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A Fuzzy Based Approach for Denoising of ECG Signal using Wavelet Transform

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Abstract

Electrocardiogram (ECG) is a vital biomedical signal for diagnosing heart diseases, but now it has many other applications like stress recognition, biometric recognition etc. but ECG signal gets noisy from various sources like as muscle noise, electrode artifacts, baseline drift noise and respiration. As wavelet transform shows a good performance in de-noising the ECG signal, however the selection of appropriate mother wavelet functions and number of wavelet decomposition levels is still an issue to remove the various kinds of noises from the input signal. It is essential to denoise the ECG signal to get appropriate features of ECG signal.This research work analyze and compare the removal of noise and distortion in ECG signal using five wavelets(Daubechies, Coiflet, Haar, Biorthogonal and Symmlet) with four thresholding rules(SURE, Hybrid,Universal and Minimax) and various decomposition levels using Fuzzy Inference System .The parameters used for performance analysis are Signal to Noise Ratio,Mean Square Error(MSE) and variance.

Keywords: *ECG, DWT, MSE, PSD, SNR, SURE, UWT.*

1. Introduction

Cardiac pathology ECG has been widely used to detect heart disease. ECG is the measurement of the electrical activity of cardiac muscle. In the time domain the ECG signal is categorized by different waves viz., P, Q, R, S, T and U as shown in fig. 1.

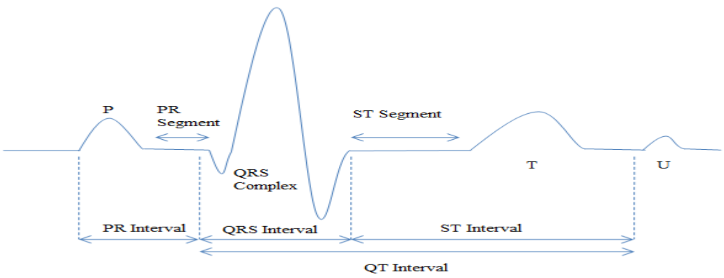

Figure 1. ECG Waveform

The P wave represents atrial depolarization. The Q, R & S waves together make up QRS complex, which represents ventricular depolarization whereas T wave characterizes the ventricular repolarisation. The interval between S wave and the beginning of the T wave is called the ST segment. In some ECG an extra wave can be seen at the end of the T wave, called as U wave [1]. In frequency domain ECG signal varies from 0.05 Hz to 100Hz whereas the associated amplitude values vary from 0.02 mV to 5 mV. The amplitude values of the human body bioelectrical signals are measured in micro volts (μV) [2]. ECG analysis and processing is used to extract the characteristic features of the ECG signal. This signal gets corrupted by different kinds of artifacts such as muscle

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noise, electrode artifacts, baseline drift noise and respiration. These artifacts affect's the ST segment, reduces the signal quality and frequency resolution that produces large amplitude signals in ECG which resembles PQRST waveforms and hides very small features that may be vital for diagnosis and clinical monitoring. The goal of ECG signal enhancement is to separate the valid signal components from the noise, so as to create an ECG signal that facilitates an easy and accurate interpretation[3]. There are many methods for denoising of ECG signals like digital filters, adaptive filters, Empirical Mode Decomposition (EMD) and Wavelets. Digital filters and adaptive filters can be applied to the signal whose statistical characteristics are stationary in many cases. ECG signal is one of the signal that is considered as a non-stationary signal which needs more time and effort for denoising [4]. An efficient method for processing such a non-stationary signal is the wavelet transform. It supersedes the problem of nonlinear phase and time delay caused by filters. Furthermore, it decomposes the signal into a frequency time scale which makes it appropriate to be used for non-stationary signals, like ECG[5]. An effective de-noising technique should minimize the noise content in the signal and also ensure that the important features in the signal are not lost or altered. The basic aim is to improve the Signal to Noise Ratio (SNR) ratio of the signal,minimize the (Mean Square Error) MSE using various practices [6].

Wavelet Transform is a signal processing technique which uses a fully scalable modulated window to solve the signal-cutting problem by shifting the window along with the signal and for every position of the spectrum [7]. By virtue of its multi-resolution representation capability, the wavelet transform has been used effectively in vital applications such as transient signal analysis [8], numerical analysis [9], computer vision [10], and image compression [11], along with many other audiovisual applications. A wavelet is basically a small wave which has energy concentrated in time as shown in Figure 2.

