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Mathematical Modelling of Change of Temperature in a Pulsating Heat Pipe with Multiple Turns

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Abstract

In this article, it has been shown that temperature decreases exponentially with respect to time in pulsating heat pipes with multiple turns in the evaporator section. In addition, it has been found that the rate of change of temperature also increases with increase in the number of turns. Finally, it has been validated statistically that the instantaneous failure rate increases exponentially as the number of turns increases.

Key words: *Heat pipe heat exchanger, waste heat recovery, instantaneous failure rate.*

1. Introduction

A heat pipe heat exchanger (HPHE), consisting of a bundle of individual heat pipes with vaporizing and condensing sections at the respective ends might be unfit for large-scale needs in industrial applications. However, modifications were added, and the HPHE has received much attention since it was launched into industry at the beginning of the eighties [1]. Heat pipes are two-phase heat transfer devices with high thermal conductivity. As a heat recovery system, the HPHE is very efficient, lightweight and compact. It is a self-contained energy recovery device. It works as a homogeneous flow model, and so it is named as *pulsating* heat pipe.

Thermal performance of an ordinary heat pipe has been studied extensively ([2], [3], [4], [5], [6], [7], [8], [9], [10], [11]). Large quantities of hot gases are generated from boilers, furnaces etc. If some of this waste heat could be recovered, a lot of primary fuel could be saved. Recovery of waste heat has a direct effect on the efficiency of the process.

Heat exchangers made of heat pipes are very effective devices for waste heat recovery. Large quantities of heat can be transported through a small cross-sectional area over a considerable distance with no additional power. Due to the high heat transport capacity, heat exchangers with heat pipes have become much smaller than traditional heat exchangers in handling high heat fluxes. Simplicity of design and manufacturing, wide temperature application range and ability to control and transport heat at various temperature levels are unique in heat pipes. The use of the heat pipe heat exchanger would reduce energy consumption, reducing carbon dioxide production thereby.

An HPHE has no moving parts and hence requires low maintenance. They are silent and reversible in operation and require no external energy other than the thermal energy they transfer. It does not need input power for its operation and is free from cooling water and lubrication systems. The heat pipe heat recovery systems are capable of operating at high temperature, 315 degrees centigrade, with quite high, 60% to 80%, heat recovery capability.

The increasing demand for energy efficiency in domestic appliances is the main drive for introducing heat recovery systems in these appliances. Heat transfer efficiency in such systems is the primary factor for efficient performance of the whole systems. The heat pipe, as a high efficiency heat transfer element, is used in the electronics cooling industry and energy efficiency sectors. They can be embedded with aluminium heat sinks to enhance cooling efficiency, compactness of cooling devices and can also be assembled with a fin stack for fluid heat transfer.

The heat transfer effectiveness of heat pipe heat exchangers was studied by Peretz [12], who found that the heat transfer effectiveness (HTE) of the heat exchanger depends upon the HTE of a single heat pipe and the number of rows parallel to the flow. The thermal performance of heat pipe heat recovery system was investigated by Azad *et al*. [6]. A model for the system was developed to predict the temperature distribution in longitudinal rows of the heat exchanger.

It is well known that water is excellent as a working fluid for heat pipes for its high latent heat, easy availability, and its high resistance to decomposition and degradation. Water has been used particularly successfully in copper heat pipes for low temperature applications. Problems of incompatibility for iron pipes have also been reported ([13], [14], [15], [7], [8], [16], [17], [18]. The studies confirm that hydrogen is evolved inside iron-water heat pipes, so that the condenser eventually becomes flooded with this noncondensable gas, making the heat pipe inoperative.

Water was selected as the working fluid of the heat pipe system developed by Akyurt *et al.* [19]. Because of the problem of incompatibility with iron, copper was selected initially as the container material. They developed a copper-water heat pipe system using the analysis presented by Lamfon *et al.* [20].

Mathematical models considering the heat transfer effects on operation of a pulsating heat pipe with open end was proposed by Zhang and Faghri [21]. They further studied ([22]) numerically the oscillatory flow in pulsating heat pipes. Numerical study was also done by Lin *et al.* [23].

