Mathematical Modelling of Change of Temperature in Pulsating Heat Pipes with Single Loops

Pranab K. Barua
Department of Energy
Tezpur University
Assam, India
pranab_kb03@yahoo.com

D. Deka
Department of Energy
Tezpur University
Assam, India
dhanapati@tezu.ernet.in

U. S. Dixit
Department of Mechanical Eng.
Indian Institute of Technology
Guwahati, Assam, India
uday@iitg.ernet.in

Abstract

Recovered heat that would have otherwise been wasted can serve useful purposes. Efficiency of heat pipes used in heat recovery would depend on the mathematical rules that govern the process of fall of temperature with respect to time. We therefore need to establish the exact mathematical model followed by such a system. In this article, we have shown that in pulsating heat pipes with single loops, temperature decreases exponentially in time. It could not however be conclusively established that for a fixed diameter of the pipe, fall of temperature depends on the length of the evaporator section. Finally, it has been found that for a fixed length of the evaporator section, temperature falls faster for smaller diameter of the pipe.

Keywords: *Heat pipe heat exchanger, waste heat recovery.*

1. Introduction

Waste heat is heat generated by fuel combustion or chemical reaction, and then wasted even though it could possibly be reused for some useful purpose. If such waste heat could be recovered and used for some useful purpose, that would always be of help in terms of economics. The strategy of recovering this heat depends on the temperature of the waste heat gases and the economics involved therein [1]. The heat pipe heat exchanger (HPHE) is a very efficient lightweight compact waste heat recovery system. It is a self-contained passive energy recovery device.

An HPHE consisting of a bundle of individual heat pipes with vaporizing and condensing sections at the respective ends is unfit for large-scale needs in industrial applications. However, modifications were added, and the HPHE has received much attention since it was launched into industry at the beginning of the eighties [2].

The heat pipe, as a high efficiency heat transfer element, is widely used in the electronics cooling industry and energy efficiency sectors. They can be embedded with aluminium heat sinks to enhance cooling efficiency and/or compactness of cooling devices [3]. Heat pipes are also widely used in energy recovery systems in domestic and industrial applications, such as in some domestic appliances for improving the efficiency of the drying cycles. However, the design of the heat recovery systems with heat pipe units is the key to providing a heat exchanger system to work as efficient as expected [4].

The heat pipe heat exchanger has many advantages in comparison with conventional heat exchangers, such as large quantities of heat transported through a small cross-sectional area with no additional power input to the system, less pressure drop of fluid, advanced maintainability, high reliability, simpler structure and smaller volume. Researches in this line have been reported by Firouzfar and Attaran [5].

Akachi [6] proposed a loop type heat pipe with a check valve for directing the heat carrying fluid flow. In his patent, he described 24 different embodiments of loop type heat pipes. He later developed [7] loop type heat pipes without check valves. Eventually, based on that idea two-phase thermal control devices called pulsating heat pipes (PHP) were developed [8].

Mathematical models considering the heat transfer effects on operation of a pulsating heat pipe with open end was proposed by Zhang and Faghri [9]. They further studied [10] numerically the oscillatory flow in pulsating heat pipes. Numerical study was also done by Lin *et. al.* [11].

Khandekar *et. al.* [12] experimented on a flat plate closed loop pulsating heat pipe structures. Khandekar and Groll [13] constructed a closed loop pulsating heat pipe to study internal hydraulics of the system. They found that gravity effects in systems with low number of turns. Khandekar and Groll [14] found that internal diameter of the closed loop pulsating heat pipe, volumetric filling ration of the working fluid, input heat flux, total number of turns, operational orientation, and thermo-physical properties of the working fluid effect the performance of a pulsating heat pipe. In the same year Khandekar *et. al.* [15] presented a mathematical model of pulsating heat pipe which helps in predicting two-phase flow parameters in each sub-section of the device.

Gravity assisted heat pipe has found numerous applications in heat recovery systems ([16], [17], [18]). Nearly one-fourth of the energy engines generate dissipating in the form of exhaust loss energy [19]. If the exhaust gas enters into surroundings directly, it will not only waste energy but also damage the environment more or less. Air from the carriage to be heated is introduced into a heat pipe heat exchanger by a fan and warmed by the heat from the exhaust gas there from. Then, the air flows into the carriage to keep comfortable temperature [20].

In many circumstances heat otherwise wasted can be recovered and reused. Heat pipes with excellent thermal merits have emerged as devices that are efficient in transporting heat [21]. A heat recovery system based on a looped heat pipe was proposed by Lamfon *et. al.* [22] to reuse waste heat from a gas turbine engine. Thermal performance of an ordinary heat pipe has been studied extensively ([23], [24], [25], [26], [27], [28], [16], [29], [30], [31]).

The maximum heat transfer of a two-phase thermosyphon was investigated by Pioro ([32], [33]). Theoretical studies on the steady-state characteristics and stability thresholds on closed two-phase thermosyphons were presented by Dorban [34]. Gross and Hahne [35] investigated the heat transfer performance of closed thermosyphons over wide ranges of pressure and inclination angles. Joudy *et. al.* [36] also reported improved heat transfer performance with an internal wall that separated the liquid and vapour streams.

The heat transfer effectiveness of heat pipe heat exchangers was studied by Peretz [37], who found that the HTE (heat transfer effectiveness) of the heat exchanger depends upon the HTE of a single heat pipe and the number of rows parallel to the flow. The thermal performance of heat pipe heat recovery system was investigated by Azad *et. al.* [27]. A model for the system was developed to predict the temperature distribution in longitudinal rows of the heat exchanger.

The effectiveness of gravity-assisted air-to-air heat exchangers was studied by Wadowski *et. al.* [38]. Water-to-air gravity-assisted heat pipe heat exchangers were also investigated ([26], [39]). The overall effectiveness of heat exchanger was calculated for a wide range of parameters. The design procedure was presented by Azad and Geoola [24], and models for heat transfer resistances in heat pipes were also given. Optimization of heat pipe heat exchangers has been studied by Peretz and Horbaniuc [40].

It is well known that water is excellent as a working fluid for heat pipes for its high latent heat, easy availability, and its high resistance to decomposition and degradation. Water has been used particularly successfully in copper heat pipes for low temperature applications. Problems of incompatibility for iron pipes have also been reported ([41], [42], [43], [28], [16], [44], [45], [46]). The studies confirm that hydrogen is evolved inside iron-water heat pipes, so that the condenser eventually becomes flooded with this noncondensable gas, making the heat pipe inoperative.

Water was selected as the working fluid of the heat pipe system developed by Akyurt *et. al.* [47]. Because of the problem of incompatibility with iron, copper was selected initially as the container material. They developed a copper-water heat pipe system using the analysis presented by Lamfon *et. al.* [22].