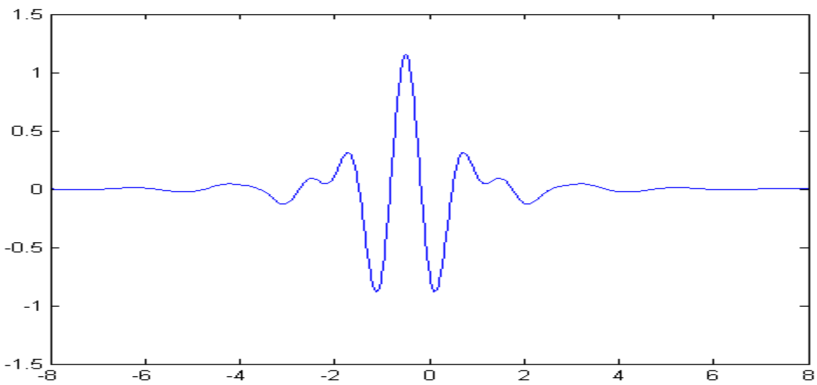

Figure 2. Wavelet Function

Scaled and shifted copies of the main pattern, so-called the "mother wavelet", are known as wavelets. The mother wavelet function Ψ{s,T(t)} is defined by equation (1), where T is translation parameter and, s as scale parameter.

$$\Psi j,k(t)=1\frac{1}{\sqrt{sj}}\Sigma f(n)\ \Psi(s-jn-kt)\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Mother wavelet Ψ(t) is a function of zero.

$$\int \Psi\,(t)dt = 0\ldots\ldots\ldots\ldots(2)$$

Continuous Wavelet Transform of the signal f (t) is calculated as

$$W(s,T)=1/\sqrt{s}\int f(t)\ \Psi(t\text{-}T/s)dt \qquad (3)$$

After denoising, the original signal is then reconstructed using inverse discrete wavelet transform which is defined as:

$$f(t)=\iint W(s,t)\Psi s,T\ (t)dTds \qquad (4)$$

Discrete version of continuous wavelet transform is known as discrete wavelet transform and is obtained by using discrete steps j of scale factor s,and discrete steps k of translation factor T.Discrete mother wavelet is defined as

$$\Psi j,k(t)=1\frac{1}{\sqrt{sj}}\Sigma f(n)\ \Psi(s\text{-}jn\text{-}kt) \qquad (5)$$

For removal of noise, the coefficients obtained from the wavelet decompositions are further processed using threshold selection techniques. The selection of suitable thresholding rule is a significant point which must be considered whenever any wavelet function is being used. Wavelet thresholding is the signal estimation practice to meet the capabilities of signal denoising. Standard wavelet thresholding rules, consists of hard thresholding and soft thresholding functions [12]. The signal if the threshold value is too large or too small cannot be estimated precisely.

$$Wht=\begin{cases} W, & W \leq t \\ 0, & W > 0 \end{cases} \qquad (6)$$

$$Wst=\begin{cases} [sign(w)(|w|-t)], & |W| \geq t \\ 0, & W > 0 \end{cases} \qquad (7)$$

where W is wavelet coefficient,t is value of threshold which is applied on wavelet coefficients. In this research work thresholding rules used are SURE, hybrid, universal and minimax.Sure thresholding is used in the soft threshold estimator,to estimate the risk for a particular threshold value and gives a decision accordingly [13].

$$Sure(x,T)=\sum_{m=0}^{n-1}(x_b[m]), \qquad (8)$$

With

$$C(u)=\begin{cases} \mu^2-\sigma^2, & u \leq T \\ \sigma^2-T^2, & u > T \end{cases} \qquad (9)$$

$$T = agr_t \min\ Sure(X,T) \qquad (10)$$

Minimax Scheduling uses fixed threshold and it gives performance for Mean Square Error (MSE) against an ideal events. The extreme value estimator can apprehend minimized of maximum mean square error for a given function.

$$Wtm=\begin{cases} o.3936+0.1829*(\log n)/(\log 2) & |n| > 32 \\ 0 & |n| \leq 0 \end{cases} \qquad (11)$$

In this method, the threshold value will be selected by obtaining a minimum error between wavelet coefficient of noise signal and original signal. Universal thresholdingcan be used as an alternative to the useof minimax threshold. It is bigger in magnitude than the minimaxthreshold[14] . The value of the threshold is calculated as:

$$T=sqrt(2*log(length(X)) \qquad (12)$$

Where, T is the threshold value and X is noisy signal. The Hybrid thresholding is a combination of SURE and global thresholding method. If the signal-to noise ratio of the signal is very small, then the SURE method approximation will have more amount of noise. In this kind of situation, the fixed form threshold is selected by means of global thresholding rule is used.

