Comparative Study between Wavelet Thresholding Techniques (Hard, Soft and Invariant-translation) in Ultrasound Images

R. Bouchouareb and D. Benatia

University of Batna, Faculty of Engineering Sciences, Department of Electronic, Batna, ALGERIA
rachidabouchouareb@gmail.com

Abstract

In this paper method has been applied in the speckle noise reduction in medical imaging. Speckle Noise reduces the ability of an observer to distinguish fine details in diagnostic testing. It also limits the effective implementation of image processing such as edge detection, segmentation and volume rendering in 3 D. Therefore; treatment methods of speckle noise were sought to improve the image quality and to increase the capacity of diagnostic medical ultrasound images.

The methods we used in this work are new, they are used in image processing, among their applications noise reduction in medical imaging.

The use of wavelets has brought a new breath in the field of signal and image processing since its appearance. They make it possible to analyze and identify discontinuities of a signal to one or two dimensions and at different scales. This feature is used for image denoising.

In the field of medical imaging, denoising seeks to discern relevant information in the medical field as the shape, contour, etc. Thresholding by wavelet, we can denoise an image by eliminating fine details.

Keywords: Wavelets transform, image processing, ultrasound image, filtering

1. Introduction

In the literature of image processing, different denoising methods have been proposed and developed. First, the spatial filtering methods [1, 2], these are to reduce noise in areas that do not exhibit artifacts and enhance the perception of structures of interest. These filtering techniques use a low-pass filter to remove high frequencies, which has the disadvantage mitigate the edges of the image. To overcome these problems, new techniques, more efficient, have emerged in the years 80 and 90; statements include approaches based on PDEs variational [3, 4, 5], approaches using Markov random fields and approaches based on multiscale transforms, including wavelet transform. Recently, these approaches have demonstrated their power in the context of the statistical estimation. Through these sparse transformed, the energy of the desired signal is concentrated on a small number of coefficients, thereby providing a non-linear natural to estimate the signal. Indeed, simply thresholding the coefficients of the observed image and reverse transformed to obtain an estimate of the useful signal [6].

One solution is the use of wavelet bases in image processing has become widespread during the last twenty years [7]. Interest in compression and noise reduction has been shown since they have integrated the latest compression standard JPEG 2000 digital images. Their application to medical imaging dates back to 1992 and has spread widely since [8, 9]. In this context, the wavelets are used for compression and noise reduction, but also for the functional analysis of medical data (for diagnosis), the local tomography, segmentation and image enhancement, or the description of texture [10].

2. Denoising Using Wavelet Transform

Denoising can be viewed as an estimation problem trying to recover a true signal component X from an observation Y where the signal component has been degraded by a noise component N:

$$X = Y + N \qquad (1)$$

The estimation is computed with a thresholding estimator in an orthonormal basis:

$$\hat{X} = \sum_{m=0}^{N-1} \rho_m \big(\langle X, g_m \rangle \big) g_m \qquad (2)$$

Where ρ_m is a thresholding function that aims at eliminating noise components (via attenuating of decreasing some coefficient sets) in the transform domain while preserving the true coefficients of the signal. If the function ρ_m is modified to rather preserve or increase coefficient values in the transform domain, it is possible to enhance some features of interest in the true signal component with the framework of Equation (2).