Traditionally, the application of a heat-pipe air-preheater for the dryer is unable to use its waste heat with a closed system and does not employ the relative humidity in any drying systems. In the application of closed-loop oscillating heat-pipe with check valves large quantities of heat are transported through a small cross-section area. The closed-loop oscillating heat-pipe with check valves is a very effective heat-transfer device invented by Akachi *et. al*. [8]. It has a simple structure and fast thermal-response. The closed-loop oscillating heat-pipe with check valves consists of a long capillary tube bent into many turns, and the evaporator section, adiabatic section, and condenser section are located at these turns, with the ends joined to form a closed loop. It incorporates one or more direction-control one-way check valves in the loop so that the working fluid can circulate in the specified direction only. Miyazaki *et. al.* [48] studied the oscillating heat pipe with check valves. It was found that the closed-loop oscillating heat-pipe with check valves has a high rate of heat transfer. Pipatpaiboon [49] studied the effect of inclination angle, working fluid and the number of check valves on the characteristics of heat transfer in a closed-loop oscillating heat-pipe with check valves. It was found that the closed-loop oscillating heat-pipe with check valves equipped with two check valves has the highest heat-transfer capacity. Rittidech *et. al.* [50] studied the closed-ended oscillating heat-pipe air-preheater for energy thrift in the dryer. It was found, from the experimental results, that the thermal effectiveness increases and the closed-ended oscillating heat-pipe air-preheater achieves energy thrift. Wu *et. al.* [51] studied the application of heat-pipe exchangers for humidity control in an air-conditioning system. It was observed that this type of heat exchanger can be an advantageous replacement for a conventional reheat coil, resulting in energy saving and enhancing the cooling capability of the cooling coils with little or no external energy need.

The principle of operation is similar to that of an air-preheater by the closed-loop oscillating heat-pipe with check valves, which is widely accepted as the most efficient heat-transfer device for high heat-loads [8]. It has the capability of operating in any position and has operational flexibility. At present, the closed-loop oscillating heat-pipe with check valves does not sufficiently solve energy problems. So, an improvement of the heat-pipe air-preheater for the dryer is needed.

2. Objectives of the Study

To check for feasibility and efficiency of a heat pipe heat exchanger, it is important to study the manner in which it helps in removing heat from a system concerned. It is obvious that the higher the heat transfer rate, the more efficient the heat pipe is. However, first of all we need to know the rate of heat transfer with respect to time in such a system. If the differential equation governing heat transfer is established based on experimental data, the question of efficiency could be answered properly. For that we need to establish the mathematical model of fall of temperature with respect to time. So in our work, it has been chosen to establish mathematical models of fall of temperature in the condenser section due to natural cooling from a certain higher temperature to the room temperature with respect to time for various diameters of the pipes and for various lengths of the evaporator sections for single loops. This ultimately translates itself into the heat removal rate from the pipe. In the present experimental work, the heat pipe has been selected to be of the *pulsating* type as we are going to use water as the working fluid in both of its phases viz. liquid and vapor, inside the same compartment. In other words, our setup is of a *homogeneous* flow model.

We propose to find answers to the following questions:

i) What is the general expression of falling temperature as a function of time?
ii) What is the effect of the length of the evaporator section of a heat pipe on the cooling rate?
iii) What is the effect of the diameter of a heat pipe on the cooling rate?

3. Fabrication and Experimental Details

For efficient operation of a heat pipe, numerous parameters should be controlled. As such, it makes it imperative to conduct experimental investigations as elaborately as it is required. The first part of the experimentation requires analysis of performance characteristics of a single loop pulsating heat pipe. Water has been used particularly successfully in copper heat pipes for low temperature applications. We were interested to study how temperature falls. For that we needed to fabricate simple heat pipes to serve our purpose only. Indeed, we were not aiming for construction of *efficient* heat pipes. Water was selected as the working fluid of the heat pipe system as was done in the case of Akyurt *et. al.* [47]. Because of the problem of incompatibility with iron, they selected copper as the container material, and later have developed a copper-water heat pipe system using the analysis presented by Lamfon *et. al*. [22]. Hence for our case, copper was selected as the material for the pipes as it has a very high thermal conductivity.

Copper pipes of 3 different diameters viz. ¼ inch (= 0.6354 cm.), ½ inch (= 1.2708 cm.) and 5/8 inch (= 1.5885 cm.) were constructed. We shall however present in our discussions the unit of the diameters in inch, because with such specifications only are they available in the market. Each pipe was of the shape of a square. The length of the evaporator zone varied for each of the diameter types and was alternately selected as 30 cm, 35 cm and 40 cm respectively. We thus have fabricated 9 different heat pipe setups.

Each pipe contains two cutout sections of length 5 cm welded into the main setup. One of them acts as water inlet while the other acts as an exit route for air pockets. The pipes were partially filled with water and then the openings of the inlet and the outlet sections were plugged with a sealing agent and an adhesive tape, the combination of which proved effective enough.

To reduce loss of heat from the pipes baring the evaporator and the condenser sections, glass wool was utilized as an insulator. Adhesive tape was used to bind the material with the pipe. We have already stated that we are interested to establish mathematical models of fall of temperature in heat pipes. The mathematical models are what we are interested in, and we are not really interested on the operability of the entire heat exchanger set up. That is why we have used insulating materials just for minimizing the heat loss without actually evaluating the critical radius of insulation as it would not really affect the mathematical models. We have used the software called *Daisy Lab* for acquisition of data. This software is connected to a thermocouple to measure the temperature which gets displayed on the computer screen.

4. Statistical Tools to be Used for Data Analysis

We have mentioned in our objectives that we are interested to establish the general expressions of falling temperature with respect to time in a heat pipe both for one single loop in the evaporator section. In other words, we are interested in mathematical modeling of falling temperature as a function of time. For that we would need to use certain statistical tools to verify the possible acceptability of certain hypothesized mathematical functions probabilistically. The hypotheses concerned would be framed only after we scrutinize the observed data with the help of diagrammatic representations. Indeed, for our purpose, we would need certain very standard statistical procedures to arrive at some data dependent conclusions.

We would use the Method of Least Squares Estimation to fit the hypothesized curves based on the observed data. The significance of the estimated values of the parameters involved in the fitted equations would then be tested statistically using the standard techniques included in Regression Analysis. After statistical validation of the estimated values of the parameters, we would find the expected values of falling temperature in every case concerned. Finally, to validate the functional patterns and to find numerically by what part the model fits the data, we would take help of the Analysis of Variance for Linear Regression.

5. Observed Average Temperatures

To fit a mathematical model regarding the general expression of falling temperature as a function of time we would need sufficient amount of data. To arrive at a parameter dependent general expression we need to collect time dependent data in equal interval of time on falling temperature for different diameters of the pipes and for different lengths of the evaporator section to verify whether that kind of a general expression really works for various combinations of diameters and in lengths. We therefore collected data in time intervals equal to 1 minute. In what follows, in Tables 1 to 3, considering diameters ¼ inch, ½ inch and 5/8 inch respectively, we are going to compare the fall of observed average temperature for different lengths of the evaporator section.

