International Journal of Bio-Science and Bio-Technology
Vol.7, No.5 (2015), pp.187-202
http://dx.doi.org/10.14257/ijbsbt.2015.7.5.18

Design Auxiliary Sliding Variable Sliding Mode Controller for Robot-Assisted Ophthalmic Surgery

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Abstract

Recent development of robot technology is revolutionizing the medical field. The concept of using robot assistance in medical surgery has been receiving more and more recognition throughout the world. Robot-assisted surgery has the advantage of reducing surgeons' hand tremor, decreasing post-operative complications, reducing patients' pains, and increasing operation dexterity inside the patients' body. Robotic assistants have been broadly used in many medical fields such as orthopedics, neurology, urology and cardiology, and robot assisted surgery is keeping expanding its influences in more general medical field.

Refer to this research, auxiliary sliding variable sliding mode controller is proposed for multi DOF joint with application in surgical robot manipulator. The main problem in this research is design robust chattering free sliding mode controller. The chattering phenomenon problem is reduced in certain/uncertain system by using auxiliary sliding variable. The simulation results exhibit that the sliding mode controller with auxiliary sliding variable works well in certain and uncertain condition.

Keywords*: Robot-assisted surgery, Ophthalmic surgery, Multi-degrees of freedom joints, sliding mode controller, auxiliary sliding variable*

1. Introduction and Background

Robot-assisted surgery has become a burgeoning field in recent years. An interdisciplinary subject involves both robot technologies and medical intervention. Because of its potentials to improve precision, enhance dexterity, eliminate tremor, reduce complication rates, and enable novel procedures not previously achievable, robot-assisted surgery has drawn broad attentions from the robotics research community and the medical world over the past decade. One successful implementation of robot-assisted surgery is the *Da-Vinci* surgical system of Intuitive Surgical. It utilizes a tele-operation control mode with a master controlled by the surgeon and a slave surgical assistant operating on the patient. Despite its capability of performing abdominal procedures, the *Da Vinci* system is too bulky, and lacks the precision and dexterity required for delicate applications that require higher accuracy. Therefore, researchers are actively developing and implementing novel robotic systems to accommodate more demanding surgical procedures, *e.g.*, micro ophthalmic surgery.

Ophthalmic surgery is one type of microsurgery that constitutes one of the most delicate surgical fields. Typical ophthalmic surgical procedures include cataract surgery, glaucoma surgery, refractive surgery, retina and vitreous surgery, *etc*. Due to the micron scale and delicacy of retinal tissues, only highly experienced ophthalmic surgeons can successfully perform these demanding yet common procedures. This limitation leads to a need for robotic assistance in order to achieve better ophthalmic surgical outcomes and complete specific procedures that are beyond surgeons' capabilities.

ISSN: 2233-7849 IJBSBT

Robot-assisted ophthalmic surgery presents many challenges, out of which dual-arm manipulation, high-precision dexterous operation, distal tool dexterity, insertion depth perception and contact force feedback are major concerns. Researchers have started to investigate some of these aforementioned concerns by developing robotic assistants, but a comprehensive robotic system that is capable of assisting general ophthalmic surgical procedures and addressing existing surgical challenges is still missing. Besides, there are also many interesting robotics-related theoretical problems to be investigated under the light of ophthalmic surgery, *e.g.*, multi-arm manipulation, robot performance evaluation, high-precision robot design, force sensing implementation, *etc*. Multi-degrees-of-freedom (DOF) actuators are wide used in a number of Industries. Currently, a significant number of the existing robotic actuators that can realize multi-DOF motion are constructed using gear and linkages to connect several single-DOF motors in series and/or parallel. Not only do such actuators tend to be large in size and mass, but they also have a decreased positioning accuracy due to mechanical deformation, friction and backlash of the gears and linkages. A number of these systems also exhibit singularities in their workspaces, which makes it virtually impossible to obtain uniform, high-speed, and high-precision motion. For high precession trajectory planning and control, it is necessary to replace the actuator system made up of several single-DOF motors connected in series and/or parallel with a single multi-DOF actuator. The need for such systems has motivated years of research in the development of unusual, yet high performance actuators that have the potential to realize multi-DOF motion in a single joint. One such actuator is the spherical motor. Compared to conventional robotic manipulators that offer the same motion capabilities, the spherical motor possesses several advantages. Not only can the motor combine 3-DOF motion in a single joint, it has a large range of motion with no singularities in its workspace. The spherical motor is much simpler and more compact in design than most multiple single-axis robotic manipulators. The motor is also relatively easy to manufacture. The spherical motor have potential contributions to a wide range of applications such as coordinate measuring, object tracking, material handling, automated assembling, welding, and laser cutting. All these applications require high precision motion and fast dynamic response, which the spherical motor is capable of delivering. Previous research efforts on the spherical motor have demonstrated most of these features. These, however, come with a number of challenges. The spherical motor exhibits coupled, nonlinear and very complex dynamics [1-7].

