|\ \Phi_n(x) - \Phi(x)\ | \to 0$.

This theorem was in fact for probability laws and therefore for laws of randomness. For a general Lebesgue-Stieltjes measure too, it would be applicable, and this is what we are going to do in what follows.

Let us consider now two spaces (Ω_1, A_1, Π_1) and (Ω_2, A_2, Π_2) where Ω_1 and Ω_2 are real intervals $[\alpha, \beta]$ and $[\beta, \gamma]$ respectively. Let $x_1, x_2, \ldots, x_n$, and $y_1, y_2, \ldots, y_n$, realizations in $[\alpha, \beta]$ and $[\beta, \gamma]$ respectively.

So for n fuzzy intervals $[x_1, y_1]^{(T/n)}, [x_2, y_2]^{(T/n)}, \ldots, [x_n, y_n]^{(T/n)}$

all with membership value equal to T/n everywhere, superimposition of the intervals would give us $[x_1, y_1]^{(T/n)}$ (S) $[x_2, y_2]^{(T/n)}$ (S) ……… (S) $[x_n, y_n]^{(T/n)}$

$$= [x_{(1)}, x_{(2)}]^{(T/n)} \cup [x_{(2)}, x_{(3)}]^{(2T/n)} \cup \ldots\ldots\ldots \cup [x_{(n-1)}, x_{(n)}]^{((n-1)T/n)}$$

$$\cup [x_{(n)}, y_{(1)}]^{(T)} \cup [y_{(1)}, y_{(2)}]^{((n-1)T/n)} \cup \ldots\ldots\ldots \cup [y_{(n-2)}, y_{(n-1)}]^{(2T/n)} \cup [y_{(n-1)}, y_{(n)}]^{(T/n)},$$

where for example $[y_{(1)}, y_{(2)}]^{((n-1)T/n)}$ represents the uniformly fuzzy interval $[y_{(1)}, y_{(2)}]$ with membership $((n-1)T/n)$ in the entire interval, $x_{(1)}, x_{(2)}, \ldots\ldots\ldots, x_{(n)}$ being values of $x_1, x_2, \ldots\ldots\ldots, x_n$ arranged in increasing order of magnitude, and $y_{(1)}, y_{(2)}, \ldots\ldots\ldots, y_{(n)}$ being values of $y_1, y_2, \ldots\ldots\ldots, y_n$ arranged in increasing order of magnitude.

Recall that for the fuzzy intervals $[x_1, y_1]^{(T/n)}, [x_2, y_2]^{(T/n)}, \ldots, [x_n, y_n]^{(T/n)}$,

all with constant membership T/n, the values of membership of the superimposed fuzzy intervals are $T/n, 2T/n, \ldots, (n-1)T/n, T, (n-1)T/n, \ldots, 2T/n$, and T/n.

These values of membership considered in two parts as

$$(0, T/n, 2T/n, \ldots, (n-1)T/n, T),$$

and

$$(T, (n-1)T/n, \ldots, 2T/n, T/n, 0),$$

would suggest that they can define an empirical distribution and a *complementary* empirical distribution on $x_1, x_2, \ldots\ldots\ldots, x_n$, and $y_1, y_2, \ldots\ldots\ldots, y_n$, respectively. In other words, for realizations of the values of $x_{(1)}, x_{(2)}, \ldots\ldots\ldots, x_{(n)}$ and of $y_{(1)}, y_{(2)}, \ldots\ldots, y_{(n)}$, we can see that if we define

$$\Psi_1(x) = 0, \text{ if } x < x_{(1)},$$
$$= (r-1)\, T/n, \text{ if } x_{(r-1)} \le x \le x_{(r)}, r = 2, 3, \ldots, n,$$
$$= T, \text{ if } x \ge x_{(n)},$$

$$\Psi_2(y) = T, \text{ if } y < y_{(1)},$$
$$= T - (r-1)\, T/n, \text{ if } y_{(r-1)} \le y \le y_{(r)}, r = 2, 3, \ldots, n,$$
$$= 0, \text{ if } y \ge y_{(n)},$$

then we are assured that

$$\Psi_1(x) \to \Pi_1 [\alpha, x], \alpha \le x \le \beta,$$
$$\Psi_2(y) \to T - \Pi_2 [\beta, y], \beta \le y \le \gamma.$$