Table 1. Average temperature in the condenser section after τ minutes of operation: Diameter 1/4 inch, for different Lengths

τ	Temperature for Length 30 cm	Temperature for Length 35 cm	Temperature for Length 40 cm
0	65	62	60
1	50	50	50
2	41.1	45.6	46.2
3	36.5	41.4	43
4	34.6	39.1	39.9
5	33.1	36.8	37.9
6	32	34.9	35.6
7	31.5	34.1	34.6
8	30.9	33	33.1
9	30.7	32.1	32.3
10	30.1	31.5	31.1
11	29.9	31.2	30.8
12	29.5	30.7	30.4
13	29.2	30.4	30.1
14		30	29.9
15		29.9	29.7
16		29.7	29.5

Table 2. Average temperature in the condenser section after τ minutes of operation: Diameter 1/2 inch, for different Lengths

τ	Temperature for Length 30 cm	Temperature for Length 35 cm	Temperature for Length 40 cm
0	75	75	75
1	62	66	65
2	56.6	59	60.8
3	52.5	55.3	56.4
4	50	52.9	52.1
5	48	50	50
6	46.4	48.2	47.8
7	45	46.6	46
8	43.1	45.2	44.1
9	41.5	43.9	42.5
10	40.4	42.6	41.6
11	39	41.3	40
12	38.2	40	38.8
13	37	38.6	38.4
14	36.5	37.8	37.7
15	35.9	37	37
16	35.1	36.3	36.7
17	34.9	35.6	36
18	34.4	35	35.4
19	34.1	34.5	34.8
20	33.3	34	34.5
21	33	33.5	34.3
22	32.6	33.1	34
23	32.1	32.8	33.5
24	31.8	32.4	33.2
25		32.2	33

Table 3. Average temperature in the condenser section after τ minutes of operation: Diameter 5/8 inch, for different Lengths

τ	Temperature for Length 30 cm	Temperature for Length 35 cm	Temperature for Length 40 cm
0	80	80	85
1	73.8	76	80
2	67.8	73.1	76.8
3	62.7	69.5	74.1
4	60.2	65.6	72.3
5	58.4	62.8	70.7
6	56.8	60.2	69.2
7	55.2	58.1	67.6
8	54.2	56.3	65.8
9	53.1	55.3	64
10	52.1	54.3	62.1
11	51.2	53.2	60.3
12	50	52.2	59.1
13	48.5	51.1	58
14	47.3	50	57
15	46.1	48.9	56
16	44.9	47.7	55
17	43.8	46.4	53.8
18	42.9	44.9	52.6
19	42	43.7	51.3
20	41.3	42.5	50
21	40.6	41.3	48.9
22	40	40.3	48.1
23	39.3	39.5	47.3
24	38.6	38.8	46.4
25	37.9	38.1	45.3
26	37.2	37.5	44.3
27	36.6	36.9	43.2
28	36.1	36.5	42.2
29	35.7	36.1	41.6
30	35.2	35.8	40.8
31	34.9	35.5	40
32	34.5	35.2	39.5
33		34.9	39.1
34		34.7	38.6
35			37.9
36			37.2
37			36.5
38			36
39			35.6
40			35.2

6. Analysis of the Data

Numerical and statistical analysis of the data collected would now lead us to certain conclusions related to heat transfer. This is the most important step in fitting data dependent mathematical models in any situation. First, we would need certain parametric hypotheses which we would test statistically or otherwise, based on the observed data. The following are our hypotheses:

i. Hypothesis - 1: We hypothesize that temperature T decreases exponentially in time τ following $T = C + \alpha e^{\beta\tau}$, where α and β are parameters to be estimated, $\alpha > 0$ and $\beta < 0$, and C is the controlled temperature beyond which there would be no more cooling possible.

ii. Hypothesis - 2: We hypothesize that for a fixed diameter of the pipe, fall of temperature possibly depends on the length of the evaporator section. This hypothesis would *not* however be tested statistically. In this case, we would try to draw conclusions based on the fitted curves only.

iii. Hypothesis - 3: We hypothesize that for a fixed length of the evaporator section, temperature falls faster for smaller diameter of the pipe. This hypothesis too would *not* be tested statistically. In this case too, we would try to draw conclusions based on the fitted curves only.

In effect, we would like to study whether feasibility of using heat pipes would be dependent on: smallness of the diameter of the pipe used, and smallness of the length of the evaporator section.

We shall now describe the statistical analysis performed on the data collected. We would proceed as follows. The hypothesized equation being $T = C + \alpha e^{\beta\tau}$, $\alpha > 0$ and $\beta < 0$, we have

$$log_e(T\text{-}C) = log_e \alpha + \beta \tau.$$

C in our case was 29.0 degrees centigrade, the room temperature. Now, ln α and β can be estimated ([52], pp 350 – 352), using the Method of Least Square Estimation. It may be noted here that for an exponential decay curve expressed as a probability density function of the type $f(t) = \lambda e^{-\lambda t}$, $\lambda \geq 0$, $t \geq 0$, λ is better known as the instantaneous failure rate ([52], pp 319). In our case, in the hypothesized decay curve $T - C = \alpha e^{\beta\tau}$, the positive quantity $(-\beta)$ is therefore the instantaneous failure rate.

In Tables 4 and 5, we are going to describe the Test of Significance ([52], pp 202) for the Regression Parameters and Analysis of Variance ([52], pp 360) of the log linear fit respectively, for pipe diameter ¼ inch and Length 30 cm.

Table 4. Test of Significance for the Regression Parameters: Diameter ¼ inch, Length 30 cm

Parameters	Estimated Values	Calculated $\lvert t \rvert$	Tabulated t, 12 d.f.
Intercept $log_e\,\alpha$	3.275531	25.45784	$t_{0.05}$ =1.782
Slope β	-0.33842	20.1175	$t_{0.025}$=2.179

We would now like to test the null hypothesis $H_0^1: \beta = 0.0$ against the two sided alternative hypothesis $H_1^1: \beta \neq 0.0$. It can be seen that the calculated value of $\lvert t \rvert$ (= 20.1175) is very much larger than the tabulated value of t (= 2.179) for 12 degrees of freedom at 5% level of significance ([52], pp 166). Hence we conclude that the hypothesis H_0^1 is not acceptable. Therefore the alternative hypothesis H_1^1 is true. In other words, in this case $\beta \neq 0.0$.

We would like to test the null hypothesis H_0^2: $log_e\ \alpha = 0.0$ against the one sided alternative hypothesis H_1^2: $log_e\ \alpha > 0.0$. It can be seen that the calculated value of $|t|$ (= 25.45784) is very much larger than the tabulated value of t (= 1.782) at 5% level of significance. Hence we conclude that the hypothesis H_0^2 is not acceptable. Therefore the alternative hypothesis H_1^2 is true. In other words, in this case $\log_e \alpha > 0.0$.

