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MHD Peristaltic Transportation of a Conducting Blood Flow with Porous Medium through Inclined Coaxial Vertical Channel

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Abstract

In this research, we investigate the MHD peristaltic transportation of a conducting blood flow with porous medium through inclined coaxial vertical channel. Applying wave frame analysis, exact analytic solutions have been obtained for the axial velocity. Expressions for the pressure gradient, pressure rise and the shear stress are also obtained and the numerical results are presented graphically for different values of the physical parameters of interest. It is found that the axial velocity decreases with increase in magnetic field and the axial velocity increases with increase in Da and α and η. It has been observed that the pressure gradient decreases with increase in Da and $\overline{Q}$. The shear stress at the lower wall increases with increase in Da, η and α. The shear stress at the wall $y = h_2$ has an opposite behavior compared with shear stress at the wall $y = h_1$.

Keywords: *Peristaltic fluid flow, porous medium, magnetic field, inclined coaxial vertical channel*

1. Introduction

Peristaltic transport is a well known process of a fluid transport which is induced by a progressive wave of area contraction or expansion along the length of distensible tube containing the fluid. It is used by many systems in the living body to propel or to mix the contents of a tube. The peristalsis mechanism usually occur in urine transport from kidney to bladder, swallowing food through the esophagus, chyme motion in the gastrointestinal tract, vasomotion of small blood vessels and movement of spermatozoa in the human reproductive tract. There are many engineering processes as well in which peristaltic pumps are used to handle a wide range of fluids particularly in chemical and pharmaceutical industries. Many modern mechanical devices have been designed on the principle of peristaltic pumping for transporting fluids without internal moving parts. The idea of peristaltic transport in mathematical point of view was first coined by Latham [1]. The initial mathematical model of peristalsis obtained by train of sinusoidal waves in an infinitely long symmetric channel or tube has been investigated by Shapiro, *et al.*, [2] and Rathod and Asha [3] have studied the effect of magnetic field and an endoscope on peristaltic motion in uniform and non-uniform annulus.

The MHD characteristics are useful in the development of magnetic devices, cancer tumor treatment, hyper thermia and blood reduction during surgeries. Hence several scientists having in mind such importance extensively discussed the peristalsis with magnetic field effects (Reddy and Raju [4], Hayat, *et al.*, [5], Abd elmaboud and Mekheimer [6] and Hayat, *et al.*, [7]). Further, Singh and Rathee [8, 9] discussed the blood flow in the presence of an applied magnetic field and also motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic

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field, and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid (Ferraro, [10]). The magnetohydrodynamic (MHD) flow of a fluid in a channel with elastic, rhythmically contracting walls (peristaltic flow) is of interest in connection with certain problems of the movement of conductive physiological fluids, *e.g.*, the blood, blood pump machines and with the need for theoretical research on the operation of a peristaltic MHD compressor. Agarwal and Anwaruddin [11] studied the effect of moving magnetic field on blood flow. They studied a simple mathematical model for blood through an equally branched channel with flexible outer walls executing peristaltic waves. Ravikumar [12] studied the peristaltic transportation with effect of magnetic field in a flexible channel under an oscillatory flux. Ravikumar, *et al.*, [13] studied the magnetohydrodynamic couple Stress Peristaltic flow of blood Through Porous medium in a flexible channel at low Reynolds number. Peristaltic flow of blood through coaxial vertical channel with effect of magnetic field: Blood flow study has studied by Ravikumar [14]. The result revealed that the velocity of the fluid increases with an increase in the magnetic field. Peristaltic transport of a Johnson-Segalman fluid under the effect of a magnetic field was developed by Elshahed and Haroun [15]. The peristaltic flow of a MHD fourth grade fluid in a planer channel has studied by Hayat, *et al.*, [16]. The magnetohydrodynamics effects on blood flow through a porous channel have been studied by Ramamurthy and Shankar [17].

Flow through a porous medium has been of significant interest in recent years particularly among geophysical fluid dynamicity. Examples of natural porous media are beach sand, sandstone, limestone, rye bread, wood, the human lung, bile duct, gall bladder with stones and in small blood vessels. The fluid motion through a porous medium has been studied by many authors: Raptis, *et al.*, [18], Raptis and Peridikis [19], and El-Dabe and El-Mohandis [20]. Pressure rise increases as the permeability decreases. This is because of the resistance caused by the porous medium. In the case of ureters stones this causes renal colic (ureteric colic) Ayman [21]. Habtu alemaychu and Radhakrishnamacharya [22] dispersion of a Solute in Peristaltic Motion of a couple stress fluids through a porous medium with slip condition. Reddy and Venkata Ramana [23] have studied the peristaltic transport of a conducting fluid through a porous medium in an asymmetric vertical channel. Krishna Kumari, *et al.*, [24] studied the peristaltic pumping of a Casson fluid in an inclined channel under the effect of a magnetic field. The peristaltic fluid flow through flexible channels has been studied by Ravikumar, *et al.*, [25-31]. Krishna Kumari, *et al.*, [32] studied the peristaltic pumping of a Jeffrey fluid in a porous tube. Ravi Kumar, *et al.*, [33] studied the unsteady peristaltic pumping in a finite length tube with permeable wall. However, the peristaltic transport of micro polar fluids in an inclined channel in the presence of magnetic field has not been studied. Some of the studies on couple- stress fluid just mentioned considered the blood as a couple stress fluid and they were carried out using no slip conditions, although in real systems there is always a certain amount of slip. There are two extremely different types of fluids that appear to slip. One class contains the rarefied gases (Kawang-Hua [34]), while the other fluids have a much more elastic character. In such fluid, some slippage occurs under a large tangential traction. It has been claimed that slippage can occur in non-Newtonian fluid, concentrated polymer solution, and molten polymer. Furthermore, in the flow of dilute suspensions of particles, a clear layer is sometimes observed next to the wall. Poiseuille, in a work that won a prize in experimental physiology, observed such a layer with a microscope in the flow of blood through capillary vessels (Coleman, *et al.*, [35]). Several investigators considered the effect of slip (Kwang, *et al*., [36], Srinivas, *et al*., [37]).

2. Formulation of the Problem and Analytic Solution

Consider the unsteady hydromagnetic flow of a viscous, incompressible and electrically conducting couple-stress fluid through a two-dimensional channel of non-uniform thickness with a sinusoidal wave travelling down its wall. The plates of the channel are assumed to be electrically insulated. We choose a rectangular coordinate system for the channel with x along centerline in the direction of wave propagation and y transverse to it.

