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Analysis of Unsteady Blood Flow through Stenosed Artery with Slip Effects

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Abstract

Present mathematical model represents the unsteady blood flow through constricted artery in the presence of velocity slip. Here the rheology of blood is characterized by Bingham plastic fluid model. An appropriate perturbation scheme has been adopted to solve the equations governing the fluid flow when the womerseley frequency parameter is small. Important flow parameters such as velocity, shear stress and flow rate have been computed. Graphical representation shows that the axial velocities, wall shear stress, flow rate decreases when the time increases along the axial distance.

***Keywords**: Unsteady flow, Bingham plastic fluid, Velocity Slip, stenosis*

1. Introduction

1.1. Atherosclerosis

Atherosclerosis occurs when the nature of blood flow changes from its usual state to a distributed flow condition due to the presence of stenosis in an artery. To understand the flow pattern in stenosed arteries, several authors have analyzed the flow of blood through an arterial stenosis [5, 6, 13]. The altered hemodynamic may further influence the development of the disease and arterial deformity and change the regional blood rheology [14]. Dechant [11] have presented a perturbation model for the oscillatory flow of a Bingham plastic in rigid and periodically displaced tubes.

Initial understanding of blood flow dynamics was done by considering blood as a Newtonian fluid. But theoretical and experimental investigations indicated that blood cannot be treated as a single phase homogenous viscous fluid while flowing through small arteries. A model has been presented to study the axially symmetric, laminar, steady, one-dimensional flow of blood through narrow stenotic vessel by considering blood as Bingham plastic fluid and shown that resistance to flow and wall shear stress increase with the size of stenosis but these increase are, however, smaller due to the non-Newtonian nature of blood [20]. The problem of non-Newtonian and non-linear blood flow through stenosed artery has been presented by [15]. Finite difference scheme has been used to analyze and to solve the non-linear Navier-Stoke's equation. Singh [2,3] formulated a mathematical model to study the effects of shape parameter and stenosis length on the resistance to flow and wall shear stress under stenotic conditions by considering, laminar, steady, one dimensional, non-Newtonian and fully developed flow of blood through axially symmetric but radially non-symmetric stenosed artery.

1.2. Slip Effect on Blood Flow

Experimental results on blood flow clearly indicate the existence of slip in velocity at the tube wall. Misra and Shit [8], Ponalgusamy [17] have developed mathematical models for blood flow through stenosed arterial segments, by taking a velocity slip condition at the constricted wall. Tanwar and Varshney [21] have investigated an effect of pulsatile

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flow of Hershel-Bulkley fluid with stenoses artery with body acceleration subject to a slip velocity condition at the constricted wall. Ponalgusamy and Selvi [18] have presented a Mathematical model for blood flow through stenosed arteries with axially variable peripheral layer thickness and variable slip at the wall.

Venkateswarlu and Rao [10] have studied the unsteady blood flow through an indented tube with atherosclerosis in the presence of mild stenosis, numerically by using finite difference method. Mandal *et al.* [16] have considered a model for showing the effect of body acceleration on unsteady pulsatile flow of non-Newtonian fluid through a stenosed artery. Srikanth *et al.* [7] have studied the effects of ω-shaped stenosis on the physiological parameters of the blood flow that is modeled as couple stress fluid through a catheterized tapered artery and also examined the effects of velocity slip at the constricted wall.

Gaur and Gupta [12] have discussed a Casson fluid model for steady flow with slip effect through a stenosed porous blood vessel in which authors explained that the axial velocity, volumetric flow rate and pressure gradient increase with the increase in slip velocity and decrease with growth in yield stress. Bhatnagar and Srivastava [1] have developed a mathematical model for the analysis of blood flow through a multiple stenosed artery in the presence of slip velocity. Maruthi Prasad *et al.* [9] have worked on the Mathematical model for the steady flow of Herschel-Bulkely fluid through a tube having overlapping stenosis and obtained the solutions for mild stenosis and they discussed that the resistance to the flow increases with heights of the stenoses, yield stress and power law index but decreases with stress ratio parameter. Mallik *et al.*, [4] have studied a non-Newtonian fluid model for blood flow using power law through an atherosclerotic arterial segment having slip velocity. Kumar and Diwakar [19] obtained a mathematical model of power law fluid with an application of blood flow through an artery with stenosis.

In the present analysis we have considered the unsteady blood flow through constricted artery in the presence of slip velocity. Slip velocity is an important factor in blood flow modeling since it enhances the flow velocity. We have seen the variation of time on various flow parameters.