Table 5. Analysis of Variance of the Log linear Fit: Diameter ¼ inch, Length 30 cm

Source of Variation	Degrees of Freedom	Sums of Squares	Calculated Value of F	Tabulated $F_{0.05,\ (1,12)}$
Regression	1	26.05505	404.7118	4.7472
Error	12	0.772551		
Total	13	26.82761		

The table above shows that the effect due to regression on the total variations is very highly significant ([52], pp 174) because the calculated value of the F–statistic (=404.7118) is very much higher than the tabulated value of $F_{0.05,\ (1,\ 12)}$ (= 4.7472). We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is given by (26.05505 / 26.82761)*100 = 97.12 ([53], pp 377). In other words, 97.12% of the variations are due to this mathematical relationship, while the rest 2.88% only is due to randomness. We therefore conclude that for Diameter ¼ inch, Length 30 cm. we have $\alpha = 26.45727$, and $\beta = -0.33842$, and therefore

$$T = 29 + 26.45727\ e^{-0.33842\ \tau}.$$

For pipe diameter ¼ inch and Length 35 cm., the following are the analyses concerned.

Table 6. Test of Significance for the Regression Parameters: Diameter ¼ inch, Length 35 cm

Parameters	Estimated Values	Calculated $\|t\|$	Tabulated t, 15 d.f.
Intercept $log_e\ \alpha$	3.266205	85.54	$t_{0.05}$ =1.753
Slope β	-0.22985	56.25	$t_{0.025}$=2.131

It can be seen that for β the calculated value of $|t|$ is *very much larger* than the tabulated value of t at 5% level of significance. Hence we conclude that the hypothesis H_0^1 is not acceptable. In other words, in this case $\beta \neq 0.0$. Similarly for $log_e\ \alpha$ the calculated value of $|t|$ is *very much larger* than the tabulated value of t at 5% level of significance. Hence we conclude that the hypothesis H_0^2 is not acceptable. In other words, in this case $log_e\ \alpha > 0.0$.

Table 7. Analysis of Variance of the Log linear Fit: Diameter ¼ inch, Length 35 cm

Source of Variation	Degrees of Freedom	Sums of Squares	Calculated Value of F	Tabulated $F_{0.05,\ (1,15)}$
Regression	1	21.55421	3188.82	4.5431
Error	15	0.10139		
Total	16	21.65561		

The effect due to regression on the total variations is very highly significant because the calculated value of the F –statistic is very much higher than the tabulated value of $F_{0.05, (1, 15)}$. We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is 99.53. In other words, 99.53% of the variations are due to this mathematical relationship, while the rest 0.47% only is due to randomness. We therefore conclude that for Diameter ¼ inch, Length 35 cm, we have α = 26.21167, and β = -0.22985 and so

$$T = 29 + 26.21167\ e^{-0.22985\ \tau.}$$

For pipe diameter ¼ inch and Length 40 cm. the following are the analyses.

Table 8. Test of Significance for the Regression Parameters: Diameter ¼ inch, Length 40 cm

Parameters	Estimated Values	Calculated $\lvert t \rvert$	Tabulated t , 15 d.f.
Intercept $log_e\ \alpha$	3.39498	121.0198	$t_{0.05}$ =1.753
Slope β	-0.25262	84.4749	$t_{0.025}$=2.131

From the table above, it can be seen that in this case too we can conclude that the hypothesis H_0^1 is not acceptable. In other words, in this case $\beta \neq 0.0$. Similarly for $log_e\ \alpha$ also we can conclude that the hypothesis H_0^2 is not acceptable. In other words, in this case $log_e\ \alpha >$ 0.0.

Table 9. Analysis of Variance of the Log linear Fit: Diameter ¼ inch, Length 40 cm

Source of Variation	Degrees of Freedom	Sums of Squares	Calculated Value of F	Tabulated $F_{0.05, (1,15)}$
Regression	1	26.03724	7136.017	4.5431
Error	15	0.054731		
Total	16	26.09197		

The table above shows that the effect due to regression on the total variations is very highly significant because the calculated value of the F –statistic is very much higher than the tabulated value of $F_{0.05, (1, 15)}$. We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is 99.79. In other words, 99.79% of the variations are due to this mathematical relationship, while the rest 0.21% only is due to randomness. We therefore conclude that for Diameter ¼ inch, Length 40 cm. we have α = 29.81405, and β = -0.25262, and hence

$$T = 29 + 29.81405\ e^{-0.25262\ \tau.}$$

For pipe diameter 1/2 inch and Length 30 cm, the analysis is as follows.

Table 10. Test of Significance for the Regression Parameters: Diameter 1/2 inch, Length 30 cm

Parameters	Estimated Values	Calculated $\lvert t \rvert$	Tabulated t , 23 d.f.
Intercept $log_e\, \alpha$	3.525721	110.3385	$t_{0.05}$ =1.714
Slope β	-0.10462	45.8378	$t_{0.025}$=2.069

Here too, it can be seen that we can conclude that the hypothesis H_0^1 is not acceptable. In other words, in this case $\beta \neq 0.0$. Similarly for $log_e\, \alpha$ too we can conclude that the hypothesis H_0^2 is not acceptable. In other words, in this case $log_e\, \alpha > 0.0$.

Table 11. Analysis of Variance of the Log linear Fit: Diameter 1/2 inch, Length 30 cm

Source of Variation	Degrees of Freedom	Sums of Squares	Calculated Value of F	Tabulated $F_{0.05,\,(1,23)}$
Regression	1	14.22907	2101.104	4.2793
Error	23	0.15576		
Total	24	14.38483		

The table above shows that the effect due to regression on the total variations is very highly significant because the calculated value of the F –statistic is very much higher than the tabulated value of $F_{0.05,\,(1,\,23)}$. We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is 98.92. In other words, 98.92% of the variations are due to this mathematical relationship, while the rest 1.08% only is due to randomness. We therefore conclude that for Diameter 1/2 inch, Length 30 cm. we have α = 33.978263, and β = -0.10462, and therefore

$$T = 29 + 33.978263\, e^{-0.10462\, \tau.}$$

For pipe diameter 1/2 inch and Length 35 cm. the analysis is as follows.

Table 12. Test of Significance for the Regression Parameters: Diameter 1/2 inch, Length 35 cm

Parameters	Estimated Values	Calculated $\lvert t \rvert$	Tabulated t , 24 d.f.
Intercept $log_e\, \alpha$	3.618465	165.3127	$t_{0.05}$ =1.711
Slope β	-0.10089	67.1939	$t_{0.025}$=2.064

Here too, the conclusions are no different. In other words, in this case too $\beta \neq 0.0$, and $log_e\, \alpha >$ 0.0.