However, the design of the heat recovery systems with heat pipe units is the key to providing a heat exchanger system to work as efficiently as expected. Without correct design of such systems, heat pipes would not be able to transport enough heat, and may function as a poor thermal conductor in the systems. Although the operating condition of a heat pipe is simple, its appropriate design and construction is complicated. The parameters concerned should be controlled, experimental investigations are therefore very important.

To check for the feasibility and the efficiency of a heat pipe heat exchanger, it is important to study the manner in which it helps in removing heat from a system concerned. It is obvious that the higher the heat transfer rate, the more efficient the heat pipe is. In our work for multiple loops, we are interested to establish mathematical models of fall of temperature in the condenser section for different number of turns in the evaporator section. This ultimately translates itself into the heat removal rate from the pipe. In the present experimental work, the heat pipe has been selected to be of the *pulsating* type as we are going to use water as the working fluid in both of its phases viz. liquid and vapor, inside the same compartment. In other words, our setup is of a *homogeneous* flow model.

In this work, we propose to find answers to the following questions:

i) What is the general expression of falling temperature as a function of time in the case of multiple turns in the evaporator section?
ii) What is the effect of the number of turns in the evaporator section of a heat pipe on the cooling rate?
iii) Is there any specific functional relationship between the number of turns in the evaporator section of a heat pipe and the instantaneous failure rate?

Basically, we would like to find out whether the instantaneous failure rate increases with increase in the number of turns in the evaporator section of a heat pipe. We are interested to establish the mathematical model following which the instantaneous failure rate behaves as a function of the number of turns.

2. Methodology

For efficient operation of a heat pipe, numerous parameters should be controlled. As such, it makes it imperative to conduct experimental investigations as elaborately as it is required. The first part of the experimentation requires analysis of performance characteristics of a single loop pulsating heat pipe. Water has been used particularly successfully in copper heat pipes for low temperature applications. Water was selected as the working fluid of the heat pipe system as was done in the case of Akyurt *et. al*. [19]. Because of the problem of incompatibility with iron, they selected copper as the container material, and later have developed a copper-water heat pipe system using the analysis presented by Lamfon *et al.* [20]. Hence for our case, copper was selected as the material for the pipes as it has a very high thermal conductivity.

Copper pipes of diameter ¼ inch (= 0.6354 cm.) were constructed. We shall present in our discussions the unit of the diameters in inch, because with such specifications only were they available in the market. Each pipe was of the shape of a square. The length of the condenser section was kept constant at 40 cm. We thus have fabricated 5 different heat pipe setups.

Each pipe contains two cutout sections of length 5 cm welded into the main setup. One of them acts as water inlet while the other acts as an exit route for air pockets. The pipes were partially filled with water and then the openings of the inlet and the outlet sections were plugged with a sealing agent and an adhesive tape, the combination of which proved effective enough.

Figure 1. A Heat Pipe with 5 Turns

To reduce loss of heat from the pipes baring the evaporator and the condenser sections, glass wool was utilized as an insulator. Adhesive tape was used to bind the material with the pipe. We have already stated that we are interested to establish mathematical models of fall of temperature in heat pipes. The mathematical models are what we are interested in, and we are not really interested on the operability of the entire heat exchanger set up. That is why we have used insulating materials just for minimizing the heat loss without actually evaluating the critical radius of insulation as it would not really affect the mathematical models.

We used the software called *Daisy Lab* for acquisition of data which in our case is temperature. It was connected to a thermocouple to measure the temperature which gets displayed on the computer screen. Every experiment was replicated thrice, and the average temperatures in the condenser section after every minute were noted.

3. Experimental Results

In tables 1, 2, 3, 4 and 5 below, we are going to show our experimental findings in moderate temperature. The room temperature was fixed at 29^o Centigrade.