This research study proposes design a controller for multi DOF actuator for dual-arm hybrid robotic system to assist ophthalmic surgical procedures. Each arm consists of a customized dexterous tool sitting on top of a high precision parallel robot as a robotic micromanipulation stage. This robotic system is designed to assist challenging surgical procedures such as Internal Limiting Membrane (ILM) peeling, vitreous membrane dissection, retinal vascular stenting, drug delivery, retinal vascular cannulation, *etc*. Experienced surgeons can only perform some of these tasks such as ILM peeling, while other tasks such as retinal vascular stenting still remain unexplored due to technical difficulties. Aiming at filling the existing gap in ophthalmology, this work presents a complete study including controller design and optimization of new technologies for ophthalmic micro-surgery. Results obtained in this work contribute to the robotics research community and confirm the feasibility of using robotic assistance in ophthalmic surgery. These results can be extended into other surgical fields as well, such as micro-vascular reconstruction, neuro-surgery, and micro-surgery. To control of robotic assistance in ophthalmic surgery, three purposes are very important:

- **Stability**: Stability is due to the proper functioning of the system. A system is called stable if for any bounded input signal the system's output will stay bounded. Therefore limitation of output deviation is very important for any design.

- **Robust**: Robust method is caused to achieve robust and stable performance in the presence of uncertainty and external disturbance. A system is robust when it would also work well under different conditions and external disturbance.
- **Reliability:** to control of nonlinear and uncertain systems, reliability play important role and most of model-base controller are reliable.

As a result, design a controller based on these three factors are the main challenge in this work. Based on control theory; controllers for robotic assistance in ophthalmic surgery are divided into two main collections:

Conventional control theory and intelligent control theory where, conventional control theories are work based on nonlinear dynamic parameters of robotic assistance in ophthalmic surgery and these are divided into two main categories: Linear control method and nonlinear control method. Intelligent control theory is worked based on intelligent control theory and it is free of nonlinear dynamic parameters of robotic assistance in ophthalmic surgery. Conventional nonlinear control theories are highly sensitive to system's behavior and work based on cancelling decoupling and nonlinear terms of dynamic parameters of each likes in robotic assistance in ophthalmic surgery. Computed Torque Control (CTC) and Sliding Mode Control (SMC) are two nonlinear conventional controller which introduced by many researchers to control of robotic assistance in ophthalmic surgery [8-14]. In this research to design robust control of robotic assistance in ophthalmic surgery, sliding mode controller is recommended.

To eliminate the actual acceleration measurement and also the computation burden as well as have stability, efficiency and robust controller, sliding mode controller is introduced in this part. This controller works very well in certain and partly uncertain condition [1, 18-19]. This controller has two important subparts, switching part and equivalent part. Switching part of controller is used to design suitable tracking performance based on very fast switching. This part has essential role to have a good trajectory performance in all joints. However this part is very important in uncertain condition but it is caused to chattering phenomenon in system performance. Chattering phenomenon can cause some important mechanical problems [18]. The second subpart in sliding mode controller is equivalent part especially in uncertain condition. Sliding mod controller is a nonlinear model based controller and equivalent part is a dynamic formulation of robotic assistance in ophthalmic surgery, which is used in control formulation of robotic assistance in ophthalmic surgery, which is used in control formulation to eliminate the decoupling and nonlinear term of dynamic parameters of each link [15-18]. However, this part is very essential to reliability but in uncertain condition or highly nonlinear dynamic systems, it can cause some problem. However conventional sliding mode controller is a robust, stable and reliable controller but there are three main issues limiting; equivalent part related to dynamic equation of robotic assistance in ophthalmic surgery, computation of the bounds of uncertainties and chattering phenomenon.

In this research, the new technique of sliding mode controller is recommended, namely, baseline sliding mode controller. To modify the chattering in this research, model-free second order controller is recommended.

This paper is organized as follows; Section 2, is served as an introduction to the dynamic of spherical motor and sliding mode controller. Part 3 focuses on the design proposed methodology. Section 4 presents the simulation results and discussion of this algorithm applied to a spherical motor and the final section describe the conclusion.

2. Theory and Background

Dynamic and Kinematics Formulation of Spherical Joint in robotic assistance in ophthalmic surgery: Dynamic modeling of spherical joint robotic assistance in ophthalmic surgery is used to describe the behavior of spherical motor such as linear or nonlinear dynamic behavior, design of model based controller such as pure sliding mode controller which design this controller is based on nonlinear dynamic equations, and for simulation. The dynamic modeling describes the relationship between motion, velocity, and accelerations to force/torque or current/voltage and also it can be used to describe the particular dynamic effects (*e.g.*, inertia, coriolios, centrifugal, and the other parameters) to behavior of system[1-10]. Spherical joint is nonlinear and uncertain dynamic parameters and it is 3 degrees of freedom (DOF) electrical motor.

The equation of a spherical motor governed by the following equation [1-10]:

$$\boldsymbol{H(q)}\begin{bmatrix}\ddot{\alpha}\\ \ddot{\beta}\\ \ddot{\gamma}\end{bmatrix}+\mathbf{B}(\boldsymbol{q})\begin{bmatrix}\dot{\alpha}\dot{\beta}\\ \dot{\alpha}\dot{\gamma}\\ \dot{\beta}\dot{\gamma}\end{bmatrix}+\boldsymbol{C(q)}\begin{bmatrix}\dot{\alpha}^2\\ \dot{\beta}^2\\ \dot{\gamma}^2\end{bmatrix}=\begin{bmatrix}\tau_x\\ \tau_y\\ \tau_z\end{bmatrix} \quad \textbf{(1)}$$

Where τ is actuation torque, H (q) is a symmetric and positive define inertia matrix, B(q) is the matrix of coriolios torques, C(q) is the matrix of centrifugal torques.