Table 13. Analysis of Variance of the Log linear Fit: Diameter 1/2 inch, Length 35 cm

Source of Variation	Degrees of Freedom	Sums of Squares	Calculated Value of F	Tabulated $F_{0.05,\,(1,24)}$
Regression	1	14.88787	4515.015	4.2597
Error	24	0.079138		
Total	25	14.967		

The table above shows that the effect due to regression on the total variations is very highly significant because the calculated value of the F –statistic is very much higher than the tabulated value of $F_{0.05, (1, 24)}$. We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is 99.47. In other words, 99.47% of the variations are due to this mathematical relationship, while the rest 0.53% only is due to randomness. We therefore conclude that for Diameter 1/2 inch, Length 35 cm. we have α = 37.28030, and β = -0.10089, and

$$T = 29 + 37.28030\ e^{-0.10089\ \tau.}$$

The following analysis is for pipe diameter 1/2 inch and Length 40 cm.

Table 14. Test of Significance for the Regression Parameters: Diameter 1/2 inch, Length 40 cm

Parameters	Estimated Values	Calculated $\lvert t \rvert$	Tabulated t , 24 d.f.
Intercept $log_e\ \alpha$	3.535868	87.34586	$t_{0.05}$ =1.711
Slope β	-0.09192	33.1023	$t_{0.025}$=2.064

Here too, we reject the hypotheses H_0^1 and H_0^2. Hence $\beta \neq 0.0$, and $log_e\ \alpha > 0.0$.

Table 15. Analysis of Variance of the Log linear Fit: Diameter 1/2 inch, Length 40 cm

Source of Variation	Degrees of Freedom	Sums of Squares	Calculated Value of F	Tabulated $F_{0.05, (1,24)}$
Regression	1	12.35834	1095.76	4.2597
Error	24	0.27068		
Total	25	12.62902		

The table above shows that the effect due to regression on the total variations is very highly significant because the calculated value of the F –statistic is very much higher than the tabulated value of $F_{0.05, (1, 24)}$. We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is 97.85. In other words, 97.85% of the variations are due to this mathematical relationship, while the rest 2.15% only is due to randomness. We therefore conclude that for Diameter 1/2 inch, Length 40 cm. we have α = 34.32479, and β = -0.09192, and

$$T = 29 + 34.32479\ e^{-0.09192\ \tau.}$$

For pipe diameter 5/8 inch and Length 30 cm. the analysis is as follows.

Table 16. Test of Significance for the Regression Parameters: Diameter 5/8 inch, Length 30 cm

Parameters	Estimated Values	Calculated $\lvert t \rvert$	Tabulated t , 31 d.f.
Intercept $log_e\ \alpha$	3.782269	234.5462	$t_{0.05} \approx 1.697$
Slope β	-0.06439	74.3402	$t_{0.025} \approx 2.042$

Here too, it can be seen that we have to reject the hypotheses H_0^1 and H_0^2. So in this case too $\beta \neq 0.0$, and $log_e\, \alpha > 0.0$.

Table 17. Analysis of Variance of the Log linear Fit: Diameter 5/8 inch, Length 30 cm

Source of Variation	Degrees of Freedom	Sums of Squares	Calculated Value of F	Tabulated $F_{0.05, (1,31)}$
Regression	1	12.40351	5526.461	≈ 4.1709
Error	31	0.069576		
Total	32	12.47308		

The table above shows that the effect due to regression on the total variations is very highly significant because the calculated value of the F –statistic (= 5526.461) is very much higher than the tabulated value of $F_{0.05, (1, 31)}$ (≈ 4.1709). In this case, we have taken the value of $F_{0.05, (1, 30)}$ as $F_{0.05, (1, 31)}$ is not available in the table of F. We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is given by (12.40351 / 12.47308)*100 = 99.44. In other words, 99.44% of the variations are due to this mathematical relationship, while the rest 0.56% only is due to randomness. Hence, it can be statistically concluded that the observed data on temperature follow the negative exponential law. We therefore conclude that for Diameter 5/8 inch, Length 30 cm. we have α = 43.91557, and β = -0.06439, and so

$$T = 29 + 43.91557\, e^{-0.06439\, \tau}.$$

For pipe diameter 5/8 inch and Length 35 cm. too the outcome of the analysis was similar.

Table 18. Test of Significance for the Regression Parameters: Diameter 5/8 inch, Length 35 cm

Parameters	Estimated Values	Calculated $\lvert t \rvert$	Tabulated t, 33 d.f.
Intercept $log_e\, \alpha$	3.894716	248.3167	$t_{0.05} \approx 1.697$
Slope β	-0.06541	82.4578	$t_{0.025} \approx 2.042$

Here also we have found α = 49.14209, and β = -0.06541 acceptable.

Table 19. Analysis of Variance of the Log linear Fit: Diameter 5/8 inch, Length 35 cm

Source of Variation	Degrees of Freedom	Sums of Squares	Calculated Value of F	Tabulated $F_{0.05, (1,33)}$
Regression	1	15.27195	6799.284	≈ 4.1709
Error	33	0.074122		
Total	34	15.34607		

The table above shows that the effect due to regression on the total variations is very highly significant because the calculated value of the F –statistic is very much higher than the tabulated value of $F_{0.05, (1, 33)}$. In this case, we have taken the value of $F_{0.05, (1, 30)}$ as $F_{0.05, (1, 33)}$

is not available in the table of *F*. We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is 99.51. In other words, 99.51% of the variations are due to this mathematical relationship, while the rest 0.49% only is due to randomness. We therefore conclude that for Diameter 5/8 inch, Length 35 cm. we have

$$T = 29 + 49.14209\ e^{-0.06541\ \tau.}$$

For pipe diameter 5/8 inch and Length 40 cm. the analysis is:

Table 20. Test of Significance for the Regression Parameters: Diameter 5/8 inch, Length 40 cm

Parameters	Estimated Values	Calculated $\|t\|$	Tabulated t , 39 d.f.
Intercept $log_e\ \alpha$	4.041256	244.6794	$t_{0.05} \approx 1.684$
Slope β	-0.05266	74.0968	$t_{0.025} \approx 2.042$

Here too, we found the null hypotheses not acceptable, and hence α = 56.89776, and β = -0.05266 are statistically valid.

Table 21. Analysis of Variance of the Log linear Fit: Diameter 5/8 inch, Length 40 cm

Source of Variation	Degrees of Freedom	Sums of Squares	Calculated Value of F	Tabulated $F_{0.05,\ (1,39)}$
Regression	1	15.92043	5490.335	$\approx$ 4.0848
Error	39	0.113089		
Total	40	16.03352		

The table above shows that the effect due to regression on the total variations is very highly significant because the calculated value of the F –statistic is very much higher than the tabulated value of $F_{0.05,\ (1,\ 39)}$. In this case, we have taken the value of $F_{0.05,\ (1,\ 40)}$ as $F_{0.05,\ (1,\ 39)}$ is not available in the table of *F*. We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is 99.29. In other words, 99.29% of the variations are due to this mathematical relationship, while the rest 0.71% only is due to randomness. Hence, it can be statistically concluded that the observed data on temperature follow the negative exponential law. We therefore conclude that for Diameter 5/8 inch, Length 40 cm. we have

$$T = 29 + 56.89776\ e^{-0.05266\ \tau.}$$

In what follows, in Figures 1 to 3, considering diameters ¼ inch, ½ inch and 5/8 inch respectively, we are going to compare the fall of average observed temperature for different lengths.