Figure (1) illustrates a multi-scale enhancement and de-noising framework using wavelet transforms. An over complete dyadic wavelet transform using bi-orthogonal basis is used. Notice that since the DC-cap contains the overall energy distribution, it is usually kept untouched during the procedure. As shown in this figure, thresholding and enhancement functions can be implemented independently from the wavelet filters and easily incorporated into the filter bank framework
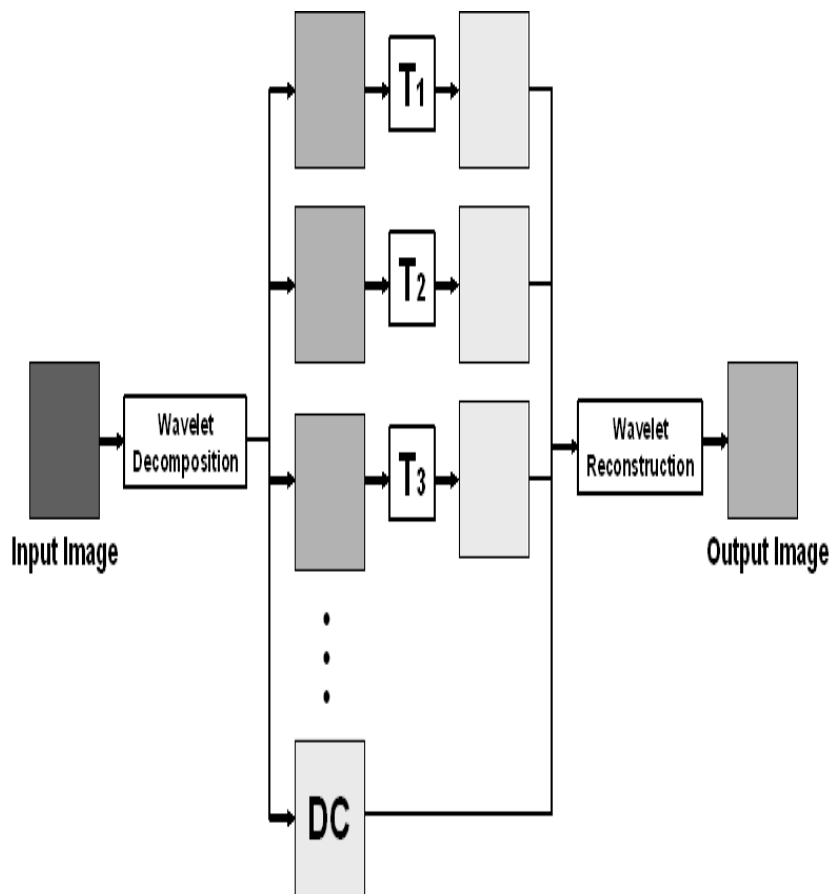
Figure 1. A Multi-scale Framework of De-noising and Enhancement using Discrete Dyadic Wavelet Transform. A Three Level Decomposition was

Shown

3. Thresholding Operators for De-noising

As a general rule, wavelet coefficients with larger magnitude are correlated with salient features in the image data. In that context, de-noising can be achieved by applying a thresholding operator to the wavelet coefficients (in the transform domain) followed by reconstruction of the signal to the original image (spatial domain).

Typical threshold operators for de-noising include:

hard thresholding:

$$\rho_T(x) = \begin{cases} x, si\ |x| > T \\ 0, si\ |x| \leq T \end{cases} ; \tag{3}$$

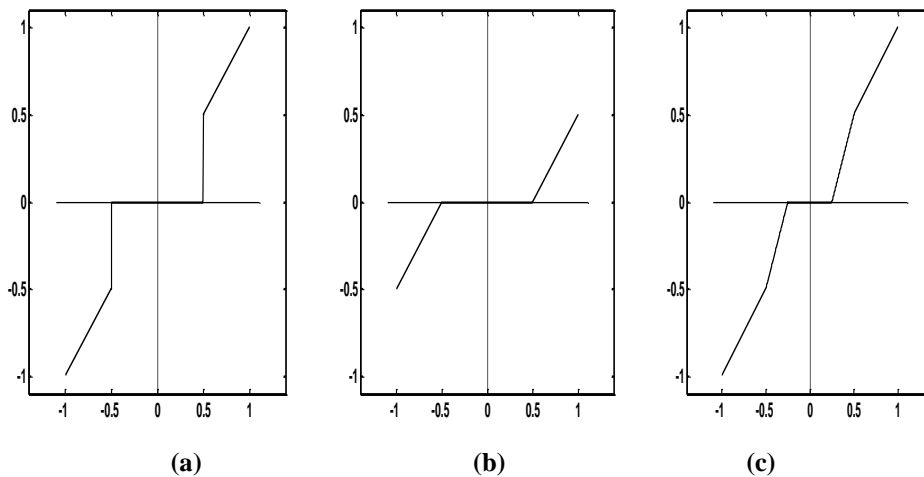soft thresholding (wavelet shrinkage)

$$\rho_T(x) = \begin{cases} x - T, si\ x \geq T \\ x + T, si\ x \leq -T \\ 0\ , si\ |x| < T \end{cases} \tag{4}$$

and affine(firm) thresholding:

$$\rho_T(x) = \begin{cases} x, si\ |x| \geq T \\ 2x + T, si -T \leq x \leq -T/2 \\ 2x - T, si\ T/2 \leq x \leq T \\ 0, si\quad |x| < T \end{cases} \tag{5}$$

The shapes of these thresholding operators are illustrated in Figure Figure (2).