2. Formulation of Problem

Let us consider unsteady flow of non-Newtonian incompressible blood, through a circular tube having an axially symmetric stenosis. Blood is assumed to be behaves like Bingham plastic fluid. The geometry of stenosed artery is shown below and described by equation (2.1).

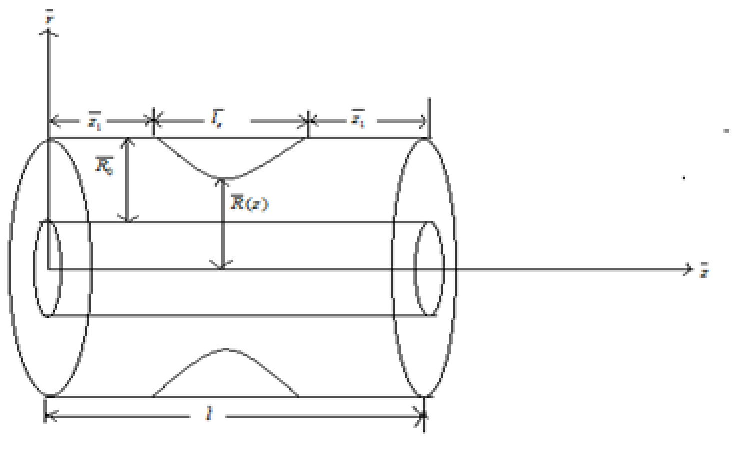

Figure 1. Geometry of Stenosed Artery

$$\bar{R}(\bar{z}) = \begin{cases} \bar{R_0} - \dfrac{\bar{\delta}}{2}\left[1+\cos\dfrac{2\pi}{\bar{l_s}}(\bar{d}+\bar{l_s}-\bar{z})\right]; & \bar{d} \le \bar{z} \le \bar{d}+\bar{l_s} \\ \bar{R_0} ; & otherwise \end{cases} \tag{2.1}$$

Where $\bar{\delta}$ is the maximum height of the stenosis, $\bar{l_s}$ is the length of the stenosis and $\bar{d}$ is the location of the stenosis, $\bar{L}$ represents the vessel length. $\bar{R}(\bar{z})$ and $\bar{R_0}$ is the radius of the artery with and without stenosis respectively.

Considering the above assumptions, equation of motion governing the fluid flow can be written as;

$$\bar{\rho}\frac{\partial \bar{v_B}}{\partial \bar{t}} = -\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\bar{\tau_B}\right) \tag{2.2}$$

$$\frac{\partial \bar{p}}{\partial \bar{r}} = 0 \tag{2.3}$$

Where $\bar{\rho}$ is the density of the blood, $\bar{p}$ is the pressure at any point at time $\bar{t}$ and $\bar{\tau_B}$ is the shear stress.

The constitutive equations for Bingham plastic fluid are,

$$\bar{\tau_B} = \bar{\tau_y} - \bar{\mu}\left(\frac{\partial \bar{v_B}}{\partial \bar{r}}\right); \qquad if \ \ \bar{\tau_B} \ge \bar{\tau_y} \tag{2.4}$$

$$\frac{\partial \bar{v_B}}{\partial \bar{r}} = 0; \qquad if \ \ \bar{\tau_B} \le \bar{\tau_y} \tag{2.5}$$

where $\bar{\tau_y}$ denotes yield stress, $\bar{v_B}$ is the axial velocity of the blood and $\bar{\mu}$ denotes the viscosity of the blood.

Boundary conditions are:

$$\left.\begin{array}{lll} \bar{v_B} = \bar{v_s} & at & \bar{r} = \bar{R}(\bar{z}) \\ \bar{\tau_B} \ is \ finite & at & \bar{r} = 0 \end{array}\right\} \tag{2.6}$$

Where $\bar{v_s}$ is the slip velocity in the axial direction,

As the pressure gradient is the function of $\bar{z}$ and $\bar{t}$, we take

$$\frac{\partial \bar{p}}{\partial \bar{z}}\left(\bar{z},\bar{t}\right) = A_0(\bar{z}) + A_1(\bar{z})\cos\left(\bar{\omega}\bar{t}\right), \quad t \geq 0, \tag{2.7}$$

where A_0 is the steady state pressure gradient, A_1 is the amplitude of the fluctuating component, $\bar{\omega} = 2\pi \bar{f}$, where $\bar{f}$ is the pulse rate frequency.