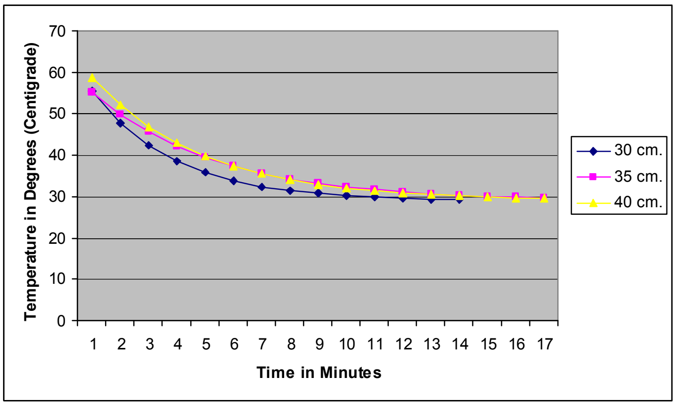

Figure 1. Fall of Expected Temperature in the condenser section for different Lengths of the evaporator section: Diameter 1/4 inch

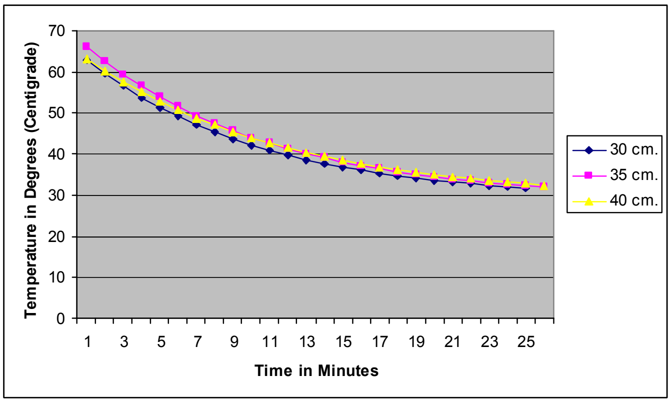

Figure 2. Fall of Expected Temperature in the condenser section for different Lengths of the evaporator section: Diameter 1/2 inch

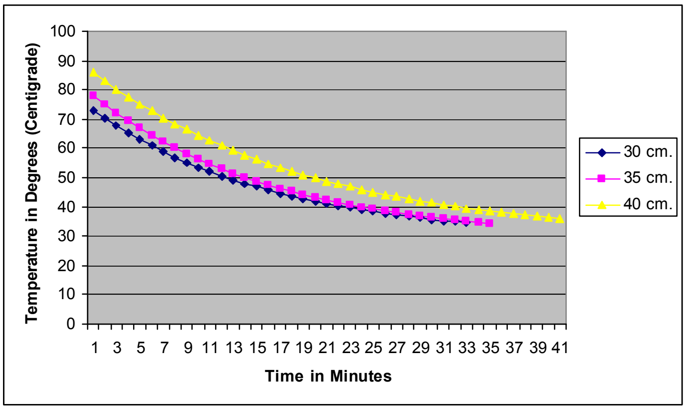

Figure 3. Fall of Expected Temperature in the condenser section for different Lengths of the evaporator section: Diameter 5/8 inch

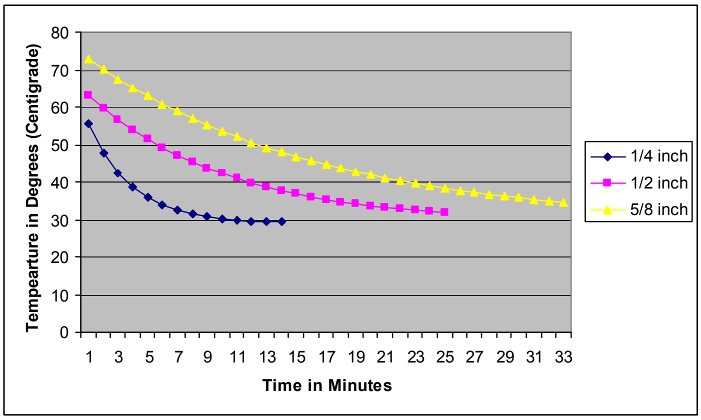

Figure 4. Fall of Expected Temperature in the condenser section for different Diameters of the pipe: Length 30 cm

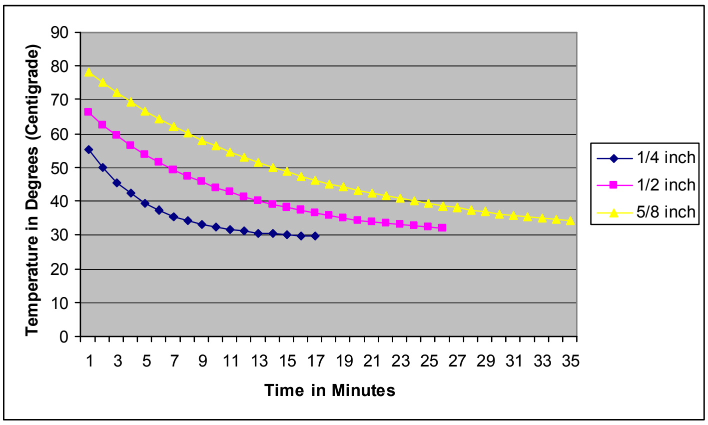

Figure 5. Fall of Expected Temperature in the condenser section for different Diameters of the pipe: Length 35 cm

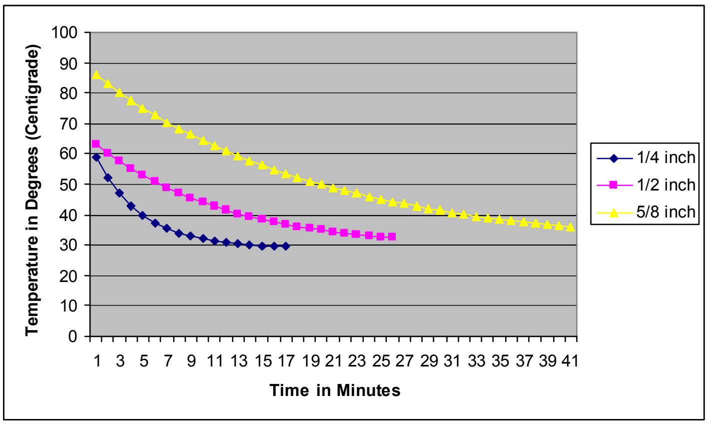

Figure 6. Fall of Expected Temperature in the condenser section for different Diameters of the pipe: Length 40 cm

In the Figures 4 to 6 above, considering lengths of the evaporator section 30 cm, 35 cm and 40 cm respectively, we are going to compare the fall of average observed temperature for different diameters.