Thus the existence of two densities in the intervals $[\alpha, \delta]$ and $[\xi, \gamma]$, $\xi \le \beta \le \delta$, is *sufficient* for the construction of a *subnormal* fuzzy number $[\alpha, \beta, \gamma]$. Hence, two distributions with reference to two Lebesgue – Stieltjes measures defined on two disjoint spaces can construct the fuzzy membership function of a subnormal fuzzy number. If $T = 1$, we get a normal fuzzy number with reference to two probability measures. For this however one needs to look into the matters through application of the Glivenko – Cantelli theorem of order statistics on superimposed uniformly fuzzy intervals. The distributions may be geared to the measure theoretic definition, and therefore to the broader definition, of randomness. What we mean is that the variable concerned need not be associated with any error term as in statistics. Even when the values of the variable are already ordered, such a construction would still be valid.

As an application of the above theorem, we illustrate the following example. For the uniform density function $f(x) = 1 / (\delta - \alpha)$, $\alpha \le x \le \beta \le \delta$,

the distribution function is given by $F(x) = \int_{\alpha}^{x} f(x)\, dx = (x - \alpha) / (\delta - \alpha)$.

$T = (\beta - \alpha) / (\delta - \alpha)$ here. Similarly, for the uniform density function

$$g(x) = 1/ (\gamma - \xi),\ \xi \le \beta \le x \le \gamma,$$

the distribution function is given by $G(x) = (x - \beta)/ (\gamma - \xi)$.

$T = (\gamma - \beta) / (\gamma - \xi)$ here. In fact, $(\beta - \alpha) / (\delta - \alpha) = (\gamma - \beta) / (\gamma - \xi)$.

It can be seen that $F(x)$ here is the left reference function and $(T - G(x))$ is the right reference function of a subnormal triangular fuzzy number $[\alpha, \beta, \gamma]$ with membership

$$\mu (x) = F(x) = (x - \alpha) / (\delta - \alpha), \text{ if } \alpha \le x \le \beta \le \delta,$$

$$= T - G(x) = T - (x - \beta)/ (\gamma - \xi), \text{ if } \xi \le \beta \le x \le \gamma,$$

$$= 0, \text{ otherwise.}$$

Thus the existence of two uniform densities, the simplest form of all densities, in the intervals $[\alpha, \delta]$ and $[\xi, \gamma]$, is *sufficient* for the construction of a subnormal triangular fuzzy number $[\alpha, \beta, \gamma]$, $\xi \le \beta \le \delta$. Other kinds of densities would be sufficient accordingly to give rise to other kinds of subnormal fuzzy numbers. We are asserting that assumption of two densities, and hence assumption of two distribution functions in $[\alpha, \delta]$ and $[\xi, \gamma]$, would give rise to a subnormal fuzzy number. As an extension of Theorem-1, this can be stated in the form of a theorem linking subnormal fuzziness and what we are proposing here to call *sub-randomness* in the sense that the total integral under the density function is equal to some T, $0 \le T < 1$.

Theorem-2: For a *subnormal* fuzzy number $N = [\alpha, \beta, \gamma]$ with membership function

$$\mu_N(x) = \Psi_1(x), \text{ if } \alpha \le x \le \beta,$$

$$= \Psi_2(x), \text{ if } \beta \le x \le \gamma, \text{ and}$$

$$= 0, \text{ otherwise,}$$

such that $\Psi_1 (\alpha) = \Psi_2 (\gamma) = 0, \Psi_1 (\beta) = \Psi_2 (\beta) = T$, $0 \le T \le 1$, where $\Psi_1(x)$ is the distribution function of a random variable defined in the interval $[\alpha, \delta]$, and $\Psi_2(x)$ is the complementary distribution function of another random variable defined in the interval $[\xi, \gamma]$ for $\xi \le \beta \le \delta$.

That was about how to define the membership function in terms of two density functions with reference to *sub-randomness*. As for the definition of the complement of such a fuzzy set, our earlier definition holds. For a subnormal fuzzy number $N = [\alpha, \beta, \gamma]$, the complement N^C will have the membership function $\mu_N{}^C (x)$, where $\mu_N{}^C (x) = 1$, $- \infty < x < \infty$,

with the condition that $\mu_N{}^C (x)$ is to be counted from $\Psi_1(x)$, if $\alpha \le x \le \beta$, from $\Psi_2(x)$, if $\beta \le x \le \gamma$, and from 0, otherwise, so that there happens to be a difference between the fuzzy membership function and the concerned fuzzy membership values. We therefore need to extend Axiom-1 as follows:

Axiom-2: The fuzzy membership function of the complement of a *generalized* fuzzy number is equal to 1 for the entire real line, with the membership values counted not from zero but from the membership function of the fuzzy number concerned.