Table 1. Average temperature in the condenser section after τ minutes of operation: Number of turns = 1

τ	Temperature	τ	Temperature
0	65	10	36.8
1	60.3	11	35.5
2	56	12	34.7
3	52	13	34
4	48	14	33
5	45	15	32.4
6	42.1	16	31.9
7	39.9	17	30.9
8	38.9	18	30.1
9	37.9	19	29.2

Table 2. Average temperature in the condenser section after τ minutes of operation: Number of turns = 2

τ	Temperature	τ	Temperature
0	65	10	35.6
1	60.2	11	34.9
2	54	12	34
3	50.8	13	33.1
4	47.3	14	32.3
5	44.3	15	32
6	41.2	16	31.5
7	39.2	17	30.3
8	37.8	18	29.7
9	36.8	19	29.2

Table 3. Average temperature in the condenser section after τ minutes of operation: Number of turns = 3

τ	Temperature	τ	Temperature
0	65	10	34.5
1	58.4	11	33.7
2	52.1	12	32.8
3	48.7	13	32.1
4	44.6	14	31.7
5	40.3	15	31.1
6	38.1	16	30.6
7	37.2	17	30.1
8	36.1	18	29.6
9	35.1	19	29.3

Table 4. Average temperature in the condenser section after τ minutes of operation: Number of turns = 4

τ	Temperature	τ	Temperature
0	65	10	34.3
1	57.8	11	33.3
2	52	12	32.7
3	48.2	13	32.1
4	44.1	14	31.6
5	40.3	15	31.1
6	37.9	16	30.5
7	37	17	30.1
8	36.1	18	29.6
9	35	19	29.2

Table 5: Average temperature in the condenser section after τ minutes of operation: Number of turns = 5

τ	Temperature	τ	Temperature
0	65	10	34.1
1	57.7	11	33.1
2	51.7	12	32.6
3	48	13	32
4	43.6	14	31.6
5	40.1	15	31
6	37.8	16	30.5
7	36.4	17	30.1
8	35.8	18	29.6
9	34.9	19	29.2

Numerical and statistical analysis of the data collected would now lead us to certain conclusions related to heat transfer. The analysis in detail has been reported in a monograph by Barua [24]. In the next Section, we are going to discuss the numerical and the statistical analytical matters with reference to mathematical modeling of the data.

4. Statistical Analysis of the Data

First, we would like to discuss in short about instantaneous failure rate, known also as hazard rate in Reliability Engineering. The hazard rate is the probability that an item will fail in the next instant of time divided by its reliability to that instant [25, page 319]. For the exponential probability law defined by the density

$$f(t) = \lambda \exp(-\lambda x),\ \lambda > 0,\ x \geq 0,$$

the hazard rate is λ. Here $f(t)$ is proportional to $\exp(-\lambda x)$. Accordingly, if it can be established that a quantity, in our case temperature T, decreases negative exponentially in time τ, following

$$T = C + \alpha\, e^{\beta\tau},$$

$\alpha > 0$ and $\beta < 0$, and C is the controlled temperature beyond which there would be no more cooling allowed, we can see that $(-\beta)$ in the model would be the hazard rate of the system, because here $(T - C)$ would be proportional to $e^{\beta\tau}$. We have mentioned earlier that we are interested basically to find the mathematical model defining hazard rate as a function of number of turns in the evaporator section of the heat pipe.

4.1 The Hypotheses:

The following two are our hypotheses:

i. **Hypothesis -1**: We hypothesize that temperature T decreases exponentially in time τ following $T = C + \alpha\, e^{\beta\tau}$, where α and β are parameters to be estimated.
ii. **Hypothesis - 2**: We hypothesize that the instantaneous failure rate φ where $\varphi = (-\beta)$, β being the log linear regression coefficient in $T = C + \alpha\, e^{\beta\tau}$, exponentially increases when the number of turns in the evaporator section increases.

4.2 Tests of Significance

We now proceed to fit the hypothesized equations from the observed data using the standard method of least squares. For this, we would have to fit the equation

$$log_e(T - C) = log_e\,\alpha + \beta\tau$$

estimating the parameters α and β. Thereafter we would proceed to test for regression coefficients used in the model. Once the coefficients are found non-rejectable statistically, we would proceed to find the level of acceptance of the models using the technique of analysis of variance.

In Tables 6 and 7, we are going to show the Test of Significance for the Regression Parameters and Analysis of Variance of the log linear fit respectively, number of turns in the evaporator section being 1.