(a)　　　　　　　　(b)　　　　　　　　(c)

Figure 2. Example of Thresholding Functions, Assuming that the Input Data was Normalized to the Range of [-1,1]. (a) Hard Thresholding. (b) Soft Thresholding. (c) Affine Thresholding. The Threshold Level was Set to *T*=0.5

4. Selection of Threshold Value

Given the basic framework of de-noising using wavelet thresholding as discussed in the previous sections, it is clear that the threshold level parameter T plays an essential role. Values too small cannot effectively get rid of noise component, while values too

large will eliminate useful signal components. There are a variety of ways to determine the threshold value T as we will discuss in this section.

Depending on whether or not the threshold value T changes across wavelet scales and spatial locations, the thresholding can be:

1. **Global Threshold:** a single value T is to be applied globally to all empirical wavelet coefficients at different scales *T=const.*

2. **Level-Dependent Threshold:** a different threshold value T is selected for each wavelet analysis level (scale). *T=T(j),j=1,...,J.* J is the coarsest level for wavelet expansion to be processed.

3. **Spatial Adaptive Threshold:** the threshold value T varies spatially depending on local properties of individual wavelet coefficients. Usually, T is also level-dependent. *T=Tj (x,y,z).*

While a simple way of determining T is a percentage of coefficients maxima, there are different adaptive ways of assigning the T value according to the noise level (estimated via its variance σ):

1. **Universal Threshold:** $T = \sigma\sqrt{2\log n}$, with n equal to the sample size. This threshold was determined in an optimal context for soft thresholding with random Gaussian noise. This scheme is very easy implement, but typically provides a threshold level larger than with other decision criteria, therefore resulting in smoother reconstructed data. Also such estimation does not take into account the content of the data, but only depends on the data size.

2. **Minimax Threshold:** $T=\sigma Tn$, where *Tn* is determined by a minimax rule such that the maximum risk of estimation error across all locations of the data is minimized. This threshold level depends on the noise and signal relationships in the input data.

3. **Stein Unbiased Estimated of Risk (SURE):** Similar as minimax threshold but *Tn* is determined by a different risk rule.

4. **Spatial Adaptive Threshold:** $T=\sigma^2/\sigma x$, where σx is the local variance of the observation signal, which can be estimated using a local window moving across the image data or, more accurately, by a context-based clustering algorithm.

In many automatic de-noising methods to determine the threshold value T, an estimation of the noise variance σ is needed. Donoho and Johnstone proposed a robust estimation of noise level σ based on the median absolute value of the wavelet coefficients as:

$$\sigma = \frac{median(|W_1(x,y,z)|)}{0.6745}$$
(6)

Where W_1 is the most detailed level of wavelet coefficients. Such estimator has become very popular in practice and is widely used [11].

5. Experiment Results

For learning, a species of ultrasound images 292x400 pixels is used. we load our image and resize it where (N=256) is the number of pixels which consist of two different grey level values, and we add White Gaussian Noise with different values of sigma to obtain denoised one.

To determine the threshold value T, the assignment of the value of noise as a function of T (through its variance σ) is applied to different types of thresholding.

For this method, the orthogonal 2D wavelet transform on the noisy image is applied by adding a white Gaussian noise with different values of sigma as shows the figures (3 and 4) and the reconstruction using hard and soft thresholding as shows the figures (5) and (6).

We try to compare the results obtained using the translation invariant wavelet transform applied to a noisy and binary image. The results are shown in the figure (7) and (8).

To measure the quality between the original image and the filtered, another comparison is performed by calculating the PSNR calculates the peak signal- to- noise ratio between two images. The higher PSNR means the quality of the image treated is better. The PSNR is calculated using the following equation:

$$PSNR = 10 log_{10}\left(\frac{R^2}{MSE}\right)$$

(7)