Wavelets used in this research paper are Daubechies, Haar, Coiflet, Symmlet and Biorthogonal. Here Daubechies Wavelet is a family of orthogonal wavelets defining a discrete wavelet transformand categorized by a maximal number of vanishing moments for some given support. With each wavelet type of this class, there is a scaling function (called the father wavelet) which generates an orthogonal multiresolution analysis [15].

$$\phi(x)=\sqrt{2}\sum_{k=0}^{L-1} h_k\,\phi(2x-k)\ldots\ldots\ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots(13)$$

where $\phi(x)$ is normalized

$\int_{-\infty}^{\infty}\phi(x)dx = o$ and wavelet $\Psi(x)$is defined in terms of scaling function

$$\Psi(x) = \sqrt{2}\sum_{k=0}^{L-1} g_k\phi(2k\text{-}x)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

Where $\phi(x)$ is the scaling function.,$\Psi(x)$ is the wavelet.In this wavelet, the scaling signals and wavelets have slightly longer supports, *i.e.*, they produce averages and differences using just a few more values from the signal. This slight change, however, provides a marvelous improvement in the proficiencies of these new transforms. The Haar Waveletis a sequence of rescaled "square-shaped" functions which combineform a wavelet family[16]. The technical disadvantage of the Haar wavelet is that it is not continuous, and therefore not differentiable. This property can, however, be an advantage for the analysis of signals with sudden transitions, such as monitoring of tool failure in machines[17].For every pair n, k of integers in Z, the Haar function $\psi n, k$ is defined on the real line R by the formula:

$$\Psi_{n,k}(t) = 2^{\frac{n}{2}}\Psi(2^n t - k) \qquad t \in R\ldots\ldots \quad \ldots.(15)$$

The Haar wavelet's mother wavelet function$\Psi(t)$ can be described as:

$$\Psi(t) = \begin{cases} 1 & 0 \le t < 1 \\ -1 & 0.5 \le t < 1\ldots\ldots\ldots\ldots \quad \ldots\ldots.......(16) \\ 0 & \text{otherwise} \end{cases}$$

Its scaling function $\phi(t)$ can be described as:

$$\phi(t)=\begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots.(17)$$

In Coiflet Wavelets both the scaling function (low-pass filter) and the wavelet function (High-Pass Filter) must be normalized by a factor $1/\sqrt{2}$. Its scaling function $\phi(t)$ can be described as$\phi(t)=2\sum_{k\epsilon Z} h_k\ \phi(2\text{-}k)$. Biorthogonal Wavelets are families of compactly supported symmetric wavelets. The symmetry of the filter coefficients is often desirable since it results in linear phase of the transfer function[18][19].

$$\Psi_{j,k}(x) = 2^{-\frac{j}{2}}\Psi\left(2^{-j}x^{-k}\right) \quad j,k \in Z\ldots\ldots\ldots\ldots\ldots \quad \ldots(18)$$

The amended version of Daubechies wavelet with increased symmetry and have similar properties are known as symmlet wavelets. There are 7 different Symlet functions from sym2 to sym8. To achieve optimum performance in the preprocessing stage of the

ECG signal, it is very crucial to select the best wavelet function for the given ECG and also the related features like threshold selection rule, decomposition level, *etc.*. In the previous researches authors worked on wavelet functions like daubechies, coiflet, symmlet on very few decomposition levels with very less thresholding rules. [19] P. Karthikeyan et. Al.(2012) [20] considered the DWT for denoising using three wavelet functions ("db4", "coif5" and "sym7") and four different Thresholding rule were used to denoise the ECG signals. The experimental result shows the significant reduction of above considered noises and it retains the ECG signal morphology effectively. Results show that the "coif5" wavelet and “rigersure” thresholding rule is optimal for better SNR ratio for the real time ECG signals.JS Sørensen(2011) [21] compares the ability to preserve information and noise reduction of the ECG for five wavelets. Computation times and SNR improvements for different noise coverages were calculated and compared. In a clinical setting where the amount of noise is unknown, IIR filters appears to be preferred for consistent performance. Oscar Hernández (2009) [22] uses Undecimated Wavelet Transform (UWT) with db06 wavelet for ECG noise removal. This research paper aims to select the optimal wavelet function and related features to attain the best noise free signal. In addition to previous research, we further included here, biorthogonal wavelet with hybrid, minimax and universal thresholding rules with more decomposition levels. The results are also compared for UWT and DWT wavelet transforms. Four different performance measures were considered for analysis, *i.e.*, SNR, MSE, variance and periodogram of Power Spectral Density (PSD).