By using the following Non dimentional quantities

$$\left.\begin{aligned} & v_B = \frac{\bar{v}_B}{A_0 \bar{R}_0^2 / 4\bar{\mu}},\ z = \frac{\bar{d} + \bar{l}_s - \bar{z}}{\bar{d}},\ R(\mathrm{z}) = \bar{R}(\bar{\mathrm{z}}) / \bar{R}_0,\ r = \bar{r} / \bar{R}_0,\ t = \bar{t}\bar{\omega}, \\ & H = \bar{\delta} / \bar{R}_0,\ v_s = \frac{\bar{v}_s}{A_0 \bar{R}_0^2 / 4\bar{\mu}},\ \tau_B = \frac{\bar{\tau}_B}{A_0 R_0 / 2},\ \alpha^2 = \frac{\bar{R}_0^2 \bar{\omega}\bar{\rho}}{\bar{\mu}}, \\ & e = A_1 / A_0,\ \theta = \frac{\bar{\tau}_y}{A_0 R_0 / 2} \end{aligned}\right\} \tag{2.8}$$

Where α is called Womersley frequency parameter also known as pulsatile Reynold number and e represents the amplitude of the flow.

The geometry of stenosis in the non-dimensional form is given by

$$\bar{R}(\bar{z}) = \begin{cases} 1 - H\cos^2 \pi z, & 0 \leq z \leq 1 \\ 1 \quad , & otherwise \end{cases} \tag{2.9}$$

The equation of motion (2.2) in the non-dimensional form is written as

$$\alpha^2 \frac{\partial v_B}{\partial t} = -2\phi + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_B) \tag{2.10}$$

Where $\phi = \phi(t) = 1 + e\cos t$

The non-dimensional constitutive equation for the Bingham plastic fluid is

$$\tau_B = \theta - \frac{1}{2}\frac{\partial v_B}{\partial r} \qquad \tau \geq \theta \tag{2.11}$$

$$\frac{\partial v_B}{\partial r} = 0 \qquad \tau_B < \theta \tag{2.12}$$

The dimensionless boundary conditions are

$$\left.\begin{aligned} & v_B = v_s && at \quad r = R(z) \\ & \tau_B \ is\ finite && at \quad r = 0 \end{aligned}\right\} \tag{2.13}$$

3. Method of Solution

We have used perturbation method for getting the required solution of the problem. In the present method we have taken $\alpha^2 << 1$ to maintain the non-Newtonian nature of blood in which a plug flow region is developed through the stenosed arteries in small blood vessels.

Let the velocity u and shear stress τ can be expressed in the following form

$$v_B = v_{B0} + \alpha^2 v_{B1} + \alpha^4 v_{B2} + ... \quad (3.1)$$

$$v_p = v_{p0} + \alpha^2 v_{p1} + \alpha^4 v_{p2} + ... \quad (3.2)$$

$$\tau_B = \tau_{B0} + \alpha^2 \tau_{B1} + \alpha^4 \tau_{B2} + ... \quad (3.3)$$

$$r_p = r_{p0} + \alpha^2 r_{p1} + \alpha^4 r_{p2} + ... \quad (3.4)$$

Where $r_p = \dfrac{\overline{r_p}}{\overline{R_0}}$ is the non-dimensional plug core radius.
Using equations (3.1) and (3.3) in equation (2.10), and comparing the constant terms and coefficient of α^2, we have

$$\frac{\partial v_{B0}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{B1}) \quad (3.5)$$

$$\frac{\partial}{\partial r}(r\tau_{B0}) = 2r\phi \quad (3.6)$$

Substituting equation (3.1) and (3.3) in equation (2.11), we have

$$-\frac{\partial v_{B0}}{\partial r} = 2\left[\tau_{B0} - \theta\right] \quad (3.7)$$

$$-\frac{\partial v_{B1}}{\partial r} = 2\tau_{B1} \quad (3.8)$$

Using equation (3.1), the boundary conditions (2.13) reduce to

$$v_{B0} = v_s \;\; and \;\; v_{B1} = 0 \;\; at \;\; r = R(z) \quad (3.9)$$

Solving equations (3.5) to (3.8) and using boundary equation (3.9) we have

$$\tau_{B0} = r\phi \quad (3.10)$$

$$v_{B0} = v_s - 2\theta(R - r) + \phi(R^2 - r^2) \quad (3.11)$$

$$v_{p0} = v_s - 2\theta(R - r_{p0}) + \phi(R^2 - {r_{p0}}^2) \quad (3.12)$$

$$\tau_{B1} = \frac{\theta'}{4}\left[2R^2 r - r^3\right] \tag{3.13}$$

$$v_{B1} = \frac{\theta'}{8}\left[r^4 - 4R^2 r^2\right] + \frac{3}{8}\theta' R^4 \tag{3.14}$$

$$v_{p1} = \frac{\theta'}{8}\left[{r_{p1}}^4 - 4R^2 {r_{p1}}^2\right] + \frac{3}{8}\theta' R^4 \tag{3.15}$$