7. Conclusions

We are now proceeding towards possible rejection of our three hypotheses stated earlier. We would test the first hypothesis statistically. The other two would have to be checked based on possible acceptance of the first hypothesis.

Our **first hypothesis** was that temperature T decreases exponentially in time τ following $T = C + \alpha\, e^{\beta\tau}$, where α and β are parameters to be estimated, $\alpha > 0$ and $\beta < 0$, and C is the controlled temperature, in our case the room temperature kept fixed using air conditioner, beyond which there would be no more cooling possible.

In the following table, the expressions of $\alpha\, e^{\beta\tau}$ for different lengths of the evaporator section and for different diameters of the pipes used are shown.

Table 22. Expressions for $T - C = \alpha\, e^{\beta\tau}$

Length → Diameter ↓	30 cm.	35 cm.	40 cm.
¼ inch	$26.45727e^{-0.33842\,\tau.}$	$26.21167\, e^{-0.22985\,\tau.}$	$29.81405e^{-0.25262\,\tau.}$
1/2 inch	$33.97826e^{-0.10462\,\tau.}$	$37.28030\, e^{-0.10089\,\tau.}$	$34.32479e^{-0.09192\,\tau.}$
5/8 inch	$43.91557e^{-0.06439\,\tau.}$	$49.14209\, e^{-0.06541\,\tau.}$	$56.89776e^{-0.05266\,\tau.}$

From Tables 4, 6, 8, 10, 12, 14, 16, 18 and 20, it is evident that in the log linear fit of the data, in every case the values of β were found to be very significantly different from zero, and that in every case the values of $log_e\, \alpha$ were found to be very significantly larger than zero.

From Tables 5, 7, 9, 11, 13, 15, 17, 19 and 21 it is evident that for every log linear fit, the effect due to regression was very significant. Indeed, in every case, the coefficients of determination were found to be extremely high. We conclude that temperature decreases exponentially in time.

Our **second hypothesis** was that for a fixed diameter of the pipe, fall of temperature depends on the length of the evaporator section. We have mentioned earlier that we would try to draw a conclusion based on fitted curves only.

From Figures 1, 2 and 3, it is not really clear that temperature falls faster or slower for smaller lengths of the evaporator section. A row wise comparison of the values of the instantaneous failure rates ($-\beta$) particularly for diameter ½ inch in Table 22 above, would show this. It can be seen that for lengthwise changes the values of ($-\beta$) are decreasing. For the other two cases, the values of ($-\beta$) have shown a decrease from 30 cm to 40 cm, though the decrements are not uniform. In other words, from our data, it can not be conclusively said that for a fixed diameter of the pipe, fall of temperature depends on the length of the evaporator section. It may be that in the range of temperature that we have worked in, the picture is not quite clear. Hence we would choose to remain inconclusive about the acceptability of this hypothesis.

Our **third hypothesis** was that for a fixed length of the evaporator section, temperature falls faster for smaller diameter of the pipe. We have mentioned that we would try to draw a conclusion based on fitted curves only.

From Figures 4, 5 and 6, it is very clear that temperature falls faster for smaller diameters of the pipes. A column wise comparison of the instantaneous failure rates (-β) in Table 22 above would clearly show this. We conclude that for a fixed length of the evaporator section, temperature falls faster for smaller diameter of the pipe.

References

[1] D. A. Reay, Heat Recovery Systems, E. & F. N. Span, London, 1979.

[2] C. J. Tu, Gravity Heat Pipe Heat Exchanger and Its Application to Exhausted Heat Utilization, Zhejiang University Press, Hangzhou, 1989.

[3] V. G. Pastukhov, F. Yu , C. V. Maidanik, M. A. Vershinin and M.A. Korukov, Miniature loop heat pipes for electronics cooling, Applied Thermal Engineering, 23 (9), 2003, 1125–1135.

[4] S. Lin, John Broadbent, Ryan McGlen, Numerical study of heat pipe application in heat recovery systems, Applied Thermal Engineering, 25, 2005, 127–133.

[5] E. Firouzfar and M. Attaran, A Review of Heat Pipe Heat Exchangers Activity in Asia, Proceedings of World Academy of Science, Engineering and Technology, Vol. 30, 2008.

[6] H. Akachi, Structure of a heat pipe, U.S. Patent, Patent Number 4921041, 1990.

[7] H. Akachi, U.S. Patent, Patent Number 5219020, 1993.

[8] H. Akachi, F. Polasek and P. Stulc, Pulsating heat-pipe. In: Proceedings of the 5th international heat-pipe symposium, Melbourne, Australia, 1996, 208–17.

[9] Y. Zhang and A. Faghri, Heat transfer in a pulsating heat pipe with an open end, International Journal of Heat and Mass Transfer, Vol. 45, 2002, 755-764.

[10] Y. Zhang and A. Faghri, Oscillatory flow in pulsating heat pipes with arbitrary number of turns", 8th AIAA / ASME, Joint Thermophysics and Heat Transfer Conference, St. Louis, Missouri, 2002 .

[11] S. Lin, J. Broadbent and R. McGlen, Numerical study of heat pipe application in heat recovery systems. Available online since 19 August, 2004.

[12] S. Khandekar, M. Schneider, P. Schafer, R. Kulenovic and M. Groll, Thermo fluid dynamic study of flat plate closed loop pulsating heat pipes, Micro scale Thermophysical Engineering, Vol. 6/4, 2002, 303-318.

[13] S. Khandekar, M. Groll, Pulsating heat pipes: study on a two-phase loop, 13th International Conference on Thermal Engineering and Thermogrammetry, 2003.

[14] S. Khandekar and M. Groll, An insight into thermo-hydrodynamic coupling in closed loop pulsating heat pipes, International Journal of thermal sciences, Vol. 43, Issue 1, 2004, 13-20.

[15] S. Khandekar, V. V. S. N. S. Manyam Sanka, M. Groll and Manmohan Pandey, Two-phase flow modeling in closed loop pulsating heat pipes, 13th International Conference (13th IHPC), Shanghai, China, 2004.

[16] P. D. Dunn and D. A. Reay, Heat Pipes, 2nd ed., Pergamon, New York, 1978.

[17] S. H. Noie-Baghban. and G. R. Majideian, Waste heat recovery using heat pipe heat exchanger for surgery rooms in hospital, Applied Thermal Engineering, 20, 2000, 1271–1282.

[18] K.A. Joudy. and A. M. Witwit, Improvements of gravity assisted wickless heat pipes, Energy Conversion and Management, 41, 2000, 2041–2061.

[19] Z. S. Shang and P.L. Li, Practical Technique of Heat Pipes, Publisher of Chemical Industry, Beijing, 1988.

[20] Feng Yang , Xiugan Yuan and Guiping Lin, Waste heat recovery using heat pipe heat exchanger for heating automobile using exhaust gas, Applied Thermal Engineering, 23 , 2003, 367–372.

[21] M. Akyurt, M. Beirutty, O. M. AI-Rabghi, Y. S. H. Najjar and T. Alp, Heat pipes for heat recovery, Trans. Mech. Eng., 1993, 221-234.