The geometry of the wall surface is defined as

$$H_1(X,t) = a_0 + b_1 \cos \frac{2\pi}{\lambda}(X - ct) \tag{1}$$

$$H_2(X,t) = -a_1 - b_2 \cos \frac{2\pi}{\lambda}((X - ct) + \theta) \tag{2}$$

Where b_1, b_2 be the amplitudes of $(0 \le \theta \le \pi)$ the waves, $a_0 + a_1$ is the width of the channel, λ is the wave length, θ is the phase differences $(0 \le \theta \le \pi)$, c is the propagation velocity and t is the time. We introduce a wave frame of reference (x, y) moving with velocity *c* in which the motion becomes independent of time when the channel length is an integral multiple of the wavelength and the pressure difference at the ends of the channel is a constant (Shapiro et al., (1969)). The transformation from the fixed frame of reference (X, Y) to the wave frame of reference (x, y) is given by

$x = X - ct$, $y = Y$, $u = U - c$, $v = V$ and $p(x) = P(X, t)$

Where (u, v) and (U, V) are the velocity components, *p* and *P* are pressures in the wave and fixed frames of reference, respectively.

The equations governing the flow in wave frame of reference are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] - \left[\sigma B_0^2\right](u+c) - \left[\frac{\mu}{k_1}\right](u+c) + g\sin\alpha \tag{4}$$

$$\rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] - \left[\sigma B_0^2\right]v - \left[\frac{\mu}{k_1}\right]v - g\cos\alpha \tag{5}$$

u and v are the velocity components in the corresponding coordinates p is the fluid pressure, ρ is the density of the fluid, μ is the coefficient of the viscosity, k_1 is the permeability of the porous medium and *k* is the thermal conductivity. Proceeding with the analysis, we introduce the following dimensionless parameters:

$$x^* = \frac{x}{\lambda} \quad y^* = \frac{y}{a_0} \quad u^* = \frac{u}{c} \quad v^* = \frac{\lambda v}{a_0 c} \quad p^* = \frac{a_0^2 p}{\lambda \mu c} \quad t^* = \frac{ct}{\lambda} \quad d = \frac{a_1}{a_0}$$

$$\mathrm{Re} = \frac{\rho c a_0}{\mu} \qquad \delta = \frac{a_0}{\lambda} \qquad h_1^* = \frac{H_1}{a_0} = 1 + \phi_1 \cos 2\pi x$$

$$h_2^* = \frac{H_2}{a_0} = -d - \phi_2 \cos(2\pi x + \theta) \quad \phi_1 = \frac{b_1}{a_0} \quad \phi_2 = \frac{b_2}{a_0}$$

Where δ , ε ϕ and Re designate the wave number, ratio of half width of channels, amplitude ratio and Reynolds number respectively. Utilizing the long wavelength and low Reynolds number approximation in Equations (3) - (5), we obtain:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

$$\mathrm{Re}\,\delta\left[u u_x + v u_y\right] = -\frac{\partial p}{\partial x} + \delta^2 u_{xx} + u_{yy} - M^2 u - M^2 - \frac{1}{Da}u - \frac{1}{Da} + \eta \sin\alpha \tag{7}$$

$$\mathrm{Re}\,\delta^{3}[u\,v_{x}]+\mathrm{Re}\,\delta[v\,v_{y}]=-\frac{\partial p}{\partial y}+\delta^{4}v_{xx}+\delta^{2}v_{yy}-\delta^{2}M^{2}v-\delta^{2}\frac{1}{Da}v-\eta_{1}\cos\alpha \quad (8)$$

Using long wavelength (*i.e.,* $\delta << 1$) and negligible inertia (*i.e.,* $Re \to 0$) approximations, we have

$$\frac{\partial^{2}u}{\partial y^{2}}-\left(M^{2}+\frac{1}{Da}\right)u=\frac{\partial p}{\partial x}+\left(M^{2}+\frac{1}{Da}\right)-\eta\sin\alpha \quad (9)$$

$$\frac{\partial p}{\partial y}=0 \quad (10)$$

With dimensionless boundary conditions

u = -1 at $y = h_1$ (11)

u = -1 at $y = h_2$ (12)

Solving equation (9) using the boundary conditions (11 and 12), we get

$$u=a_{1}\,Sin\,h[\alpha_{1}y]+a_{2}\,Cos\,h[\alpha_{1}y]+A \quad (13)$$

Where $\alpha_{1}=\sqrt{M^{2}+\frac{1}{D}}$

$$a_{1}=\left[\frac{-(1+A)}{Sinh(\alpha_{1}h_{1})}\right]\left[1+\left[\frac{Cosh(\alpha_{1}h_{2})}{\left[\frac{Cosh(\alpha_{1}h_{1})-Cosh(\alpha_{1}h_{2})}{Sinh(\alpha_{1}h_{1})-Sinh(\alpha_{1}h_{2})}\right]Sinh(\alpha_{1}h_{1})-Cosh(\alpha_{1}h_{1})}\right]\right]$$

$$a_{2}=\left(\frac{(1+A)}{\left[\frac{Cosh(\alpha_{1}h_{1})-Cosh(\alpha_{1}h_{2})}{Sinh(\alpha_{1}h_{1})-Sinh(\alpha_{1}h_{2})}\right]Sinh(\alpha_{1}h_{1})-Cosh(\alpha_{1}h_{1})}\right)$$

$$A=\frac{D}{M^{2}D+1}\left(\eta\,Sin\,\alpha-\frac{\partial p}{\partial x}\right)-1$$

From equation (3)

$$v=-\left[\left(\frac{a_{3}}{\alpha_{1}}\right)Cosh(\alpha_{1}y)+\left(\frac{a_{4}}{\alpha_{1}}\right)Sinh(\alpha_{1}y)\right] \quad (14)$$

3. Shear Stress, Pressure Gradient and Pressure Rise

The shear stress at the wall $y = h_1(x)$, the dimensional form is given by

$$T=\frac{\frac{1}{2}\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right]\left[1-\left(\frac{dh_{1}}{dx}\right)^{2}\right]+\left[\frac{\partial v}{\partial y}-\frac{\partial u}{\partial x}\right]\left(\frac{dh_{1}}{dx}\right)}{\left[1+\left(\frac{dh_{1}}{dx}\right)^{2}\right]}$$

and its solution is given by $T=\dfrac{\frac{1}{2}[b_{1}+b_{2}]\left[1-\left(\frac{dh_{1}}{dx}\right)^{2}\right]+[b_{3}-b_{4}]\left(\frac{dh_{1}}{dx}\right)}{\left[1+\left(\frac{dh_{1}}{dx}\right)^{2}\right]}$

The shear stress at the wall $y = h_2(x)$, the dimensional form is given by

$$T = \frac{\frac{1}{2}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]\left[1 - \left(\frac{dh_2}{dx}\right)^2\right] + \left[\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right]\left(\frac{dh_2}{dx}\right)}{\left[1 + \left(\frac{dh_2}{dx}\right)^2\right]}$$

and its solution is given by $T = \dfrac{\frac{1}{2}[b_1 + b_2]\left[1 - \left(\frac{dh_2}{dx}\right)^2\right] + [b_3 - b_4]\left(\frac{dh_2}{dx}\right)}{\left[1 + \left(\frac{dh_2}{dx}\right)^2\right]}$