Table 6. Test of Significance for the Regression Parameters: One Turn

Parameters	Estimated Values	Standard Errors	Calculated $\|t\|$	Tabulated t, 18 d.f.
Intercept $log_e\, \alpha$	3.828255	0.196341	19.49797	$t_{0.05}$ =1.734
Slope β	-0.19932	0.017668	11.2818	$t_{0.025}$=2.101

We now state a hypothesis H_0^1: $\beta = 0.0$. We would like to test this hypothesis against the *two sided* alternative hypothesis H_1^1: $\beta \neq 0.0$. It can be seen that our calculated value of $|t|$ (= 11.2818) is *very much larger* than the tabulated value ([25], page 533) of t (= 2.101) at 5% level of significance. Hence we conclude that the hypothesis H_0^1 is not acceptable. In other words, in this case $\beta \neq 0.0$.

We now state another hypothesis H_0^2: $log_e\, \alpha = 0.0$. We would like to test this hypothesis against the *one sided* alternative hypothesis H_1^2: $log_e\, \alpha > 0.0$. It can be seen that the calculated value of $|t|$ (= 19.49797) is *very much larger* than the tabulated value of t (= 1.734) at 5% level of significance. Hence we conclude that the hypothesis H_0^2 is not acceptable. In other words, in this case $log_e\, \alpha > 0.0$.

We therefore conclude that for one turn $log_e\, \alpha = 3.828255$ or $\alpha = 45.98223$, and $\beta = -0.19932$.

Table 7. Analysis of Variance of the Log linear Fit: One Turn

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	Calculated Value of F	Tabulated $F_{0.05,\,(1,18)}$
Regression	1	26.42013	26.42013	127.2792	4.4139
Error	18	3.736371	0.207576		
Total	19	30.1565			

The table above shows that the effect due to regression on the total variations is very highly significant because the calculated value of the F –statistic (= 127.2792) is very much higher than the tabulated value ([25], page 536) of $F_{0.05,\,(1,\,18)}$ (= 4.4139). We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is given by (26.42013 / 30.1565)*100 = 87.61. In other words, 87.61% of the variations are due to this mathematical relationship, while the rest 12.39% is due to randomness. Hence, it can be statistically concluded that the observed data on temperature shown in Table - 1 follows the negative exponential law. We therefore conclude that for one turn the equation

$$T = 29 + 45.98223\, e^{-0.19932\, \tau}$$

is statistically valid with coefficient of determination 87.61%.

In Tables 8 and 9, we are going to depict the Test of Significance for the Regression Parameters and Analysis of Variance of the log linear fit respectively, number of turns in the evaporator section being 2.

Table 8. Test of Significance for the Regression Parameters: Two Turns

Parameters	Estimated Values	Standard Errors	Calculated $\|t\|$	Tabulated t, 18 d.f.
Intercept $log_e\, \alpha$	3.824116	0.180756	21.15627	$t_{0.05}$ =1.734
Slope β	-0.21201	0.016265	13.0348	$t_{0.025}$=2.101

To test the hypothesis H_0^1 against the *two sided* alternative hypothesis H_1^1 in this case, it can be seen that the calculated value of $|t|$ (= 13.0348) is *very much larger* than the tabulated value of t at 5% level of significance. Hence we conclude that the hypothesis H_0^1 is not acceptable. Therefore the alternative hypothesis H_1^1 is true. In other words, in this case $\beta \neq 0.0$. As for the hypothesis H_0^2 against the *one sided* alternative hypothesis H_1^2 , it can be seen that the calculated value of $|t|$ (= 21.15627) is *very much larger* than the tabulated value of t at 5% level of significance. Hence we conclude that the hypothesis H_0^2 is not acceptable. Therefore the alternative hypothesis H_1^2 is true. In other words, in this case $log_e\, \alpha > 0.0$. Hence for two turns $log_e\, \alpha = 3.824116$ or $\alpha = 45.79230$, and $\beta = -0.21201$.