This is a decoupled system with simple second order linear differential dynamics. In other words, the component $\ddot{q}$ influences, with a double integrator relationship, only the variableq_i, independently of the motion of the other parts. Therefore, the angular acceleration is found as to be [1-11]:

$$\ddot{\boldsymbol{q}}=\boldsymbol{H^{-1}(q)}.\{\boldsymbol{\tau}-\{\boldsymbol{B}+\boldsymbol{C}\}\} \quad \textbf{(2)}$$

This technique is very attractive from a control point of view.

Study of spherical motor is classified into two main groups: kinematics and dynamics. Calculate the relationship between rigid bodies and final part without any forces is called Kinematics. Study of this part is pivotal to design with an acceptable performance controller, and in real situations and practical applications. As expected the study of kinematics is divided into two main parts: forward and inverse kinematics. Forward kinematics has been used to find the position and orientation of task frame when angles of joints are known. Inverse kinematics has been used to find possible joints variable (angles) when all position and orientation of task frame be active [1].

According to the forward kinematics formulation;

$$\boldsymbol{\Psi(X,q)=0} \quad \textbf{(3)}$$

Where $\Psi(.)\in R^n$ is a nonlinear vector function, $X=[X_1,X_2,\dots\dots,X_l]^T$ is the vector of task space variables which generally task frame has three task space variables, three orientation, $q=[q_1,q_2,\dots.,q_n]^T$ is a vector of angles or displacement, and finally n is the number of actuated joints. The Denavit-Hartenberg (D-H) convention is a method of drawing spherical motor free body diagrams. Denvit-Hartenberg (D-H) convention study is necessary to calculate forward kinematics in this motor.

A systematic Forward Kinematics solution is the main target of this part. The first step to compute Forward Kinematics (F.K) is finding the standard D-H parameters. The following steps show the systematic derivation of the standard D-H parameters.

1. Locate the spherical motor
2. Label joints
3. Determine joint rotation (θ)
4. Setup base coordinate frames.
5. Setup joints coordinate frames.
6. Determineα_i, thatα_i, link twist, is the angle between Z_i and Z_{i+1}.

7. Determine d_i and a_i , that a_i, link length, is the distance between Z_i and Z_{i+1} along X_i. d_i, offset, is the distance between X_{i-1} and X_i along Z_i axis.
8. Fill up the D-H parameters table. The second step to compute Forward kinematics is finding the rotation matrix (R_n^0). The rotation matrix from$\{F_i\}$ to $\{F_{i-1}\}$ is given by the following equation;

$$\boldsymbol{R_i^{i-1} = U_{i(\theta_i)} V_{i(\alpha_i)}} \tag{4}$$

Where $U_{i(\theta_i)}$ is given by the following equation [1-11];

$$\boldsymbol{U_{i(\theta_i)}} = \begin{bmatrix} \boldsymbol{cos(\theta_i)} & \boldsymbol{-sin(\theta_i)} & \boldsymbol{0} \\ \boldsymbol{sin(\theta_i)} & \boldsymbol{cos(\theta_i)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1} \end{bmatrix} \tag{5}$$

and $V_{i(\alpha_i)}$ is given by the following equation [1-11];

$$\boldsymbol{V_{i(\theta_i)}} = \begin{bmatrix} \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{cos(\alpha_i)} & \boldsymbol{-sin(\alpha_i)} \\ \boldsymbol{0} & \boldsymbol{sin(\alpha_i)} & \boldsymbol{cos(\alpha_i)} \end{bmatrix} \tag{6}$$

So (R_n^0) is given by [8]

$$\boldsymbol{R_n^0 = (U_1V_1)(U_2V_2) \dots\dots\dots (U_nV_n)} \tag{7}$$

The final step to compute the forward kinematics is calculate the transformation ${}_n^0T$ by the following formulation [3]

$$\boldsymbol{{}_n^0T = {}_1^0T.{}_2^1T.{}_3^2T \dots\dots.{}_n^{n-1}T} = \begin{bmatrix} \boldsymbol{R_n^0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{1} \end{bmatrix} \tag{8}$$

Sliding mode controller: Sliding mode controller (SMC) is a powerful nonlinear controller which has been analyzed by many researchers especially in recent years. The main reason to opt for this controller is its acceptable control performance in wide range and solves two most important challenging topics in control which names, stability and robustness. Sliding mode control theory for control of robot manipulator used to solve the set point problem ($\boldsymbol{\dot{q}_d = 0}$) by discontinuous method (sliding surface) in the following form [12-14];

$$\boldsymbol{\tau_{(q,t)}} = \begin{cases} \boldsymbol{\tau_i^+(q,t)} & \boldsymbol{if\ S_i} > 0 \\ \boldsymbol{\tau_i^-(q,t)} & \boldsymbol{if\ S_i} < 0 \end{cases} \tag{9}$$