Where

$$b_1 = \frac{\partial u}{\partial y} = a_1\alpha_1 \cosh[\alpha_1 y] + a_2\alpha_1 \sinh[\alpha_1 y]$$

$$b_2 = \frac{\partial v}{\partial x} = \left[\frac{\partial}{\partial x}(a_3)\right]\frac{\cosh[\alpha_1 y]}{\alpha_1} + \left[\frac{\partial}{\partial x}(a_4)\right]\frac{\sinh[\alpha_1 y]}{\alpha_1}$$

$$b_3 = \frac{\partial v}{\partial y} = -[a_3 \sinh[\alpha_1 y] + a_4 \cosh[\alpha_1 y]]$$

$$b_4 = \frac{\partial u}{\partial x} = \left[\frac{\partial}{\partial x}(a_1)\right]\sinh[\alpha_1 y] + \left[\frac{\partial}{\partial x}(a_2)\right]\cosh[\alpha_1 y]$$

$$a_3 = \frac{\partial}{\partial x}(a_1) = \frac{\partial}{\partial x}\left(\left[\frac{-(1+A)}{Sinh(\alpha_1 h_1)}\right]\left[1 + \left[\frac{Cosh(\alpha_1 h_2)}{\left[\frac{Cosh(\alpha_1 h_1) - Cosh(\alpha_1 h_2)}{Sinh(\alpha_1 h_1) - Sinh(\alpha_1 h_2)}\right]Sinh(\alpha_1 h_1) - Cosh(\alpha_1 h_1)}\right]\right]\right)$$

$$a_4 = \frac{\partial}{\partial x}(a_2) = \frac{\partial}{\partial x}\left(\left[\frac{(1+A)}{\left[\frac{Cosh(\alpha_1 h_1) - Cosh(\alpha_1 h_2)}{Sinh(\alpha_1 h_1) - Sinh(\alpha_1 h_2)}\right]Sinh(\alpha_1 h_1) - Cosh(\alpha_1 h_1)}\right]\right)$$

The rate of volume flow 'q' through each section is a constant (independent of both x and t).It is given by

$$q = \int_{h_2}^{h_1} u\, dy = \int_{h_2}^{h_1} (a_1 Sin\ h[\alpha_1 y] + a_2 Cos\ h[\alpha_1 y] + A)dy$$

$$-a_5 + a_6 + A[(h_1 - h_2) - a_5 + a_6] \tag{15}$$

Where

$$a_5 = \left[\frac{-(Cosh(\alpha_1 h_2) - Cosh(\alpha_1 h_1))}{\alpha_1 Sinh(\alpha_1 h_1)}\right]\left[1 + \left[\frac{Cosh(\alpha_1 h_2)}{\left[\frac{Cosh(\alpha_1 h_1) - Cosh(\alpha_1 h_2)}{Sinh(\alpha_1 h_1) - Sinh(\alpha_1 h_2)}\right]Sinh(\alpha_1 h_1) - Cosh(\alpha_1 h_1)}\right]\right]$$

$$a_6 = \frac{1}{\alpha_1}\left[\frac{(Sinh(\alpha_1 h_2) - Sinh(\alpha_1 h_1))}{\left(\left(\frac{Cosh(\alpha_1 h_1) - Cosh(\alpha_1 h_2)}{Sinh(\alpha_1 h_1) - Sinh(\alpha_1 h_2)}\right)Sinh(\alpha_1 h_1)\right) - Cosh(\alpha_1 h_1)}\right]$$

Hence the flux at any axial station in the fixed frame is found to be given by

$$Q = \int_{h_2}^{h_1} (u + 1)\, dy = q + (h_1 - h_2) \tag{16}$$

The average volume flow rate over one wave period ($T = \frac{\lambda}{c}$) of the peristaltic wave is defined as

$$\overline{Q} = \frac{1}{T}\int_0^T Q\, dt = q + 1 + d \tag{17}$$

The pressure gradient obtained from equation (17) can be expressed as

$$\frac{dp}{dx} = (\eta \sin \alpha) - \left(\frac{(\overline{Q} - (1 + d)) - (a_6 - a_5)}{(h_1 - h_2) + (a_6 - a_5)}\right)\left(M^2 + \frac{1}{D}\right) - \left(M^2 + \frac{1}{D}\right) \tag{18}$$

The pressure rise Δ_p in the channel of length L, non-dimensional form is given by

$$\Delta_p = \int_0^1 \left(\frac{dp}{dx}\right) dx = \int_0^1 \left((\eta \sin \alpha) - \left(\frac{(\overline{Q} - (1 + d)) - (a_6 - a_5)}{(h_1 - h_2) + (a_6 - a_5)}\right)\left(M^2 + \frac{1}{D}\right) - \left(M^2 + \frac{1}{D}\right)\right) dx \tag{19}$$

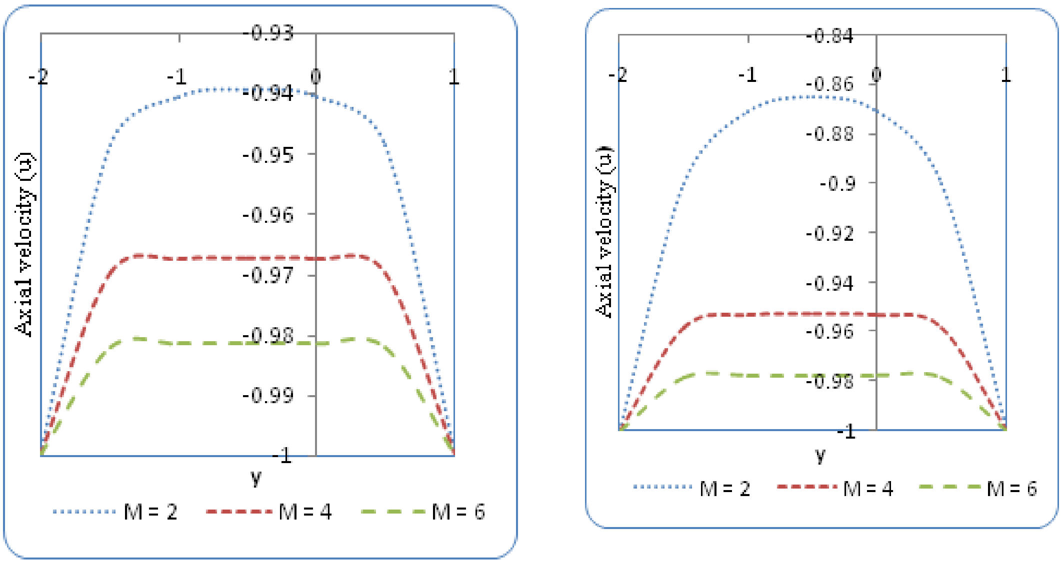
4. Results and Discussion

The primary object of this investigation has been to study magnetohydrodynamics peristaltic transportation of a conducting blood flow with porous medium through inclined coaxial vertical channel. The analytical expressions for velocity distribution, pressure gradient and shear stress have been derived in the previous section. **Mathematica** software is used to find out numerical results.