Table 9. Analysis of Variance of the Log linear Fit: Two Turns

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	Calculated Value of F	Tabulated $F_{0.05, (1,18)}$
Regression	1	29.89166	29.89166	169.9071	4.4139
Error	18	3.16673	0.175929		
Total	19	33.05839			

The table above shows that the effect due to regression on the total variations is very highly significant because the calculated value of the F –statistic is very much higher than the tabulated value of $F_{0.05, (1, 18)}$. We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is 90.42. In other words, 90.42% of the variations are due to this mathematical relationship, while the rest 9.58% only is due to randomness. Hence, it can be statistically concluded that the observed data on temperature shown in Table - 2 follows the negative exponential law. We therefore conclude that for two turns the equation

$$T = 29 + 45.79230\, e^{-0.21201\, \tau}$$

is statistically valid with coefficient of determination 90.42%.

In Tables 10 and 11, we are going to depict the Test of Significance for the Regression Parameters and Analysis of Variance of the log linear fit respectively, number of turns in the evaporator section being 3.

Table 10. Test of Significance for the Regression Parameters: Three Turns

Parameters	Estimated Values	Standard Errors	Calculated $\|t\|$	Tabulated t , 18 d.f.
Intercept $log_e\, \alpha$	3.654231	0.115552	31.62405	$t_{0.05}$ =1.734
Slope β	-0.21285	0.010398	20.471	$t_{0.025}$=2.101

It has been found that the calculated value of $|t|$ (= 20.471) is *very much larger* than the tabulated value of t at 5% level of significance. Hence we conclude that the hypothesis H_0^1 is not acceptable. Therefore the alternative hypothesis H_1^1 is true. In other words, in this case $\beta \neq 0.0$. It has also been seen that the calculated value of $|t|$ (= 31.62405) is *very much larger* than the tabulated value of t at 5% level of significance. Hence we conclude that the hypothesis H_0^2 is not acceptable. Therefore the alternative hypothesis H_1^2 is true. In other words, in this case $log_e\, \alpha > 0.0$. We therefore conclude that for three turns in the evaporator section, $log_e\, \alpha = 3.654231$ or $\alpha = 38.63779$, and $\beta = -0.21285$.

Table 11. Analysis of Variance of the Log linear Fit: Three Turns

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	Calculated Value of F	Tabulated $F_{0.05, (1,18)}$
Regression	1	30.12922	30.12922	419.0599	4.4139
Error	18	1.294149	0.071897		
Total	19	31.42337			

The table above shows that the effect due to regression on the total variations is very highly significant because the calculated value of the F –statistic is very much higher than the tabulated value of $F_{0.05, (1, 18)}$. We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is 95.88. In other words, 95.88% of the variations are due to this mathematical relationship, while the rest 4.12% only is due to randomness. Hence, it can be statistically concluded that the observed data on temperature shown in Table - 3 follows the negative exponential law. We therefore conclude that for three turns the equation

$$T = 29 + 38.63779\, e^{-0.21285\, \tau}$$

is statistically valid with coefficient of determination 95.88%.

In Tables 12 and 13, we are going to depict the Test of Significance for the Regression Parameters and Analysis of Variance of the log linear fit respectively, number of turns in the evaporator section being 4.

Table 12. Test of Significance for the Regression Parameters: Four Turns

Parameters	Estimated Values	Standard Errors	Calculated $\|t\|$	Tabulated t, 18 d.f.
Intercept $log_e\, \alpha$	3.67042	0.142865	25.69147	$t_{0.05}$ =1.734
Slope β	-0.21881	0.012856	17.0203	$t_{0.025}$=2.101

While testing the hypothesis H_0^1 against the *two sided* alternative hypothesis H_1^1 , it was found that the calculated value of $|t|$ (= 17.0203) is *very much larger* than the tabulated value of t at 5% level of significance. Hence we conclude that the hypothesis H_0^1 is not acceptable. Therefore the alternative hypothesis H_1^1 is true. In other words, in this case $\beta \neq 0.0$. For H_0^2 against the *one sided* alternative hypothesis H_1^2 , it was seen that the calculated value of $|t|$ (= 25.69147) is *very much larger* than the tabulated value of t at 5% level of significance. Hence we conclude that the hypothesis H_0^2 is not acceptable. Therefore the alternative hypothesis H_1^2 is true. In other words, in this case $log_e\, \alpha > 0.0$. We therefore conclude that for four turns $log_e\, \alpha$ = 3.67042 or α = 39.26839, and β = -0.21881.