In [1], we have discussed why the fuzzy sets could define a field if the complement of a fuzzy set is defined in our way. Following the same lines, it would now be easy to see that subnormal fuzzy numbers which are nothing but generalized fuzzy numbers conform to the definition of a field.

3. Conclusions and Discussions

In this article, we have cited an example in how misleading a manner the concept of subnormality of a fuzzy set had been equated to that of probability. When we take intersection of two normal fuzzy numbers of the types $[a, b, c]$ and $[d, e, f]$ with $a < d < c < f$, we actually get a *subnormal* fuzzy number $[d, g, c]$ where the amount of partial presence at g, the point in the interval for which the two membership curves intersect, is less than unity. We have here shown how subnormal fuzziness should be explained logically, and hence mathematically.

Two laws of randomness can define one normal law of fuzziness. Besides, the fuzzy membership function of the complement of a normal fuzzy number is always equal to unity, with the condition that membership *values* of the complement however has to be counted from the membership function of the normal fuzzy number concerned. Based on these two facts, the theory can be generalized to explain subnormal fuzziness too, and this is another point that we have discussed in this article.

Trying to infer a law of probability from a normal law of fuzziness is an exercise based on an arrogant belief that in the same interval on which a normal law of fuzziness has been defined, a law of probability also can be defined, linking fuzziness with probability thereby. In the process, the theory of probability has been misrepresented many a time. Many workers have tried to deduce *one* law of randomness from a given law of fuzziness, and vice versa. Trying to establish a conversion formula from a law of fuzziness to a law of randomness is but such an exercise only.

We have here cited just one example of misrepresentation of the theory of probability. The literature on fuzziness is full of such results in which the workers have put forward certain ideas extending the concept of probability, which would never possibly enter into the books on probability for obvious reasons. If we can establish a probability density function in an interval, we could very well go for applying the calculus of probability in that situation. The question of applying principles of fuzziness should not arise in that kind of a case. This kind of confusion can occur only when one does not know whether to go for applying the theory of fuzziness or to go for applying the theory of probability in any particular situation. Fuzziness and probability are two entirely different concepts, and hence such confusions must not be there, unless of course the user does not happen to have understood the basics of the theory of probability well enough. First asserting that probability is insufficient a tool to explain a particular situation, and that fuzziness can explain that situation more properly, we fail to understand why at all one should try to frame a conversion formula to deduce a probability density function from a given membership function!

Ours is a simple standpoint: mathematics should be rooted at logic; it must never be the other way around. When one visualizes a phenomenon, one may attempt to uncover the underlying theory behind it. This is how the laws of Nature were framed. Facts were observed, and mathematical explanations of those facts were forwarded. Once the mathematical explanation of an observed fact was validated, other examples too could be found to follow that explanation. On the contrary, if one starts framing a theory based on wrong axioms, one would invariably end up explaining matters in a wrong way. A wrong axiom is in fact nothing but a baseless belief, and just symbolic representation of such a belief must not be called mathematics, be that whatever else. Coming back to our case, for example, it is known that a penumbral eclipse occurs when our Moon passes through the Earth's penumbra. The penumbra then happens to create a partial darkening of the surface of the Moon. In fact, during a total lunar eclipse, first a special type of total penumbral eclipse can be seen. During that period, just before a total lunar eclipse starts, the Moon happens to lie within the Earth's penumbra. When that happens, the surface of the Moon appears partly darker than usual. Here then is a case of visualization of subnormal fuzziness. This is something that can actually be seen, and necessary mathematical explanations of such an example of subnormality could follow thereafter.

References

[1] Hemanta K. Baruah, The Theory of Fuzzy Sets: Beliefs and Realities, International Journal of Energy, Information and Communications, Vol. 2, Issue 2, 2011, 1-22.

[2] J. N. Sheen, (2006), Generalized Fuzzy Numbers Comparison by Geometric Moments, *Proceedings of the 5th WSEAS International Conf. on Instrumentation, Measurement, Circuits and Systems*, Hangzhou, China, 279-284.

[3] Jean Lemaire, Fuzzy Insurance, *ASTIN Bulletin*, Vol. 20, No. 1, 1990, 33 – 55.

[4] Hemanta K. Baruah, The Mathematics of Fuzziness: Realities and Beyond, LAP, Saarbrücken, Germany, 2010.

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