The Axial velocity has been calculated as a function of y from the equation (13) and plotted from the figures 1to4. Figure 1 represent the variation of axial velocity with magnetic field (M) for fixed other parameters ϕ_1, ϕ_2, η, α, dp/dx ,x, d, θ when (a) Da = 0.1and (b) Da = 0.5, we notice that the axial velocity decreases when the magnetic field increases. Figure 2 is drawn to study the effects of porous parameter Da on the axial velocity when (a) M =2 and (b) M = 4, it can be found that velocity increases with increase of M and Da. Figure 3 shows that the effect of α on the axial velocity when (a) Da = 0.1 and (b) Da = 0.5. It reveals that the velocity increases when increase in α. The effect of η on the axial velocity is shown in the Figure 4, it can be seen that axial velocity increases with increase in η when (a) Da = 0.1 and (b) Da = 0.5.

The pressure gradient has been calculated as a function of x from the equation (18) and plotted from the Figures 5 to7. Figure 5 shows the influence of Porous parameter Da on the pressure gradient *dp/dx* when (a) M =2 and (b) M = 4. It is observed that in the wider part of channel *x* ε [0, 0.3] and *x* ε [07, 1], the pressure gradient is relatively small. Hence, the flow can easily pass without imposing large pressure gradient. However, in the narrow part of channel *x* ε [0.3, 7], larger pressure gradient is needed to maintain the same flux to pass through it. It is further observed that the pressure gradient decreases by increasing the values of Da i.e. the magnitude of pressure gradient is inversely proportional to porous parameter. It is also found that when M >2, the pressure gradient gradually increases. Figure 6 represents the variation in the pressure gradient verses x when (a) Da = 0.1 and (b) Da = 0.5. It is note that the pressure gradient decreases when $\overline{Q}$ increases. It can be seen that when Da > 0.1, the variations in pressure gradient gradually decreases (figure 6(b)). The effect of magnetic field (M) on the pressure gradient is shown in the figure (7). It interested to note that the pressure gradient is increases when increase in magnetic field. It is observed that the flow can easily pass without imposing the pressure gradient in wider part of channel *x* ε [0, 0.3] and *x* ε [07, 1].It is found that the variations in the pressure gradient is gradually deceases when Da > 0.1(Figure 7(b)). Finally, it can be conclude that the effect of magnetic field on pressure gradient is inversely proportional to the effects of Da and $\overline{Q}$ on pressure gradient.

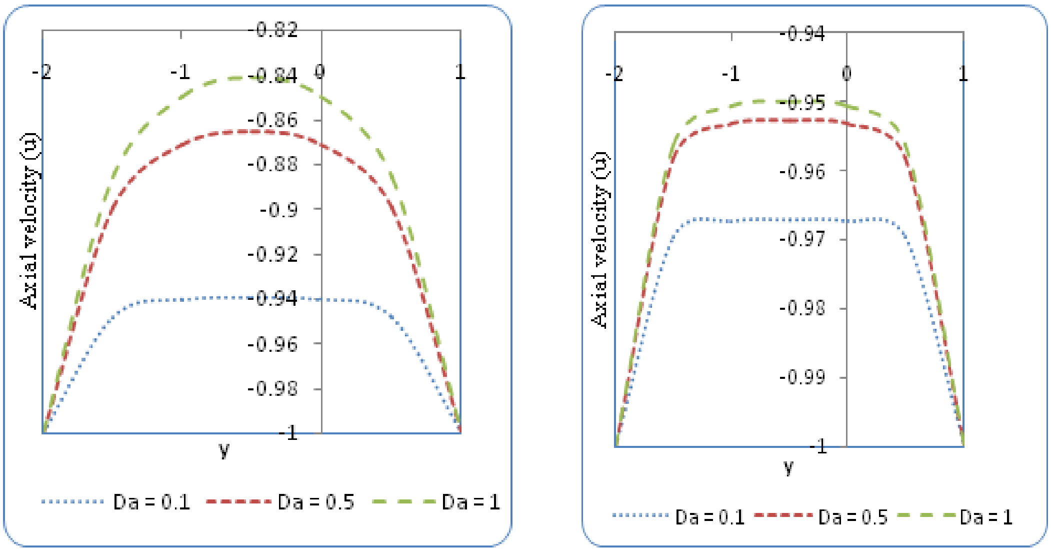
Figure (8) is plotted to study the effect of Da on shear stress at the wall y = h_2. We notice that the shear stress increases in the regions x ε [0, 0.2] and x ε [0.6, 0.8] while it decreases in the regions x ε [0.2, 0.6] and x ε [0.8, 1] when porous parameter (Da) increases. Moreover, shear stress is symmetric for M = 2 and M = 4.Figure (9) is plotted to study the effect of η on shear stress at the wall y = h_2. We notice that the shear stress increases in the regions x ε [0, 0.2] and x ε [0.6, 0.8] while it decreases in the regions x ε [0.2, 0.6] and x ε [0.8, 1] when η increases. Moreover, shear stress is symmetric for Da = 0.1 and Da =0.5. Figure (10) is plotted to study the effect of α on shear stress at the wall y = h_2. We notice that the shear stress increases in the regions x ε [0, 0.2] and x ε [0.6, 0.8] while it decreases in the regions x ε [0.2, 0.6] and x ε [0.8, 1] as α increases. Finally, we conclude that the shear stress is increases with increase in Da, η and α at the wall y = h_2. Figure (11) is plotted to study the effect of Da on shear stress at the wall y = h_1. We notice that the shear stress decreases in the regions xε[0,0.2] and xε[0.5,0.8] while it increases in the regions x ε [0.2,0.5] and x ε [0.8,1] with increase in porous parameter Da. Figures 12 and13 reveals the shear stress at the wall y = h_1. We notice that shear stress is decreases with increase in η and α.