Table 13. Analysis of Variance of the Log linear Fit: Four Turns

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	Calculated Value of F	Tabulated $F_{0.05, (1,18)}$
Regression	1	31.83774	31.83774	289.6902	4.4139
Error	18	1.978249	0.109903		
Total	19	33.81599			

The table above shows that the effect due to regression on the total variations is very highly significant because the calculated value of the F –statistic is very much higher than the tabulated value of $F_{0.05, (1, 18)}$. We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is 94.15. In other words, 94.15% of the variations are due to this mathematical relationship, while the rest 5.85% only is due to randomness. Hence, it can be statistically concluded that the observed data on temperature shown in Table - 4 follows the negative exponential law. We therefore conclude that for four turns the equation

$$T = 29 + 39.26839\, e^{-0.21881\, \tau}$$

is statistically valid with coefficient of determination 94.15%.

In Tables 14 and 15, we are going to depict the Test of Significance for the Regression Parameters and Analysis of Variance of the log linear fit respectively, number of turns in the evaporator section being 5.

Table 14. Test of Significance for the Regression Parameters: Five Turns

Parameters	Estimated Values	Standard Errors	Calculated $\|t\|$	Tabulated t, 18 d.f.
Intercept $log_e\, \alpha$	3.646031	0.140281	25.99083	$t_{0.05}$ =1.734
Slope β	-0.21847	0.012623	17.3067	$t_{0.025}$=2.101

It can be seen that the calculated value of $|t|$ (= 17.3067) is *very much larger* than the tabulated value of t at 5% level of significance. Hence we conclude that the hypothesis H_0^1 is not acceptable. Therefore the alternative hypothesis H_1^1 is true. In other words, in this case $\beta \neq 0.0$. It can also be seen that the calculated value of $|t|$ (= 25.99083) is *very much larger* than the tabulated value of t at 5% level of significance. Hence we conclude that the hypothesis H_0^2 is not acceptable. Therefore the alternative hypothesis H_1^2 is true. In other words, in this case $log_e\, \alpha > 0.0$. We therefore conclude that for five turns in the evaporator section $log_e\, \alpha = 3.646031$ or $\alpha = 38.32226$, and $\beta = -0.21847$.

Table 15. Analysis of Variance of the Log linear Fit: Five Turns

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	Calculated Value of F	Tabulated $F_{0.05, (1,18)}$
Regression	1	31.73848	31.73848	299.5235	4.4139
Error	18	1.907338	0.105963		
Total	19	33.64582			

The table above shows that the effect due to regression on the total variations is very highly significant because the calculated value of the F –statistic is very much higher than the tabulated value of $F_{0.05, (1, 18)}$. We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is 94.33. In other words, 94.33% of the variations are due to this mathematical relationship, while the rest 5.67% only is due to randomness. Hence, it can be statistically concluded that the observed data on temperature shown in Table - 5 follows the negative exponential law. We therefore conclude that for five turns the equation

$$T = 29 + 38.32226\, e^{-0.21847\, \tau}$$

is statistically valid with coefficient of determination 94.33%.