2. System Implementation

In this work, we employed different types of wavelet thresholding rules to remove noises from the ECG signal. Previous researchers have used: "db4", "coif5" and "sym7" wavelet function for genetic algorithm based denoising ECG signal [23]. We extended the work by including more thresholding rules, decomposition levels for latest wavelet functions and using Fuzzy Inference System(FIS) for system analysis. The soft thresholding method is investigated with four different thresholding rules (Sure, Hybrid, Universal and minimax). All the noises are having certain frequency characteristics and ranges, *i.e.*, power line noise (50 Hz or 60 Hz), baseline wander (>1Hz), and high frequency noises (>100). Therefore, the effect of noises in the frequency spectrum of ECG lies in between (0-500) Hz. Figure 3 shows the procedure of wavelet decomposition on the input ECG signals. On each level of wavelet decomposition, the value of threshold has been calculated by applying the threshold selection rules and the wavelet coefficient (soft thresholding). In general, the value of ECG signal frequency above 100 Hz does not have any useful information [6]. In addition, the effect of baseline wandering is usually lies in the frequency range of less than 1 Hz. After applying threshold on each level of the original signal, the effects of noises on the ECG signals were removed and then the signal is reconstructed on each level by using Inverse Discrete Wavelet Transform (IDWT). The process of the analysis is represented in fig 3.

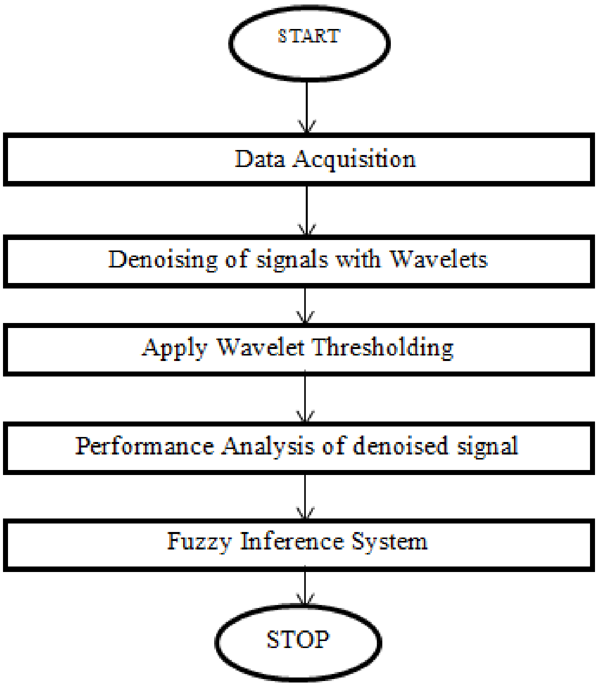

Figure 3. Process of Wavelet Denoising

3. Resul and Simulation

A Fuzzy Inference System (FIS) is designed to analyze the performance of different wavelets for de-noising of ECG signals. In this system three parameters, *i.e.* Signal to Noise Ratio (SNR), Mean Square Error (MSE) and variances are used to evaluate the performance of wavelets. In this research work, five wavelet functions with different levels and four thresholding rules are considered to analyze the performance of denoised ECG signals using the soft thresholding method. DWT based thresholding rules has been tested over ECG datasets and each with duration of (10 min).Output SNR is given by the equation:

$$\text{SNR}=10 \log [\textstyle\sum_{i-1}^{N} x(i)^2 / \sum_{i-1}^{N} (x(i) - \overline{x(i)})]^2$$

Where x(i) is the original signal, $\overline{x(i)}$ is the denoised ECG signal and N is the length of ECG signal. Mean Square Error(MSE) is estimated between de-noised ECG signal and original ECG signal given by the equation.

$$\text{MSE} = 1/\text{N} \textstyle\sum_{i=1}^{N} (x(i) - \overline{x(i)})^2$$

Variance is expected difference between original signal and noisy signal

3.1. Analysis of ECG Noise Removal in Terms of S/I Ratio :

In this research work SNR of wavelets with different decomposition levels has been calculated for UWT & DWT as shown in Figue 4 and 5.