(a) Da = 0.1

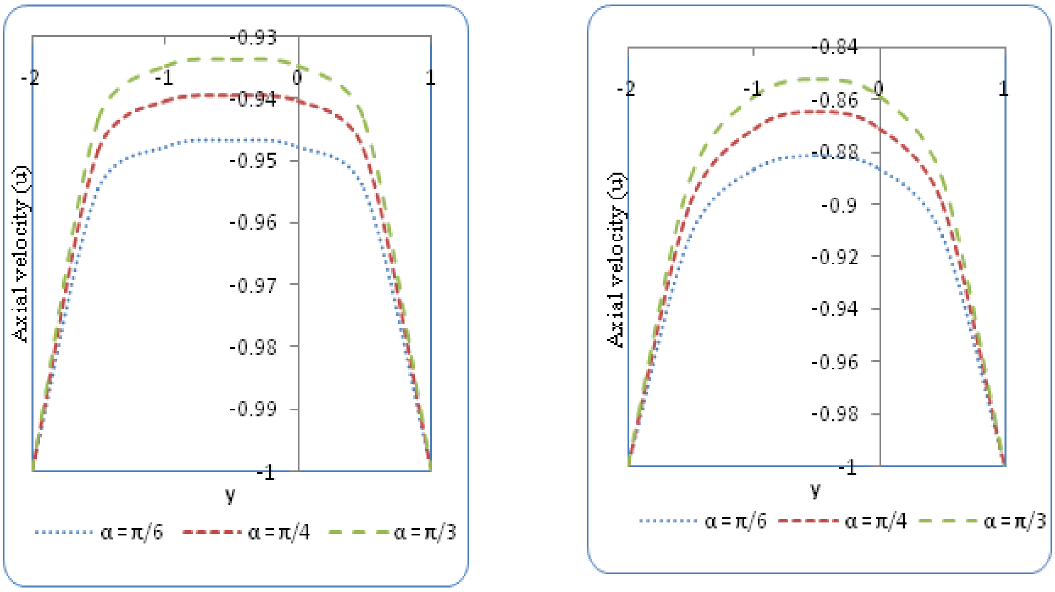
(b) Da = 0.5

Figure 1. Distribution of Axial Velocity for Different Values of M with Fixed ϕ_1 = 0.7, ϕ_2 = 1.2, η = 0.5, $\alpha = \frac{\pi}{4}$, $\frac{dp}{dx}$ = - 0.5, x = 0.25 ,d = 2,θ = 0



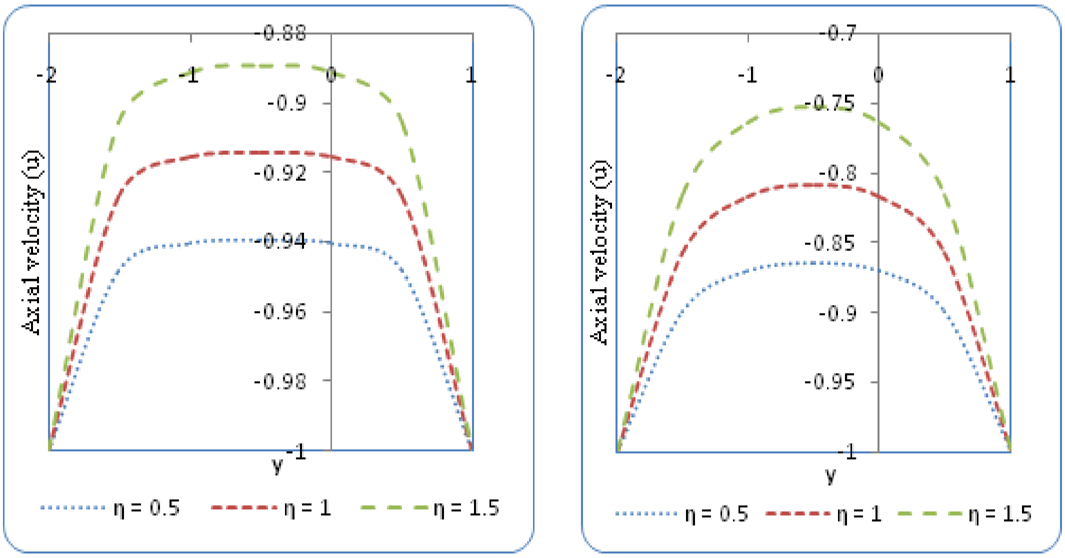
M = 2 (a) M = 4

Figure 2. Distribution of Axial Velocity for Different Values of Da with Fixed ϕ_1 = 0.7, ϕ_2 = 1.2, η = 0.5, , $\frac{dp}{dx}$ = - 0.5, x = 0.25, d = 2,θ = 0



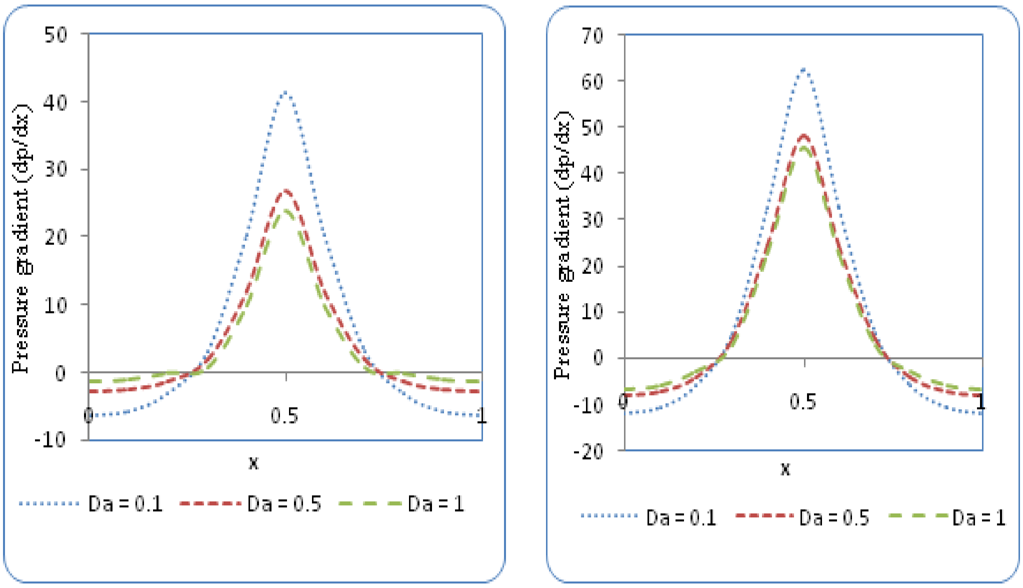
(a) Da = 0.1 (b) Da = 0.5

Figure 3. Distribution of Axial Velocity for different Values of α with Fixed M = 2, ϕ_1 = 0.7, ϕ_2 = 1.2, η = 0.5, $\frac{dp}{dx}$ = - 0.5, x = 0.25, d = 2, θ = 0