where $\boldsymbol{S_i}$ is sliding surface (switching surface), $\boldsymbol{i = 1, 2, \dots\dots, n}$ for n-DOF robot manipulator, $\boldsymbol{\tau_i(q,t)}$ is the $\boldsymbol{i^{th}}$ torque of joint. Sliding mode controller is divided into two main sub controllers: discontinues controller($\boldsymbol{\tau_{dis}}$) / sliding surface part and equivalent controller($\boldsymbol{\tau_{eq}}$). Discontinues controller/sliding surface technique causes an acceptable tracking performance at the expense of very fast switching. However this part of theory can improve the stability and robustness but caused to high frequency oscillations. High frequency oscillation can causes some problems such as saturation and heat the mechanical parts of robot manipulators or drivers. A time-varying sliding surface $s(x,t)$ in the state space R^n is given by [15]:

$$\boldsymbol{s(x,t) = (\frac{d}{dt} + \lambda)^{n-1}\,\tilde{x} = 0} \tag{10}$$

where λ is the positive constant. The main target in this methodology is kept the sliding surface slope $\boldsymbol{s(x,t)}$ near to the zero. Therefore, one of the common strategies is to find input $\boldsymbol{U}$ outside of $\boldsymbol{(x,t)}$ **.**

$$\boldsymbol{\frac{1}{2}\frac{d}{dt}s^2(x,t) \le -\zeta|s(x,t)|} \tag{11}$$

where ζ is positive constant.

$$\text{If } \mathbf{S(0)>0} \rightarrow \frac{\mathbf{d}}{\mathbf{dt}}\mathbf{S(t)} \le -\boldsymbol{\zeta} \tag{12}$$

To eliminate the derivative term, it is used an integral term from t=0 to t=$\boldsymbol{t_{reach}}$

$$\boldsymbol{\int_{t=0}^{t=t_{reach}} \frac{d}{dt}S(t) \le -\int_{t=0}^{t=t_{reach}} \eta \rightarrow S\,(t_{reach}) - S(0) \le -\zeta(t_{reach} - 0)} \tag{13}$$

Where t_{reach} is the time that trajectories reach to the sliding surface so, suppose S($t_{reach} = 0$) defined as

$$\boldsymbol{0 - S(0) \le -\eta(t_{reach}) \rightarrow t_{reach} \le \frac{S(0)}{\zeta}} \tag{14}$$

and

$$\boldsymbol{if\ S(0)} < 0 \rightarrow 0 - S(\mathbf{0}) \le -\eta(\boldsymbol{t_{reach}}) \rightarrow \boldsymbol{S(0)} \le -\zeta(\boldsymbol{t_{reach}}) \rightarrow \boldsymbol{t_{reach}} \le \frac{|\boldsymbol{S(0)}|}{\boldsymbol{\eta}} \tag{15}$$

Equation (15) guarantees time to reach the sliding surface is smaller than $\frac{|S(0)|}{\zeta}$ since the trajectories are outside of $S(t)$.

$$\boldsymbol{if\ S_{t_{reach}} = S(0) \rightarrow error(x - x_d) = 0} \tag{16}$$

A simple solution to get the sliding condition when the dynamic parameters have uncertainty is the switching control law:

$$\boldsymbol{U_{dis} = \hat{U} - K(\vec{x},t)\cdot \mathbf{sgn}(s)} \tag{17}$$

where the switching function $\mathbf{sgn(S)}$ is defined as

$$\boldsymbol{sgn(s)} = \begin{cases} \mathbf{1} & \boldsymbol{s} > 0 \\ \mathbf{-1} & \boldsymbol{s} < 0 \\ \mathbf{0} & \boldsymbol{s = 0} \end{cases} \tag{18}$$

and the $\boldsymbol{K(\vec{x},t)}$ is the positive constant.

According to above formulation, the formulation of sliding mode controller for spherical joint in robotic assistance ophthalmic surgery is [16-18];

$$\boldsymbol{\tau} = \boldsymbol{\tau_{eq}} + \boldsymbol{\tau_{dis}} \tag{19}$$

$\boldsymbol{\tau_{eq}}$ is equivalent term of sliding mode controller and this term is related to the nonlinear dynamic formulation of spherical joint in robotic assistance ophthalmic surgery. Conventional sliding mode controller is reliable controller based on the nonlinear dynamic formulation (equivalent part). The switching discontinuous part is introduced by $\boldsymbol{\tau_{dis}}$ and this item is the important factor to resistance and robust in this controller. In spherical joint in robotic assistance ophthalmic surgery the equivalent part is written as follows;

$$\boldsymbol{\tau_{eq}} = \left[\boldsymbol{H^{-1}}(\boldsymbol{q}) \times (\boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}})) + \dot{\boldsymbol{S}}\right] \times \boldsymbol{H}(\boldsymbol{q}) \tag{20}$$

The nonlinear term of $\boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is**;**

$$[\boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}})] = [\boldsymbol{V}(\boldsymbol{q},\dot{\boldsymbol{q}})] \tag{21}$$

$$[\boldsymbol{V}(\boldsymbol{q},\dot{\boldsymbol{q}})] = [\boldsymbol{b}(\boldsymbol{q})][\dot{\boldsymbol{q}}\,\dot{\boldsymbol{q}}] + [\boldsymbol{C}(\boldsymbol{q})][\dot{\boldsymbol{q}}]^{2} \tag{22}$$