[22] N. Lamfon, M. Akyurt, O. M. Al-Rabghi and Y. S. H. Najjar, Thermophysical relationships for waste heat recovery using looped heat pipes, Int. J. Energy Res., 18. 1994, 633-642.

[23] S. C. Arora, and S. Domkundwar, A Course in Heat and Mass Transfer, J. C. Kapoor, New Delhi, 1987.

[24] G. A. Asselman, and D. B. Green, Heat pipes. Philips Tech. Rev., 33, 1973, 104-113.

[25] E. Azad and F. Geoola, Design procedure for gravity-assisted heat pipe heat exchanger, J. Heat Recov. Sys., 4, 1984, 101-111.

[26] E. Azad and B. M. Gibbs, Analysis of air-to-water heat pipe heat exchanger, J. Heat Recov. Sys., 7, 1987, 351-358.

[27] E. Azad, F. Mohammadieh, and F. Moztarzadeh, Thermal performance of heat pipe recovery system, J. Heat Recov. Sys., 5, 1985, 561-570.

[28] S. W. Chi, Heat pipe theory and practice, Hemisphere, Washington, DC, 1976, 242.

[29] B. S. Huang and J. T. Tsuei, Method of analysis for heat pipe heat exchangers, Int. J. Heat Mass Transfer, 28, 1985, 553-562.

[30] M. Shiraishi, K. Kikuchi and T. Yamanishi, Investigation of heat transfer characteristics of a two-phased closed thermosyphon, J. Heat Recov. Sys., 1, 1981, 287-297.

[31] P. Stulc and L. L. Vasiliev, Heat pipe heat exchangers in heat recovery system. J. Heat Recov. Sys., 5, 1985, 415-418.

[32] I. L. Pioro, Maximum heat transfer capability two-phase thermosyphons, Heat Transfer Soy. Res., 15, 1983, 24-31.

[33] I. L Pioro., Correlation of experimental data on the maximum heat and mass transfer in two-phase thermosyphons, Heat Transfer Soy. Res., 17, 1985, 75-83.

[34] F. Dorban, Steady-state characteristics and stability thresholds of a closed two-phase thermosyphon, Int. J. Heat Transfer, 28, 1985, 949-957.

[35] U. Gross and E. Hahne, Heat transfer in closed thermosyphon over a wide range of pressures and inclinations, Ger. Chem. Eng., 9, 1986, 292-299.

[36] K. A Joudy, T. A. Husain and P. M. Abdulmajeed, Improved heat transport of heat pipes with an internal wall separating liquid and vapor streams, Energy Converg. Mgmt., 31, 1991, 141-148.

[37] R. Peretz, Heat transfer effectiveness of heat pipes, Heat Recov. Sys., 2, 1982, 165-172.

[38] T. Wadowski, A. Akbarzadeh and P. Johnson, Characteristics of a gravity-assisted heat pipe based heat exchanger, J. Heat Recov. Sys. & CHP, 11, 1991, 69-77.

[39] E. Azad and F. Moztarzadeh, Design of air-to-water co-axial heat pipe heat exchanger, J. Heat Recov. Sys., 5, 1985, 217-224.

[40] R. Peretz, and B. Horbaniuc, Optimal heat pipe heat exchanger design, J. Heat Recov. Sys., 4, 1984, 9-24.

[41] M. Akyurt, Development of heat pipes for solar water heaters, Solar Energy, 32, 1984, 625-631.

[42] M. Akyurt, The heat pipe solar water heater, J. Eng. Appl. Sci., 3, 1986, 23-38.

[43] A. Basiulis, Compatibility and reliability of heat pipe materials, Int. Heat Pipe Conf. 2nd, Eur. Space Agency, 1, 1976, 357-372.

[44] D. A. Reay, The Perkins tube--A noteworthy contribution to heat exchanger technology, J. Heat Recov. Sys., 2, 1982, 173-187.

[45] D. A. Reay, The European Community energy R & D program--Report on a recent contractors meeting on heat exchangers and heat recovery, J. Heat Recov. Sys., 3, 1983, 245-271.

[46] S. Seshan and D. Vijayalakshmi, Heat pipes--Concepts, materials, and applications, Energy Convers. Mgmt., 26, 1986, 1-9.

[47] M. Akyurt, N., J. Lamfon, Y. S. H. Najjar, M. H. Habeebullah, and T. Alp, Modeling of waste heat recovery by looped water-in-steel heat pipes Int. J. Heat and Fluid Flow, 16, 1995, 263-271.

[48] Y. Miyazaki, F. Polasek and H. Akachi, Oscillating heat-pipe with check valves. In: Proceedings of the 6th international heat pipe symposium, Chiang Mai, Thailand, 2000, 389–93.

[49] N. Pipatpaiboon, Effect of inclination angle, working fluid and number of check vales on the characteristics of heat transfer in a closed-looped oscillating heat-pipe with check valves, Proceedings of 1st international seminar on heat-pipe and heat-recovery systems, Kuala Lumpur, Malaysia, 2004, 83–87.

[50] S. Rittidech, W. Dangeton and S. Soponronnarit, Closed-ended oscillating heat-pipe air-preheater for energy thrift in dryer, Appl. Energy, 81, 2005, 198–208.

[51] Wu Xiao Ping, P. Johnson and A. Akbarzadeh, Application of heat-pipe exchangers to humidity control in air-conditioning system, Appl. Therm. Eng., 6:561–8, 1997.

[52] Irwin Guttman, Samuel Wilks and John Stuart Hunter, Introductory Engineering Statistics, John Wiley & Sons, New York, 1982.

[53] Sheldon M. Ross, Introduction to Probability and Statistics for Engineers and Scientists, Academic Press, London, 2004.

Authors

.

Pranab Kumar Barua completed his Bachelor of Technology in Mechanical Engineering from Sikkim Manipal Institute of Technology, Gangtok, India, in 2007. He completed his Master of Technology in Energy Technology from Tezpur University, India, in 2009.

Dhanapati Deka is an Associate Professor in the Department of Energy of Tezpur University, Assam, India. He did his Bachelor of Science in Agriculture from Assam Agricultural University, India in 1990, and Masters of Science in the same discipline from the same university in 1992. Subsequently, he earned his doctorate from Tezpur University. His research interests lie in the fields of Bio-energy, Biogas Technology, Biomass Gasification, Energy and Environment.

Uday S. Dixit is a Professor in the Department of Mechanical Engineering of the Indian Institute of Technology, Guwahati, Assam, India. He completed his Bachelor of Engineering from the erstwhile University of Roorkee, in 1987. He did his Master of Technology in 1993, and his PhD in 1998, both from the Indian Institute of Technology, Kanpur, in Mechanical Engineering. His research interests lie in Design and Manufacturing covering mainly the fields of Finite Element Modeling, Neural Network, Fuzzy Set Applications, and Mechatronics.