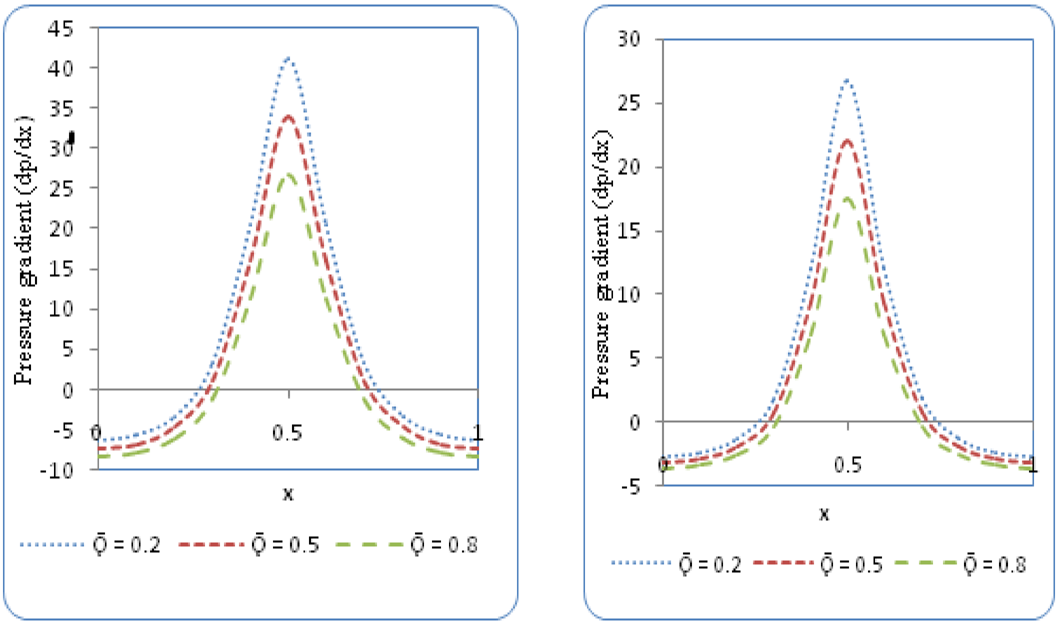
(a) Da = 0.1 (b) Da = 0.5

Figure 4. Distribution of Axial Velocity for Different Values of η with Fixed M = 2, ϕ_1 = 0.7, ϕ_2 = 1.2, $\alpha = \frac{\pi}{4}$, $\frac{dp}{dx}$ = - 0.5, x = 0.25 ,d = 2,θ = 0.



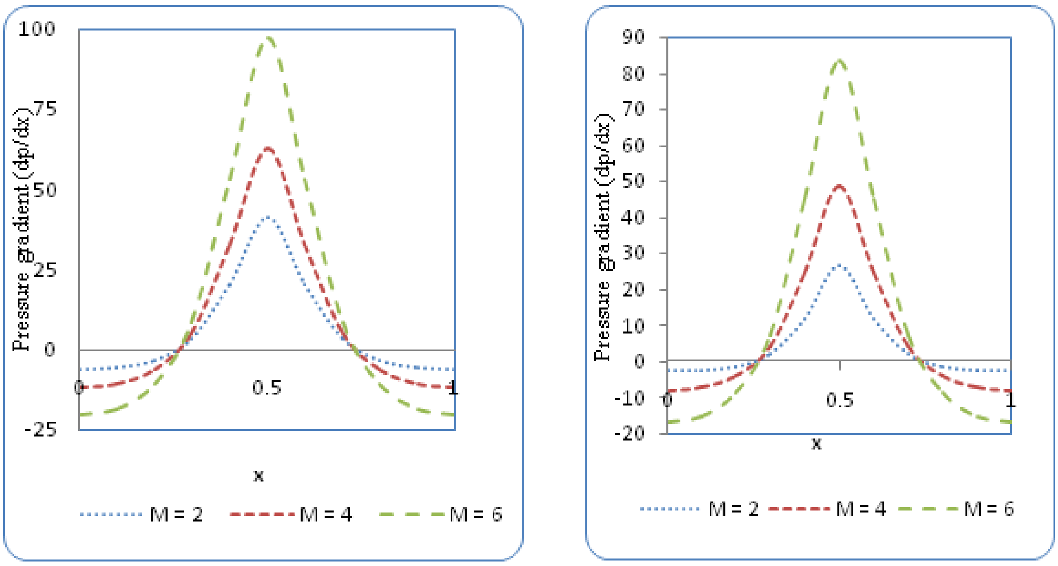
M = 2 (b)M = 4

Figure 5. Pressure Gradient ($\frac{dp}{dx}$) Versus x with Da for Fixed ϕ_1 = 0.7, ϕ_2 =1.2, $\alpha = \frac{\pi}{4}$, d = 2, θ = 0, η = 0.5, $\overline{Q}$ = 0.2



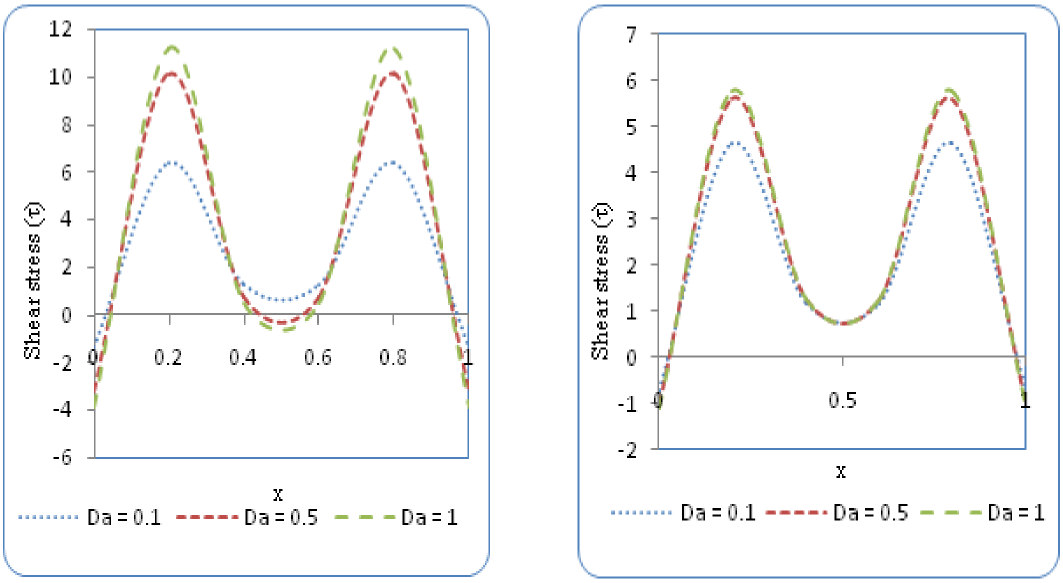
Da = 0.1 Da = 0.5

Figure 6. Pressure Gradient ($\frac{dp}{dx}$) Versus x with $\overline{Q}$ for Fixed M = 2, ϕ_1 = 0.7, ϕ_2 = 1.2 $\alpha = \frac{\pi}{4}$, d = 2, θ = 0, η = 0.5