In PD sliding surface, the change of sliding surface calculated as;

$$\boldsymbol{S_{PD}} = \boldsymbol{\lambda e} + \dot{\boldsymbol{e}} \rightarrow \dot{\boldsymbol{S}}_{\boldsymbol{PD}} = \boldsymbol{\lambda}\dot{\boldsymbol{e}} + \ddot{\mathrm{e}} \tag{23}$$

The discontinuous switching term ($\boldsymbol{\tau_{dis}}$) is computed as [16-18];

$$\boldsymbol{\tau_{dis}} = \boldsymbol{K} \cdot \mathbf{sgn}(\boldsymbol{S}) \tag{24}$$

$$\boldsymbol{\tau_{dis-PD}} = \boldsymbol{K} \cdot \mathbf{sgn}(\boldsymbol{\lambda e} + \dot{\boldsymbol{e}}) \tag{25}$$

$$\boldsymbol{\tau_{dis-PI}} = \boldsymbol{K} \cdot \mathbf{sgn}\left(\boldsymbol{\lambda e} + (\frac{\boldsymbol{\lambda}}{\mathbf{2}})^{2} \sum \boldsymbol{e}\right) \tag{26}$$

The discontinuous switching part is;

$$\boldsymbol{\tau_{dis-PID}} = \boldsymbol{K} \cdot \mathbf{sgn}\left(\boldsymbol{\lambda e} + \dot{\boldsymbol{e}} + (\frac{\boldsymbol{\lambda}}{\mathbf{2}})^{2} \sum \boldsymbol{e}\right) \tag{27}$$

$$\boldsymbol{\tau} = \boldsymbol{\tau_{eq}} + \boldsymbol{K}.\mathbf{sgn}(\boldsymbol{S}) = \left[\boldsymbol{H^{-1}}(\boldsymbol{q}) \times (\boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}})) + \dot{\boldsymbol{S}}\right] \times \boldsymbol{H}(\boldsymbol{q}) + \boldsymbol{K}.\mathbf{sgn}(\boldsymbol{S}) \tag{28}$$

The formulation of PD-SMC is;

$$\boldsymbol{\tau_{PD-SMC}} = \boldsymbol{K} \cdot \mathbf{sgn}(\boldsymbol{\lambda e} + \dot{\boldsymbol{e}}) + \left[\boldsymbol{H^{-1}}(\boldsymbol{q}) \times (\boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}})) + \dot{\boldsymbol{S}}\right] \times \boldsymbol{H}(\boldsymbol{q}) \tag{29}$$

3. Methodology

Proportional algorithm is used to responds immediately to difference of control input variables by immediately changing its influences variables, but this type of control is unable to eliminate the control input difference.

Proportional and Derivative (PD) controller is widely used in control process where the results are sensitive to exceeded of set point. This controller, like Proportional controller, has permanent variation in presence of self-limitation control. In mathematically, the formulation of Proportional-Derivative part calculated as follows;

$$\boldsymbol{U_{PD}} = \boldsymbol{K_p} \times \boldsymbol{e} + \boldsymbol{K_v}(\frac{\boldsymbol{de}}{\boldsymbol{dt}}) = \boldsymbol{K_p} \times \boldsymbol{e} + \boldsymbol{K_v}\dot{\boldsymbol{e}} \tag{30}$$

The Derivative component in this type of methodology is used to cancel outs the change process variables change in presence of quick change in controllers input.

Integral algorithm, integrate the input signal deviation over a period of time. This part of controller is used to system stability after a long period of time. In contrast of Proportional type of controller, this type of controller used to eliminate the deviation. In mathematically, the formulation of integral part calculated as follows;

$$\boldsymbol{I} = \frac{\boldsymbol{1}}{\boldsymbol{T}}\int \boldsymbol{e}.\boldsymbol{dt} = \sum \boldsymbol{e} \tag{31}$$

According to integral type of controller, it takes relatively long time. The proportional type controller used to immediately response to the input variations. The proportional-integral (PI) controller has the advantages of both proportional and integral controller; it is rapid response to the input deviation as well as the exact control at the desired input.

$$\boldsymbol{U_{PI}} = \boldsymbol{K_p} \times \boldsymbol{e} + \boldsymbol{K_i}(\frac{\boldsymbol{1}}{\boldsymbol{T}}\int \boldsymbol{e}.\boldsymbol{dt}) = \boldsymbol{K_p} \times \boldsymbol{e} + \boldsymbol{K_i}\sum \boldsymbol{e} \tag{32}$$

The combination of proportional (P) component, integral (I) component with a derivative (D) controller offered advantages in each case. This type of controller has rapid response to the input deviation, the exact control at the desired input as well as fast response to the disturbances. The PID controller takes the error between the desired joint variables and the actual joint variables to control the nonlinear system. This method does not provide sufficient control for systems with time-varying parameters or highly nonlinear systems. The formulation of PID controller calculated as follows;

$$\boldsymbol{U_{PID}} = \boldsymbol{K_p} \times \boldsymbol{e} + \boldsymbol{K_i}(\frac{\boldsymbol{1}}{\boldsymbol{T}}\int \boldsymbol{e}.\boldsymbol{dt}) + \boldsymbol{K_v}(\frac{\boldsymbol{de}}{\boldsymbol{dt}}) = \boldsymbol{K_p} \times \boldsymbol{e} + \boldsymbol{K_i}\sum \boldsymbol{e} + \boldsymbol{K_v}\dot{\boldsymbol{e}} \tag{33}$$