In the above equation, R is the maximum variation in the data type of the input image. For example, if the input image is double, then R is 1. If it is of unit 8, R is 255. MSE is the mean square error (MSE) which is the square of the accumulated error between the original with noise and filtered image. The lower value of MSE means the error is small. MSE is calculated using the following equation:
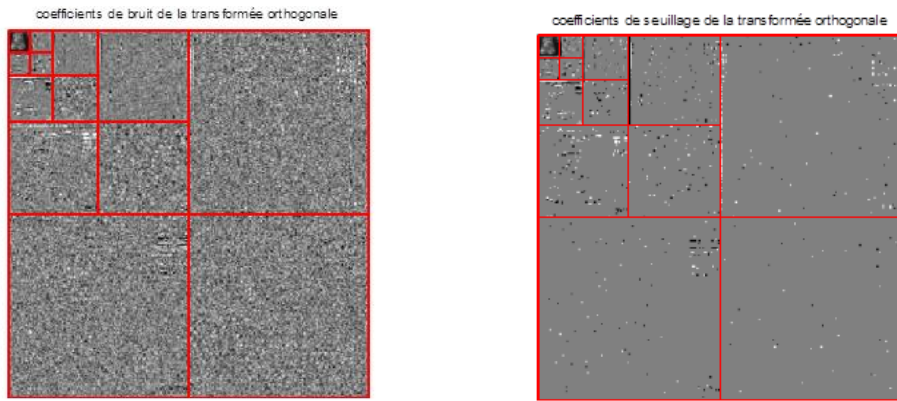
$$MSE = \frac{\sum_{M,N}[I_1(m,n) - I_2(m,n)]^2}{M \cdot N}$$

(8)

In the above equation, M and N are the number of rows and columns in the input images, respectively.

The mean square error (MSE) and peak ratio of signal to noise (PSNR) are the two error parameters used to compare the quality of image compression.
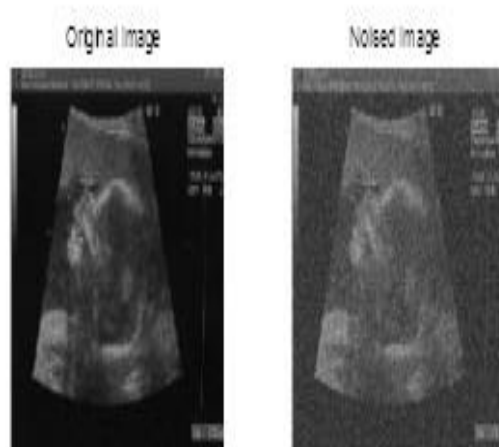
Figure 3. Original and Noised Image

Figure 4. Orthogonal 2D Wavelet Transform for the Noised and Denoised Image

Figure 5. Denoised Image (Hard Thresholding)

Figure 6. Denoised Image (Soft Thresholding)

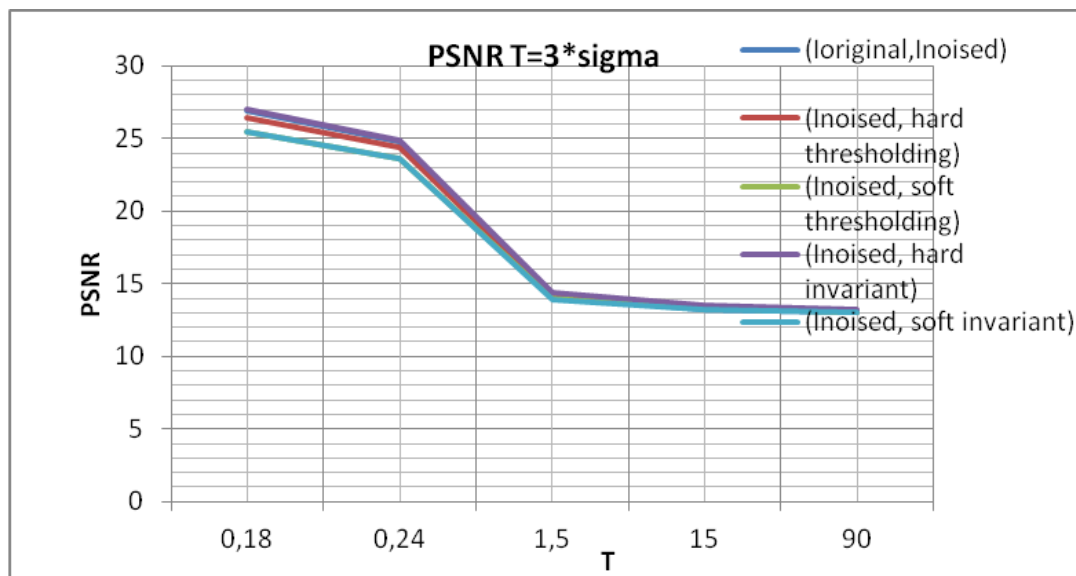**Figure 7. Denoised Image (Soft Invariant)**