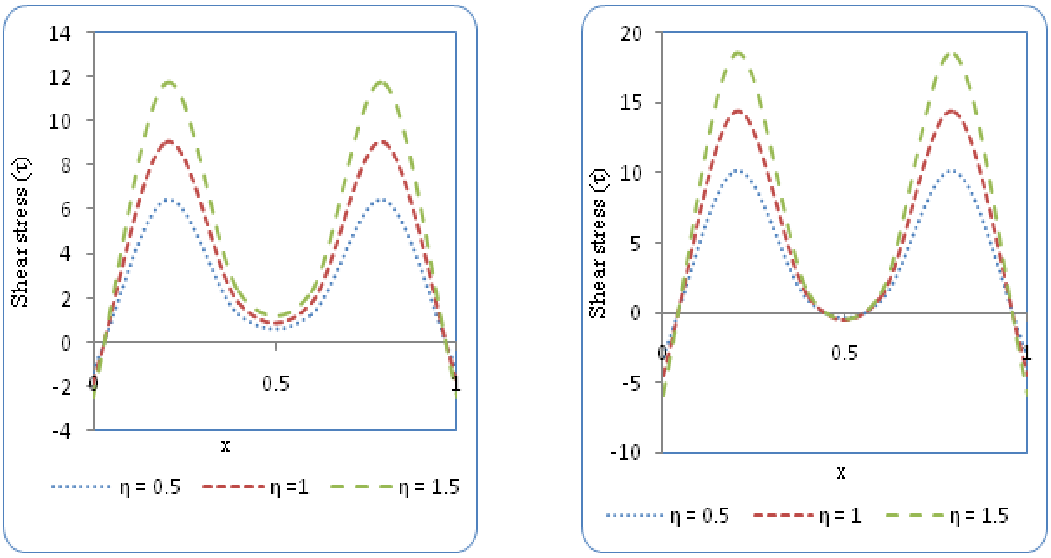
Da = 0.1 Da = 0.5

Figure 7. Pressure Gradient ($\frac{dp}{dx}$) Versus x with M for fixed ϕ_1 = 0.7, ϕ_2 = 1.2, $\alpha = \frac{\pi}{4}$, d = 2, θ = 0, η = 0.5, $\overline{Q}$ = 0.2



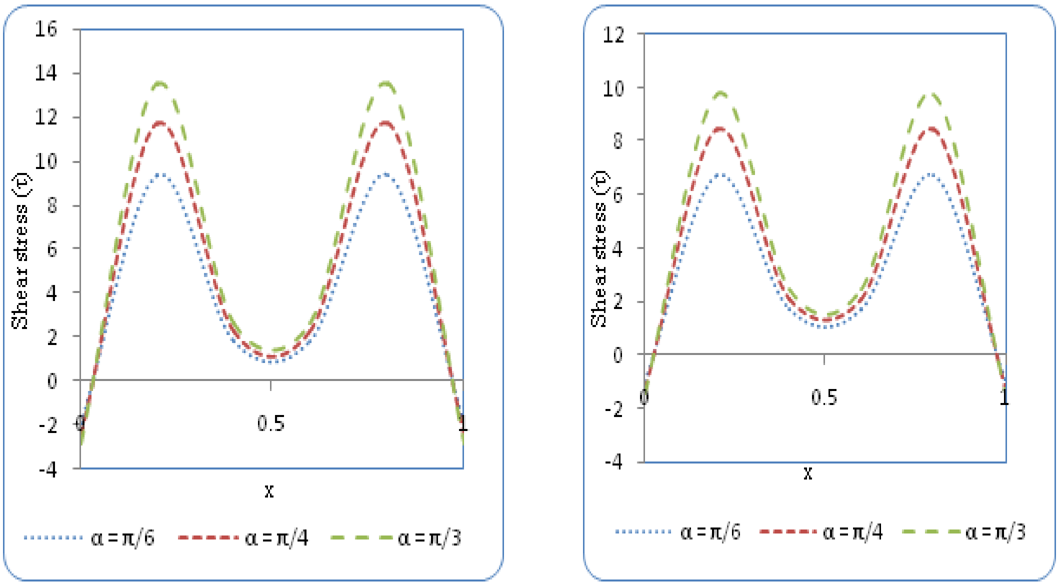
(a) M = 2 M = 4

Figure 8. Shear Stress (τ) Versus x at the Wall y = h_2 with Da for Fixed ϕ_1 = 0.7, ϕ_2 = 1.2, $\alpha = \frac{\pi}{4}$, $\frac{dp}{dx}$ = - 0.5, d = 2, θ = 0, η = 0.5



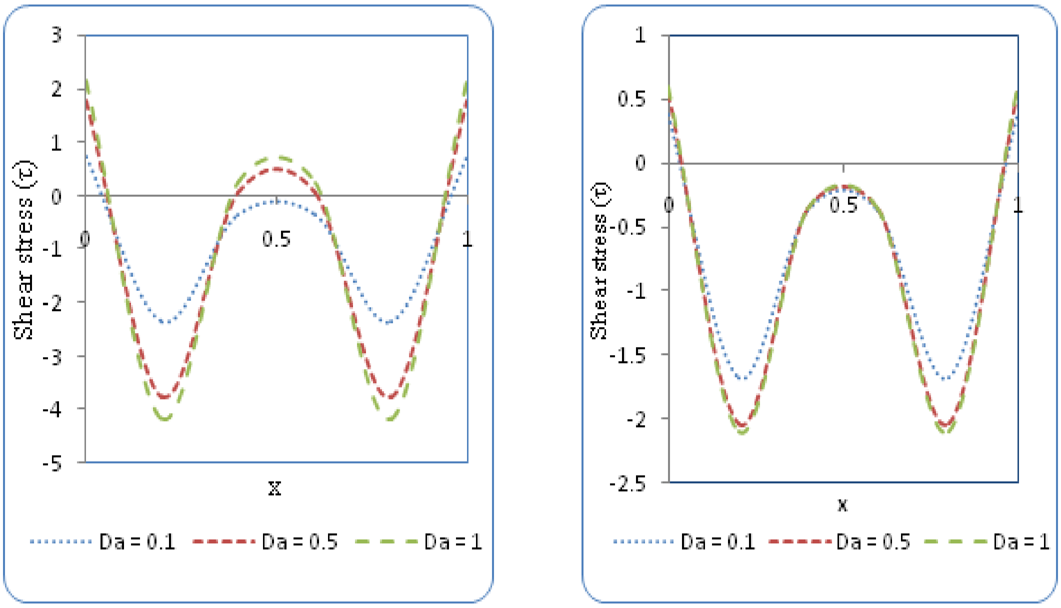
(a) Da = 0.1 **(b)** Da = 0.5

Figure 9. Shear Stress (τ) Versus x at the Wall y = h_2 with η for Fixed M =2, ϕ_1 = 0.7, ϕ_2 = 1.2, $\alpha = \frac{\pi}{4}$, $\frac{dp}{dx}$ = - 0.5, d = 2, θ = 0, η = 0.5.