Thus the axial velocity distribution for $r_p \le r \le R(z)$ is

$$v_B = v_s - 2\theta(R - r) + \phi(R^2 - r^2) + \alpha^2 \frac{\theta'}{8}\left[r^4 - 4R^2 r^2\right] + \frac{3}{8}\theta' R^4 \tag{3.16}$$

The plug flow velocity distribution for the region $0 \le r \le r_p$

$$v_p = v_s - 2\theta(R - r_{p0}) + \phi(R^2 - {r_{p0}}^2) + \alpha^2 \frac{\theta'}{8}\left[{r_{p1}}^4 - 4R^2 {r_{p1}}^2\right] + \frac{3}{8}\theta' R^4 \tag{3.17}$$

The shear stress τ_B is given as

$$\tau_B = r\phi + \alpha^2 \frac{\theta'}{8}\left[2R^2 r - r^3\right] \tag{3.18}$$

The wall shear stress τ_w is given as

$$\tau_w = R\phi + \alpha^2 \frac{\theta'}{4} R^3 \tag{3.19}$$

The non-dimensional volumetric flow rate for the region $0 \le r \le R(z)$ is defined as

Where $Q(z,t) = \dfrac{\overline{Q}(\bar{z},\bar{t})}{\pi \overline{A_0}\,\overline{R_0^4}/8\overline{\mu}}$; where $\overline{Q}(\bar{z},\bar{t})$ being the dimensional volumetric flow rate.

$$Q = 2v_s R^2 - \frac{4}{3}\theta R^3 + \phi R^4 + \frac{\phi'}{3}\alpha^2 R^6 \tag{3.20}$$

4. Results and Discussion

The expressions for various flow parameters have been found by solving the Navier-stokes equations using perturbation technique. MATLAB has been used as a tool for getting solutions of axial velocity, plug flow velocity, volumetric flow rate, wall shear stress for unsteady flow of blood through stenosed artery. Results are shown and discussed through graph for different values of stenosis height, yield stress and slip velocity.

Figure 2 represents the velocity profile for axial velocity. It shows the variation of axial velocity along the along the axial distance z for different values of time t, stenosis height H, yield stress θ and slip velocity v_s taking fixed values $e = 0.1$ and $\alpha = 0.1$. Graph shows that axial velocity of the fluid increases with the slip velocity as well as with the pulse. It is found that axial velocity decreases with the increse in time, yield stress and stenosis height along the axial distance.

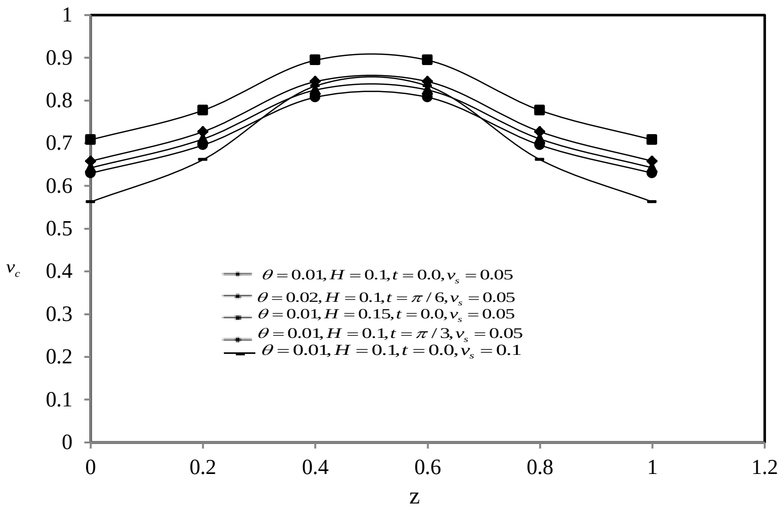

Figure 2. Variation of Axial Velocity against Axial Distance for Different Values of Time, Stenosis Height, Yield Stress and Slip Velocity

Figure 3 gives the variation of axial velocity with the radial distance r for some fixed values of pressure gradient e = 0.1 and womersley number α & for different values of stenosis height, time and slip velocity. It is found that axial velocity increses with the increase in stenosis height and slip velocity and decreases with the increase in time.