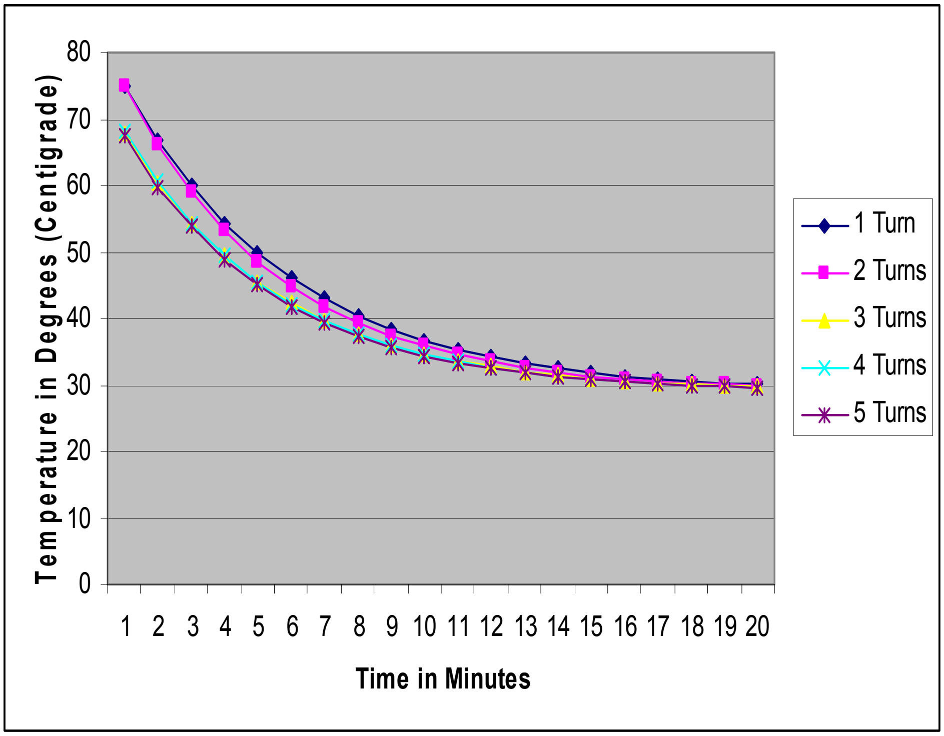

Figure 2. Fall of Expected Temperature in the condenser section: Multiple turns in the evaporator section

In Fig 2, considering pipes of diameter ¼ inch and the condenser section being 40 cm in length, we have compared the fall of expected temperature for different turns in the evaporator section. It has thus been established that temperature T decreases negative exponentially in time τ, following

$$T = C + \alpha\, e^{\beta\tau},$$

$\alpha > 0$ and $\beta < 0$, and C is the controlled temperature beyond which there is no more cooling allowed. In other words, the rate of change of temperature with respect to time is directly proportional to temperature T as well as to β. Therefore, the dependence of the cooling rate on number of turns in the evaporator section would be established only after finding out the relationship between β and the number of turns.

We now proceed to find statistically the possible relation between the log linear regression parameter β and the number n of turns in the evaporator section of the pipes. It can be observed that β decreases as the number of turns increases, and that the decrement is very slow. As soon as it has been established that decrement of temperature is negative exponential, we are already certain that in any particular number of turns in the evaporator section the hazard rate is constant ([25], pp 325), independent of time. We have hypothesized that the hazard rate ($-\beta$) increases exponentially with respect to number of turns. If this hypothesis is found to be statistically nonrejectable, it would help us in drawing a conclusion that we should not possibly go on increasing the number of turns because this would have an adverse effect on cooling. As β is negative, we would instead like to see the relationship between ($-\beta$), the instantaneous failure rates in the decay curves and n. Let us say, the instantaneous failure rate $\varphi = (-\beta)$. The hypothesized equation in this case is

$$\varphi = \xi\, e^{\psi n},\ \xi \geq 0,\ \psi \geq 0.$$

Once again, as it is transformable to the linear form of the type

$$\log_e \varphi = \log_e \xi + \psi n.$$

Now we can apply the usual method of least squares to fit this equation. In tables 16 and 17, we are going to depict the Test of Significance for the Regression Parameters $\log_e \xi$ and ψ, the hypothesized values of the Instantaneous Failure Rates, and Analysis of Variance of the concerned log linear fit respectively, for the equation stated above.

Table 16. Test of Significance for Regression Parameters in $\log_e \varphi = \log_e \xi + \psi n$

Parameters	Estimated Values	Standard Errors	Calculated $\|t\|$	Tabulated t, 3 d.f.
Intercept $\log_e \xi$	-1.61487	0.020042	80.5744	$t_{0.05}$ =2.353
Slope ψ	0.021504	0.006043	3.558641	$t_{0.025}$=3.182

We now proceed to test the hypothesis $H_0^1: \psi = 0.0$ against the *two sided* alternative hypothesis $H_1^1: \psi \neq 0.0$. It can be seen that the calculated value of $|t|$ (=3.558641) is *larger* than the tabulated value of t (=3.182) at 5% level of significance. Hence we conclude that the hypothesis H_0^1 is not acceptable. Therefore the alternative hypothesis H_1^1 is true. In other words, in this case $\psi \neq 0.0$.