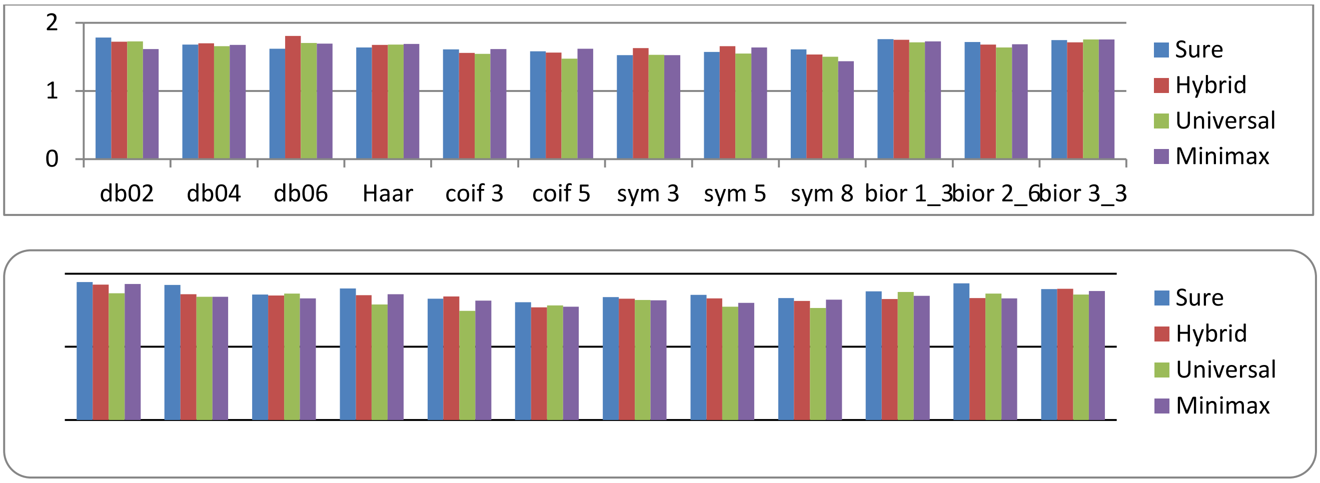

Figure 4. SNR of Different Wavelets with UWT and DWT

While calculating SNR of different wavelets, It is found that db06 wavelet(UWT type) and db02 (DWT type) gives maximum value. Among all the 12 wavelets in UWT type, db02 gives higher SNR value, *i.e.*, 1.781126 where as in DWT type, bior 3_3 gives higher value *i.e.*, 1.8864 for sure thresholding while coiflet 5 gives the worst results. Sure thresholding gives the most appropriate results as compare to other threading rules.

3.2. Analysis of ECG noise removal in terms of Mean Square Error:

Table 1 and 2 shows mean square error of different wavelets with UWT and DWT type. In this research work MSE of wavelets with different decomposition levels has been calculated for UWT & DWT. Here MSE is calculated between denoised ECG signal and noisy ECG signal. From the analysis of mean square error, it is found that it has minimum value i.e .00384for db06 waveletof UWT type and 0.00259 for db02 of DWT typeat minimaxthresholding.Bior 3_3 is giving minimum MSE with UWT type and db02 gives minimum value for the discrete type while symmletwavelet gives the worst results. Sure thresholding gives the most appropriate results as compare to other thresholding rules.

Table 1. Mean Square Error of Different Wavelets (Undecimated Type) According to Different Threshold Rules

Wavelet Type	DAUBECHIES			HAAR	COIFLET		SYMMLET			BIORTHOGONAL		
Thresholding	Db02	Db04	Db06	Haar	Coif 3	Coif 5	Sym 3	Sym 5	Sym 8	Bior 1_3	Bior 2_6	Bior 3_3
SURE	.00430	.00409	.00434	.00347	.00430	.00525	.00544	.00556	.00667	.00421	.00300	.00391
HYBRID	.00457	.00438	.00458	.00474	.00585	.00979	.00499	.00416	.00424	.00376	.00390	.00401
UNIVERSAL	.00509	.00365	.00473	.00425	.00634	.00585	.00615	.00681	.00638	.00413	.00428	.00474
MINIMAX	.00396	.00329	.00259	.00378	.00582	.00523	.00559	.00521	.00546	.00361	.00346	.00381

3.3. Analysis of ECG Noise Removal in Terms of Variance :

Table 1 and 2 shows variance of different wavelets with UWT and DWT type.Variance measures how far each number in the set is from the mean. Variance is calculated by taking the differences between each number in the set and the mean, squaring the differences (to make them positive) and dividing the sum of the squares by the number of values in the set.