In this research to improve the chattering performance as well nonlinearity challenge $(PID)^2$ controller is introduced.

$$(U_{PID})^2 = (K_p \times e + K_i \sum e + K_v \dot{e}) \times (K_p \times e + K_i \sum e + K_v \dot{e}) \tag{34}$$

Therefore;

$$U_{dis-PD-New} = K \cdot \mathrm{sgn}(\lambda e + \dot{e}) + K_v . sgn\,(U_{PID})^2 \tag{35}$$

This method used to attenuate chattering and improve the nonlinearity performance using an auxiliary sliding variable.

This method is robust and stable. It is more robust than conventional sliding mode controller according to switching function for PD type and $(\mathrm{PID})^2$ type.

The proof of Lyapunov function can be determined by the following equations. The dynamic formulation of multi DOF joints for robotic assistance ophthalmic surgery can be written by the following equation

$$\tau = H(q)\ddot{q} + V(q,\dot{q})\dot{q} \tag{36}$$

the Lyapunov formulation can be written as follows,

$$V = \frac{1}{2} S^T . H . S \tag{37}$$

the derivation of V can be determined as,

$$\dot{V} = \frac{1}{2} S^T . \dot{H} . S + S^T H \dot{S} \tag{38}$$

the dynamic equation of multi DOF joints for robotic assistance ophthalmic surgery can be written based on the sliding surface as

$$H\dot{S} = -VS + H\dot{S} + VS - \tau \tag{39}$$

it is assumed that

$$S^T(\dot{H} - 2V)S = 0 \tag{40}$$

$$\dot{V} = \frac{1}{2} S^T \dot{H} S - S^T V S + S^T(H\dot{S} + VS - \tau) = S^T(H\dot{S} + VS - \tau) \tag{41}$$

suppose the control input is written as follows

$$\hat{\tau} = \widehat{\tau_{eq}} + \widehat{\tau_{dis}} = \left[\widehat{H^{-1}}(\hat{V}) + \dot{S}\right]\hat{H} + K.sgn(S_1) + K_v sgn(S_2) \tag{42}$$

$$S_1 = (\lambda e + \dot{e})$$
$$S_2 = K_v . sgn[(K_p \times e + K_i \sum e + K_v \dot{e}) \times (K_p \times e + K_i \sum e + K_v \dot{e})]$$
$$\dot{V} = S^T(H\dot{S} + VS - \hat{H}\dot{S} - \hat{V}S - K_v sgn(S_2) - K sgn(S_1) = S^T\left(\tilde{H}\dot{S} + \tilde{V}S - K_v sgn(S_2) - K sgn(S_1)\right) \tag{43}$$

it is obvious that

(44)

$$\left|\widetilde{H}\dot{S} + \widetilde{V}S - K_v sgn(S_2) - Ksgn(S_1)\right| \\ \le \left|\widetilde{H}\dot{S}\right| + \left|\widetilde{V}S\right| + |K_v sgn(S_2)| + |Ksgn(S_1)|$$

the Lemma equation can be written as follows

$$K_u = \left[\left|\widetilde{H}\dot{S}\right| + \left|\widetilde{V}S\right| + |K_v sgn(S_2)| + |Ksgn(S_1)| + \eta\right]_i, i = 1,2,3,4,\dots \quad \textbf{(45)}$$

$$K_u \ge \left|\left[\widetilde{H}\dot{S} + \widetilde{V}S - K_v sgn(S_2) - Ksgn(S_1)\right]_i\right| + \eta_i \quad \textbf{(46)}$$

Therefore

$$\dot{V} \le -\sum_{i=1}^{n} \eta_i\, |S_i| \quad \textbf{(47)}$$

Consequently, the equation (47) guaranties the stability of the Lyapunov equation.

4. Result and Discussion

The tracking accuracy in proposed method shows in Figure 1. According to this Figure, it can follow the desired trajectory with minimum error.