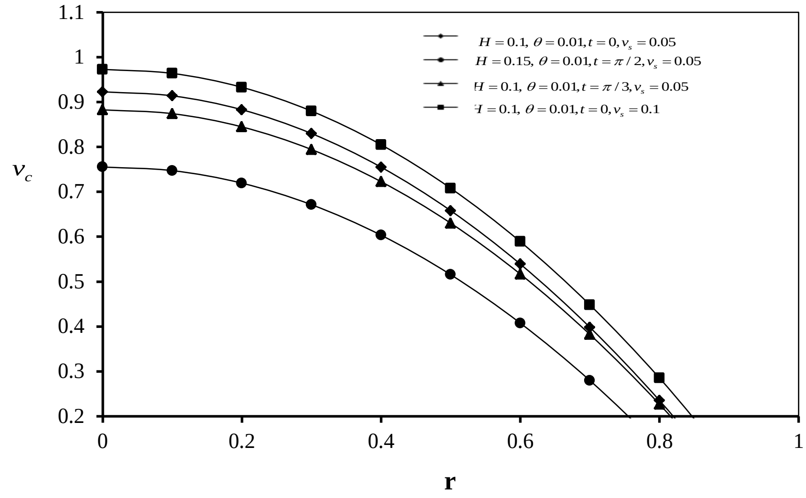

Figure 3. Variation of Axial Velocity Against Radial Distance For Different Values Of Time, Yield Stress, Slip Velocity

Figure 4 exhibit how wall shear stress varies with the axial distance z for different times and for some fixed values of pressure gradient $e = 0.1$ and yield stress $\theta = 0.01$ and $\alpha = 0.1$. Figure describes that the wall shear stress shows a wave like variation along the axial distance. Wall shear stress decreases with the increase in time.

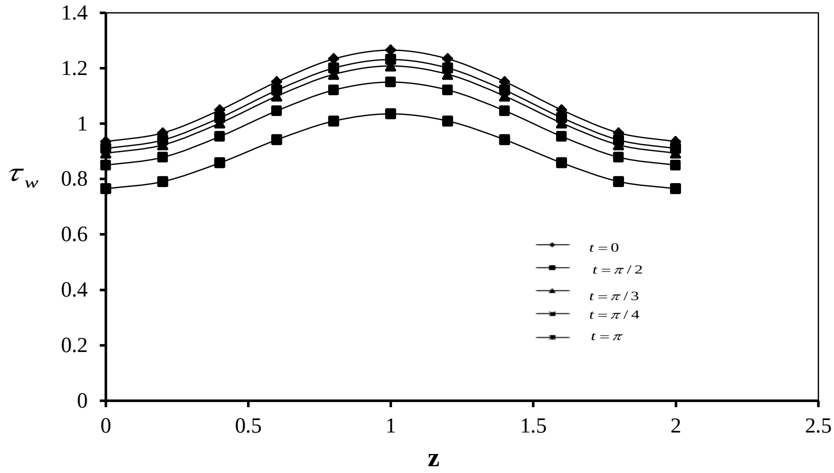

Figure 4. Variation of Wall Shear Stress Against Axial Distance for Different Values of Time

Figure 5 shows the variation of wall shear stress with time for different values of stenosis height and pressure gradient. It depicts that wall shear stress decreases with the increase in stenosis height. Wall shear stress also varies for different values of pressure gradient e.

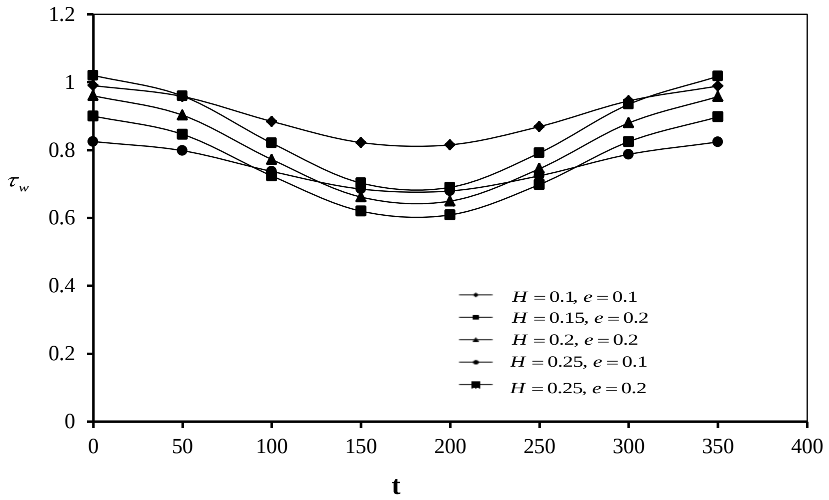

Figure 5. Variation of Wall Shear Stress with Time for Different Values of Stenosis Height and Pressure Gradient