Table 3. Variance of Different Wavelets (Undecimated Type) According to Different Threshold Method

Wavelet Type	DAUBECHIES			HAAR	COIFLET		SYMMLET			BIORTHOGONAL		
Thresholding	Db02	Db04	Db06	Haar	Coif 3	Coif 5	Sym 3	Sym 5	Sym 8	Bior 1_3	Bior 2_6	Bior 3_3
SURE	.04799	.04511	.0.0434	.0446	.0452	.04280	.0408	.0412	.04319	.04875	.04774	.049046
HYBRID	.04576	.0438	.0458	.0490	.04078	.04257	.04365	.0440	.0404	.0449	.04437	.04545
UNIVERSAL	.04837	.0438	.04956	.0442	.04222	.03988	.041936	.0437	.0412	.04318	.04677	.04600
MINIMAX	.045917	.04744	.04763	.0432	.041079	.0428	.04753	.0460	.0395	.05388	.04564	.04969

From above analysis it is analyzed that variance is maximum for bio 3_3 minimax *i.e.*, .04969 for UWT type and Haar Sure 4i.e .050177 for DWT type.It is also analyzed that it is maximum with biorthogonal wavelet for UWT type and also for Discrete type. Sure Thresholding gives the most appropriatevalue results as compare to other thresholding rules.

3.4 Analysis of ECG noise removal in terms of PSD :

PSD shows the strength of the deviations(energy) as a function of frequency. In other words, it shows at which frequencies deviations are resilient and at which frequencies variations are fragile. Figure 6 shows PSD of clean signal and noisy signal. The unit of PSD is energy per frequency(width) and you can obtain energy within a specific frequency range by integrating PSD within that frequency range. Computation of PSD is done directly by the method called FFT or computing autocorrelation function and then transforming it. Figure 5 shows PSD of clean ECG signal and Noisy ECG signal. Figure 6 shows PSD of different wavelets (a) Daubechies (b) Haar (c) Coiflet (d) Symmlet (E) Biorthogonal.