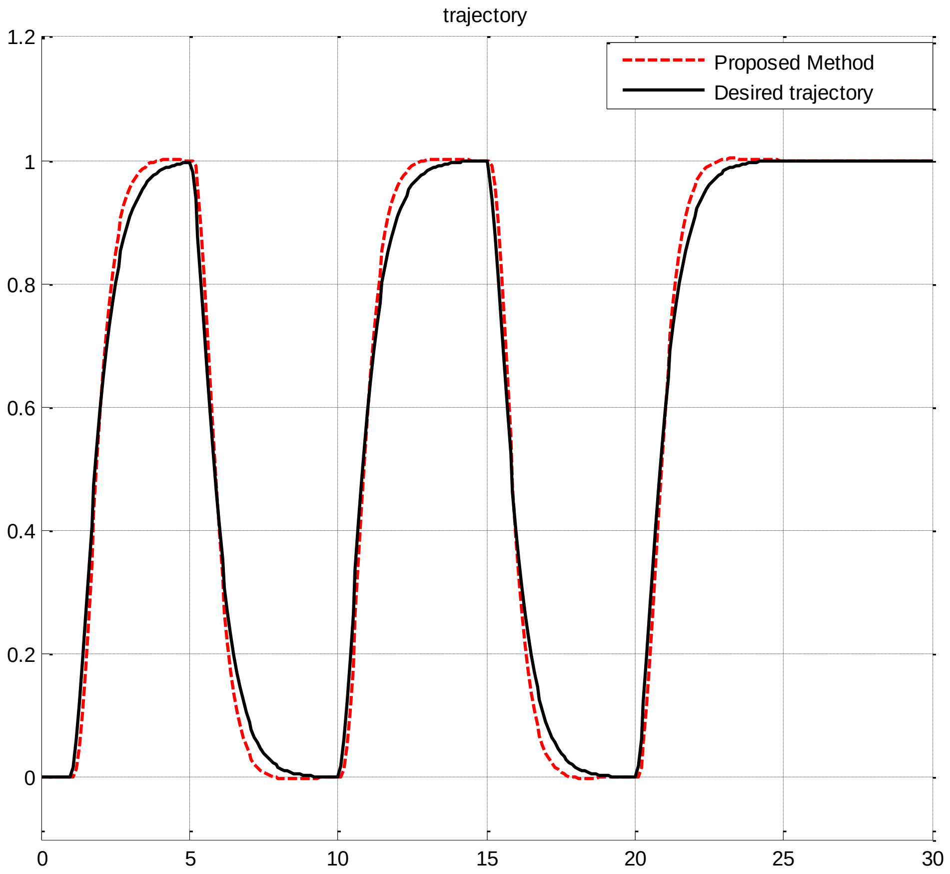

Figure 1. Trajectory Follows

Figure 2 shows the sliding surface in proposed controller. According to following graph, proposed control technique has stable sliding surface.

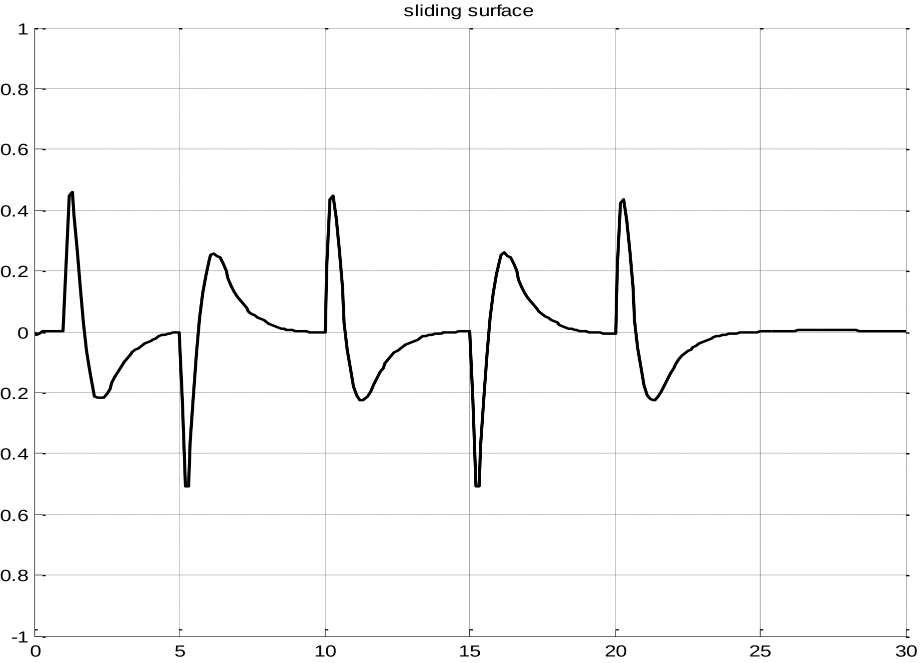

Figure 2. Sliding Surface

The control input, forces the multi DOF joints to track the desired trajectories. Figure 3 shows the torque performance in proposed controller. According to the following graph, proposed method eliminates the chattering with respect to reduce the energy level.