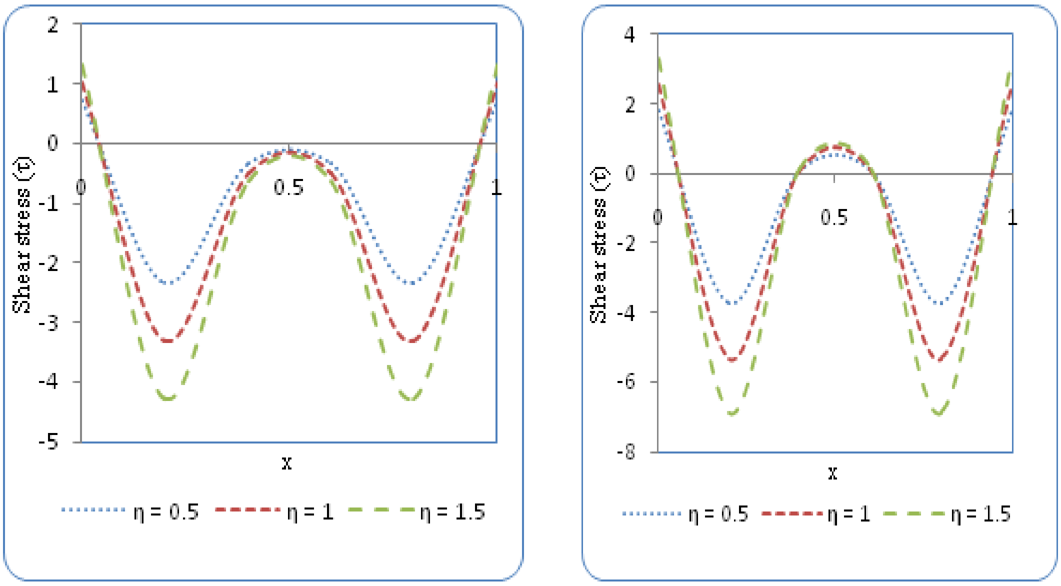
M =2 **M = 4**

Figure 10. Shear Stress (τ) Versus x at the Wall y = h_2 with α for Fixed Da = 0.1, ϕ_1 = 0.7, ϕ_2 = 1.2, $\alpha = \frac{\pi}{4}$, $\frac{dp}{dx}$ = - 0.5, d = 2,θ = 0, η = 0.5



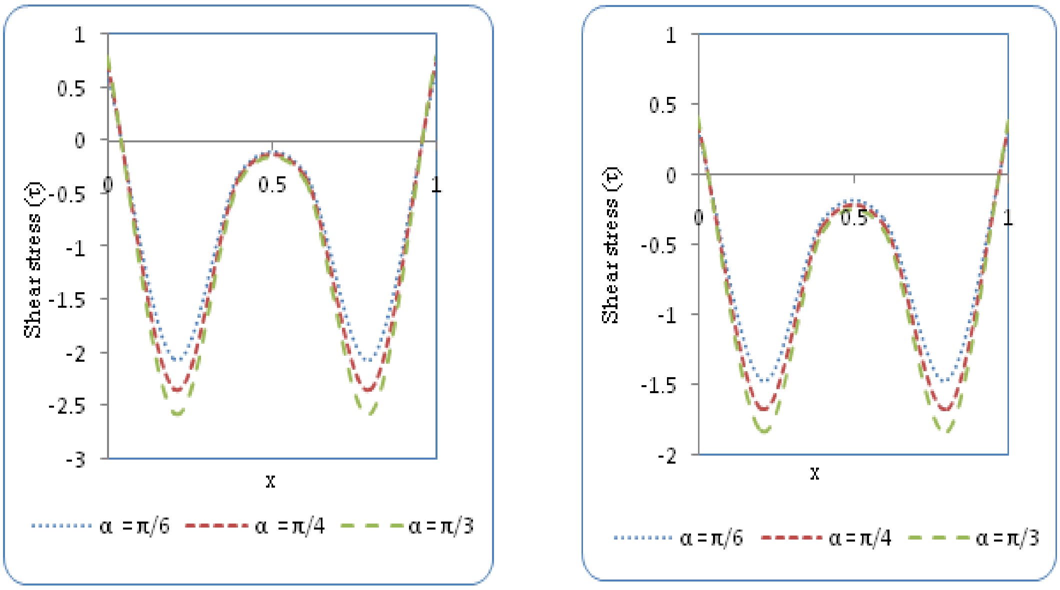
M = 2 **M =4**

Figure 11. Shear Stress (τ) Versus x at the Wall y = $h_1$with Da for Fixed ϕ_1 = 0.7,ϕ_2 = 1.2, $\alpha = \frac{\pi}{4}$, $\frac{dp}{dx}$ = - 0.5,d = 2, θ = 0, η = 0.5



Da = 0.1 Da = 0.5

Figure 12. Shear Stress (τ) Versus x at the Wall y = h_1 with η for Fixed M =2, ϕ_1 = 0.7, ϕ_2 = 1.2, $\alpha = \frac{\pi}{4}$ $\frac{dp}{dx}$ = - 0.5, d = 2, θ = 0, η = 0.5



M = 2 (a) M = 4

Figure 13. Shear stress (τ) Versus x at the Wall y = h_1 with α for Fixed Da = 0.1, ϕ_1 = 0.7, ϕ_2 = 1.2, $\alpha = \frac{\pi}{4}$, $\frac{dp}{dx}$ = - 0.5, d = 2,θ = 0, η = 0.5.

5. Conclusions

This research considered the magnetohydrodynamics peristaltic transportation of a conducting blood flow with porous medium through inclined coaxial vertical. The exact solution of simplified equations is calculated. The results are discussed through graphs. We conclude the following observations:

- The axial velocity deceases with increase in Magnetic field (M).
- With increase in Da, α and η, the axial velocity increases.
- The pressure gradient decreases when increase porous parameter (Da) and volume flow rate ($\overline{Q}$).
- Pressure gradient increases as increases magnetic field (M).
- The shear stress is increases with increase in Da, α and η at the wall $y = h_1$.
- The shear stress is decreases with increase in Da, α and η at the wall $y = h_2$.

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