Figure 6 shows the variation of volumetric flow rate along the axial distance z for different values of time t, stenosis height H, yield stress θ and slip velocity v_s. It is found that along the axial distance, the volumetric flow rate varies. The flow rate decreases with the increase in time; yield stress or stenosis height but it increases when the slip velocity increases.

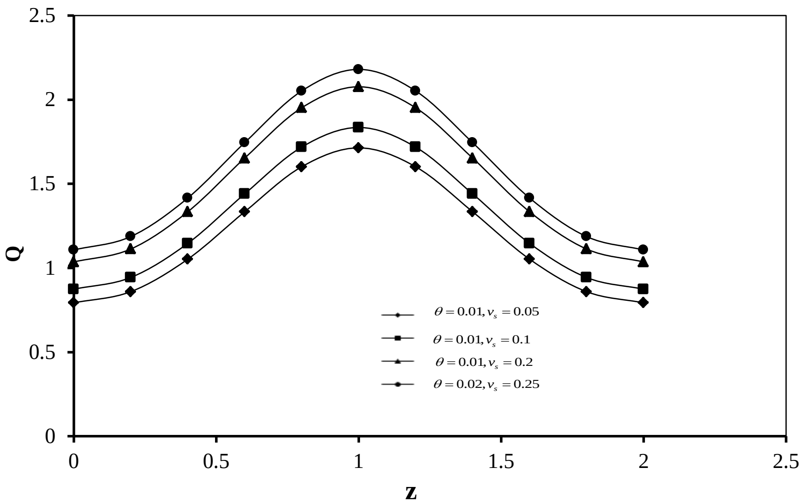

Figure 6. Variation of Volumetric Flow Rate Against the Axial Distance for the Different Values of Shear Stress and Slip

Figure 7 shows the variations of volumetric flow rate along the stenosis height for different values of time with fixed values of $e = 0.1, \alpha = 0.1$. It is observed from the graph that volumetric flow rate decreases with the increase in stenosis height and found that flow rate decreases with the increase in time.

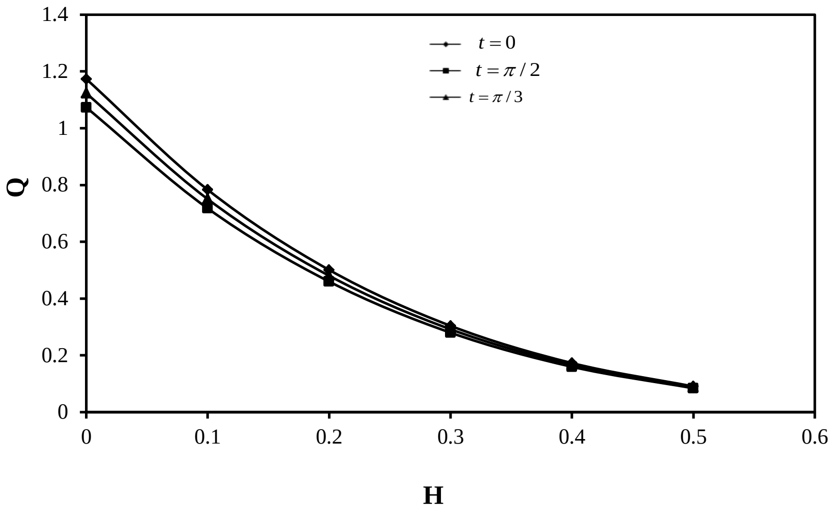

Figure 7. Variation of Volumetric Flow Rate against the Stenosis Height for Different Values of Time

5. Conclusion

Present study brings out many interesting results on rheological properties of blood flow through stenosed artery considering blood as Bingham plastic fluid model. Since high blood viscosity is very dangerous for the cardiovascular disorders. Slip velocity at the stenotic wall may be used as the major tool in reducing the blood viscosity. It is also found that the effect of stenosis reduce the flow rate. It is also noticed that the flow rate decreases with the increase in time, yield stress or stenosis height but it increases when the slip velocity increases.

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