We would now proceed to test this hypothesis $H_0^2: \log_e \xi = 0.0$ against the *one sided* alternative hypothesis $H_1^2: \log_e \xi < 0.0$. It can be seen that the calculated value of $|t|$ (=80.5744) is *very much larger* than the tabulated value of t (=2.353) at 5% level of significance. Hence we conclude that the hypothesis H_0^2 is not acceptable. Therefore the alternative hypothesis H_1^2 is true. In other words, in this case $\log_e \xi < 0.0$. We therefore are at least 95% confident that $\log_e \xi$ = -1.61487 or ξ = 0.19892, and ψ = 0.021504.

Table 17. Analysis of Variance of the Log linear Fit: Instantaneous Failure Rates

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	Calculated Value of F	Tabulated $F_{0.05, (1,3)}$
Regression	1	0.004624	0.004624	12.66393	10.128
Error	3	0.001095	0.000365		
Total	4	0.00572			

The table above shows that the effect due to regression on the total variations is significant because the calculated value of the F –statistic is higher than the tabulated value of $F_{0.05, (1, 18)}$. We conclude that the log linear equation fits the data very well. Indeed, it can be seen that the coefficient of determination is 80.84. In other words, 80.84% of the variations are due to this mathematical relationship, while the rest 19.16% is due to randomness. Hence, it can be statistically concluded that the data on instantaneous failure rates follow the exponential law. We therefore conclude that the equation

$$\varphi = 0.19892\ e^{0.021504\ n}$$

is statistically valid with coefficient of determination 80.84%. In table 18 below, we are showing the estimated values of the instantaneous failure rates for different number of turns.

Table 18. Hypothesized Values of Instantaneous Failure Rates

Number of Turns	Calculated Instantaneous Failure Rates	Hypothesized Instantaneous Failure Rates
1	0.19932	0.20324
2	0.21201	0.20766
3	0.21285	0.21217
4	0.21881	0.21678
5	0.21847	0.22149

We have thus found that instantaneous failure rate increases exponentially with respect to number of turns in the evaporator section. When plotted, the curve looks almost like a straight line. However, we have found that an exponential fit does have a very high coefficient of determination, and therefore we can now say that the positive growth is indeed exponential.

5. Conclusions

From the analysis done so far, we now make the following conclusions:

(A) Our first hypothesis was that for multiple turns, temperature T decreases exponentially in time τ following $T = C + \alpha e^{\beta\tau}$, where α and β are parameters to be estimated, $\alpha > 0$ and $\beta < 0$, and C is the controlled temperature beyond which there would be no more cooling possible.

It is evident that in the log linear fit of the data; in every case the values of β were found to be very significantly different from zero and that in every case the values of $log_e \alpha$ were found to be very significantly larger than zero.

It is evident that for every log linear fit, the effect due to regression was very significant. Indeed, in every case, the coefficients of determination were found to be extremely high. We conclude that temperature does decrease exponentially in time for multiple turns.

(B) It has been established that the instantaneous failure rate $\varphi = -\beta$ increases when the number of turns in the evaporator section increases. Our second hypothesis was that this increment is exponential in nature.

It is evident that the estimates of the parameters ξ and ψ in the equation $\varphi = \xi e^{\psi n}$, $\xi \geq 0$, $\psi \geq 0$, are statistically significant. Also, we could see that the concerned log linear regression equation is statistically valid. Hence, we conclude that the instantaneous failure rate φ is dependent on the number of turns n in the evaporator section, and that φ increases exponentially as n increases. Hence, increasing the number of turns in the evaporator section would have an adverse effect on cooling. The rate of change of temperature is directly proportional to β. Accordingly, the dependence of the rate of change of temperature on the number of turns is also evident.

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