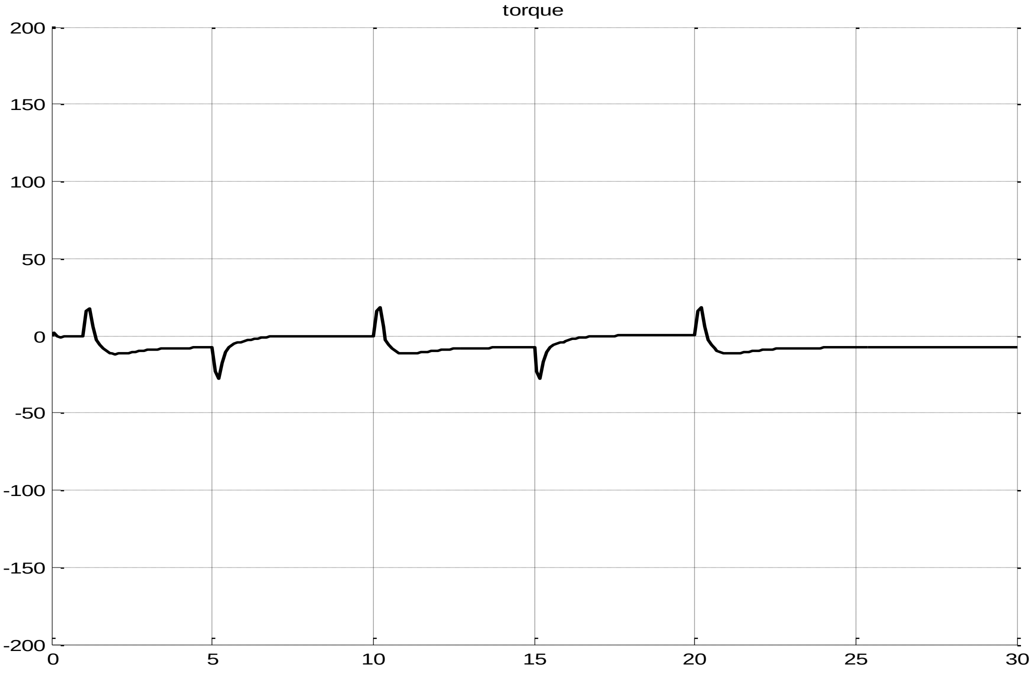

Figure 3. Torque Performance

Tracking error is used to test the controller joint variable accuracy. Figure 4 shows the steady state error in proposed controller. According to this Figure, proposed controller has steady stable.

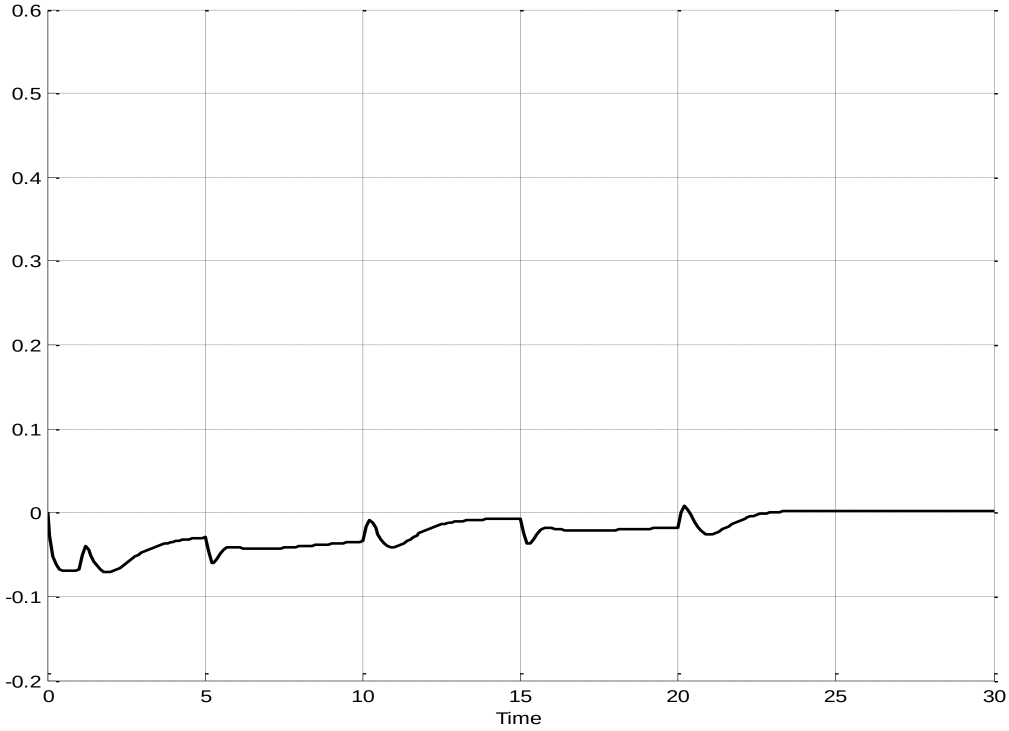

Figure 4. Error Performance

5. Conclusion

Refer to this research, auxiliary sliding variable sliding mode controller is proposed for multi DOF joint with application in surgical robot manipulator. The main problem in pure sliding mode controller was chattering phenomenon in certain and uncertain systems. The chattering phenomenon problem is reduced in certain/uncertain system by using auxiliary sliding variable. The simulation results exhibit that the sliding mode controller with auxiliary sliding variable works well in certain and uncertain condition.

The first challenge to design robust and stable sliding mode controller based on switching function is chattering phenomenon. In sliding mode controller select the desired sliding surface and $sign$ function play a vital role to system performance and if the dynamic of multi DOF joint is derived to sliding surface then the linearization and decoupling through the use of feedback, not gears, can be realized. As a result, uncertainties are estimated by discontinuous feedback control but it can cause to chattering. To reduce the chattering in presence of switching functions; nonlinear model-free controller is added to discontinuous part of sliding mode controller. Linear controller is type of stable controller as well as conventional sliding mode controller. In proposed methodology PID^2 nonlinear controller is used in parallel with discontinuous part to reduce the role of sliding surface slope as a main coefficient.

Acknowledgment

The authors would like to thank the anonymous reviewers for their careful reading of this paper and for their helpful comments. This work was supported by the Iranian

Institute of Advance Science and Technology Program of Iran under grant no. **2014-Persian Gulf-1A**.

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