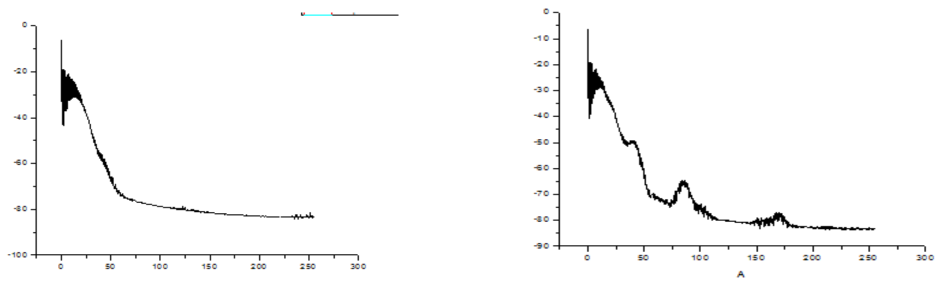

Figure 5. PSD of (a) Clean ECG Signal (b) Noisy ECG Signal

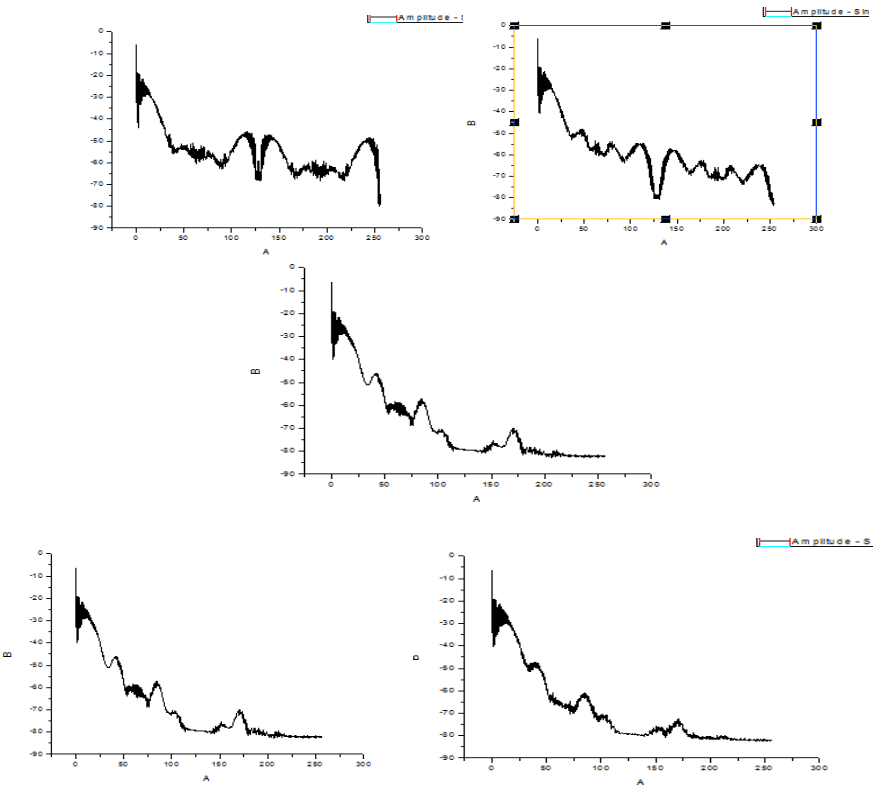

Figure 6. Power Spectral Density of Wavelets for UWT (a) Daubechies (b) Haar (c) Coiflet (d) Symmlet (e) Biorthogonal

3.5. ECG Morphology after Noise Removal

Figure 7 shows morphology of clean and noisy ECG signal.In clean ECG signal P wave is at 0.24 mV,QRS is at 1.125 mV and T is at 0.05 mV. Figure 10 and 11 shows morphology of ECG signal of different wavelets. From the study of ECG morphology, it is concluded that in biorthogonal wavelet of UWT as well as DWT type P wave is at 0.23 mV, QRS is at 1.1 mV and T is at -0.02 mV which is earest to the values of clean ECG signal. Although biorthogonal wavelet gives nearest values in DWT, it also distorts the signal.Haar wavelet in DWT type gives less nearest values as compare to biorthogonal wavelet but signal remain distortionless.

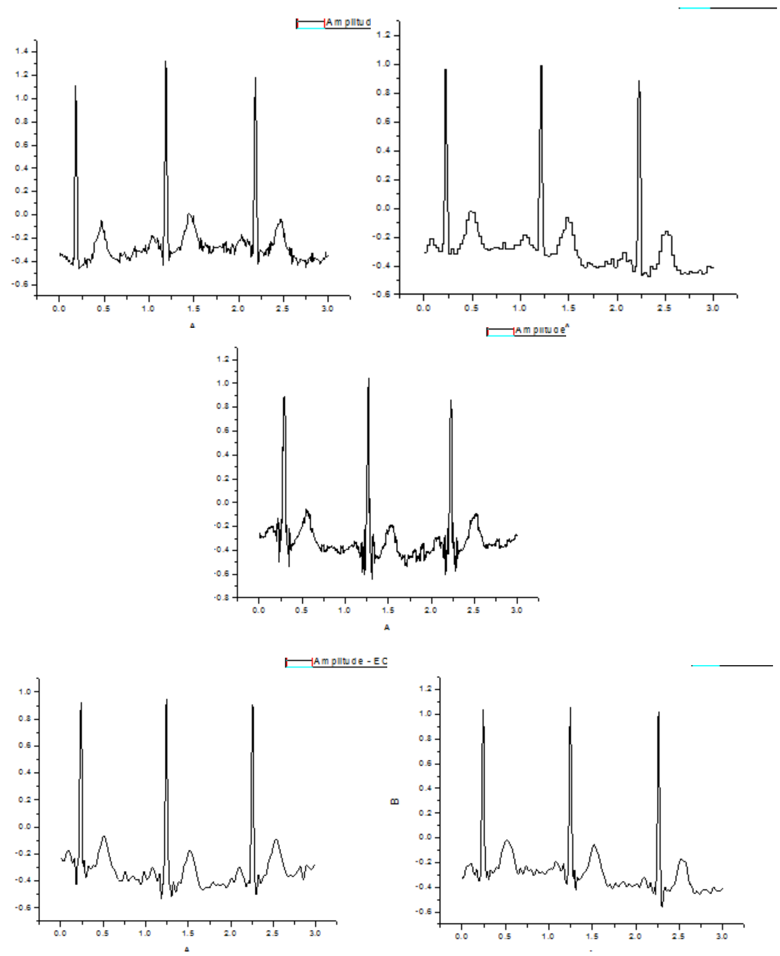

Figure 7. ECG Signal after Noise Removal for Wavelets(DWT) (a) Daubechies (b) Haar (c) Coiflet (d) Symmlet (e) Biorthogonal

In this proposed method, mamdani FIS is employed, the triangular membership function is used for converting crisp sest into fuzzy set, due to their simple formula and computational efficiency. Crisp value is performance parameter of denoised ECG signal. Fuzzification of performance parameter of denoised signal has been carried out as input variables using triangular membership functions. There are three inputs for three different parameters corresponding to which there is one output. The fuzzy sets of the input variables are given in the table below.

Table 4. Fuzzy Sets of Input Variables

Input variables	MSE	VARIANCE	SAR
Linguistic variable	Interval		
Best	[0.0015 , 0.00275]	[0.025 , 0.035]	[1.5 , 1.6]
Average	[0.00275 , 0.0045]	[0.035 , 0.05]	[1.6 , 1.7]
Not Acceptable	[0.0045 , 0.0060]	[0.05 , 0.07]	[1.7 , 2.0]

For each performance parameter, there are three membership functions (triangular) the fuzzy sets of input variable are given in table 4 and input membership functions. Output variable has also three membership functions (triangular). The fuzzy sets of output variables are given in Table 5 and its membership function.

Table 5. Fuzzy Set of Output Variable

Linguistic variable	Output
Best	[0.66 1]
Average	[0.33 0.66]
Not acceptable	[0.0 0.33]

The linguistic variable in this research work is based on the interval that refers to performance is given by experts as shown in the table. The rules determine the input and output membership functions that will be used in inference process. These rules are in linguistic form called "if-then" fuzzy rules. As there are three performance parameters and three intervals so there are 27 rules suggested by experts for performance evaluation.

In these rules output is determined by following rules which are given by experts.

1. If three/two parameters are best and one is average then the output is best.
2. If any of the two parameter are best and one is non acceptable then result is average. If one of the parameter is best and other two are average then the result is average. If all the parameters are average then the result is average.
3. In other cases which don't line in above 2 points then result is non-acceptable.

These rules correspond to the fuzzy descision on performance value of denoised system. In this paper the method mamdani fuzzy inference is employed and it operates M inference mechanism. The output of this Fuzzy Inference System is depicted in Figure 8.

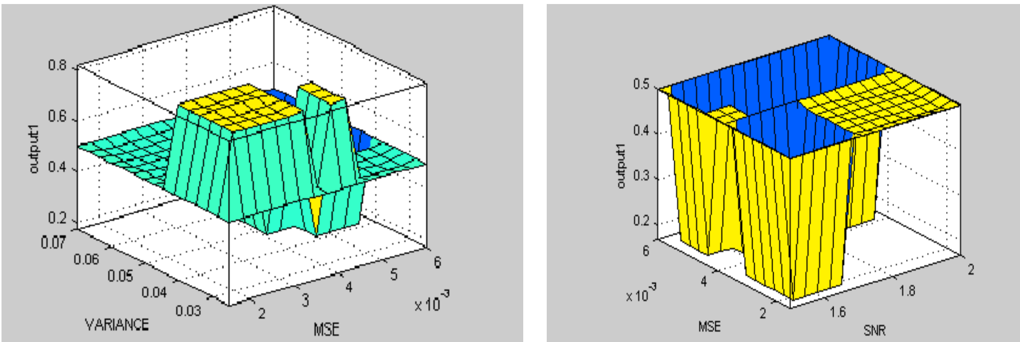

Figure 8. Output of Fuzzy Inference System

4. Conclusion

In this research work five wavelets (Daubechies, Coiflet, Haar, Biorthogonal and Symmlet) with four thresholding rules (SURE, Hybrid, Universal and Minimax) and various decomposition levels are taken. Results are analyzed with FIS system using SNR ratio, MSE, Variance, PSD and morphology of output ECG signal. Results show that in SNR and MSE db06 wavelet and db02 wavelet in DWT gives best results. Bi-orthogonal and Haar wavelet in UWT and DWT gives best results in terms of variance and PSD. The Fuzzy Inference system considered all the three parameters from which Haar Wavelet is the best wavelet whose output is 0.8 in two rules i.e Sure Thresholding and minimax Thresholding. In all the Four Thresholding rules Minimax Thresholding is the best Thresholding as it gives best output in two wavelets (0.825). So, it is concluded that Haar Wavelet and minimax Thresholding is best for denoising of ECG signals.

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