Figure 8. Denoised Image (Hard Invariant)

The comparison results are shown in the following tables and figures

Table 1. Comparaison between Original Image, Noised Image, Denoised by Wavelet (Hard and Soft Thresholding, Hard and Soft Invariant Thresholding) for T=3*Sigma

Images	PSNR Sigma=0.06	PSNR Sigma=0.08	PSNR Sigma=0.5	PSNR Sigma=5	PSNR Sigma=30
(Ioriginal,Inoised)	26.9600	24.7846	14.1023	13.2700	13.0038
(Inoised,hard thresholding) T=3*sigma	26.4067	24.3789	14.1937	13.3995	13.1472
(Inoised, soft thresholding) T=3*sigma	25.4317	23.6670	14.0398	13.3167	13.0505
(Inoised ,hard invariant) T=3*sigma	27.0244	24.8893	14.3479	13.5299	13.2727
(Inoised, soft thresholding) T=3*sigma	25.4943	23.6629	13.9070	13.2766	13.0320
T=	0.18	0.24	1.5	15	90

Figure 9. Comparaison between Original Image, Noised Image, Denoised by Wavelet (Hard and Soft Thresholding, Hard and Soft Invariant Thresholding) for T=3*Sigma
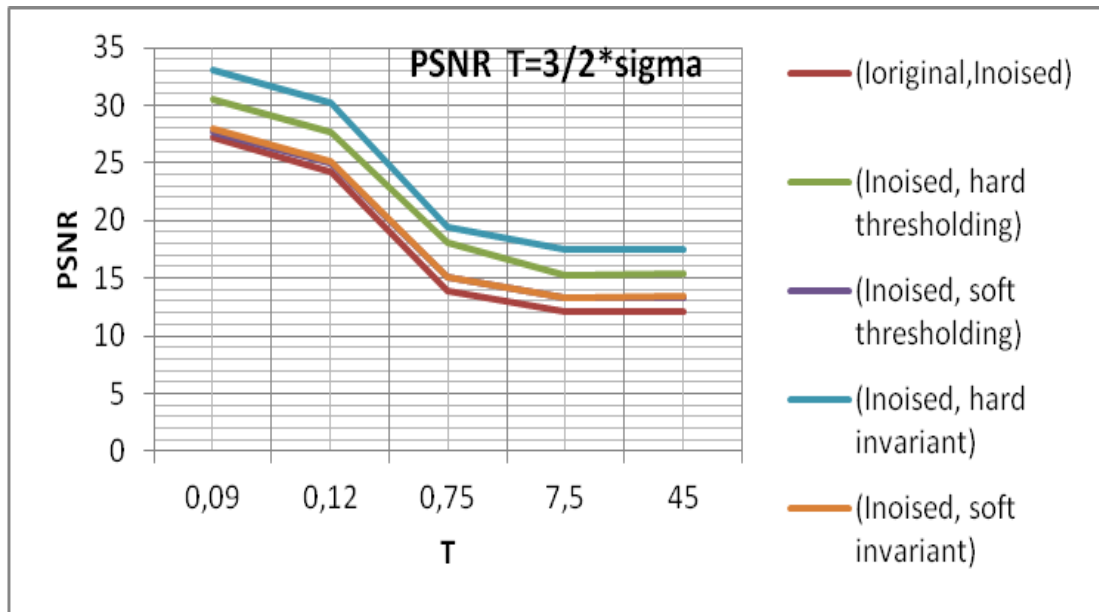
Table 2. Comparaison between Original Image, Noised Image, Denoised by Wavelet (Hard and Soft Thresholding, Hard and Soft Invariant Thresholding) for T=3/2*Sigma

| Images	PSNR Sigma=0.06	PSNR Sigma=0.08	PSNR Sigma=0.5	PSNR Sigma=5	PSNR Sigma=30
(Ioriginal,Inoised)	27.2259	24.3068	13.9578	12.0939	12.1704
(Inoised,hard thresholding) T=3/2*sigma	30.5460	27.6648	18.0594	15.3027	15.3385
(Inoised,soft thresholding) T=3/2*sigma	27.6645	24.9268	15.0579	13.2681	13.3300
(Inoised,hard invariant) T=3/2*sigma	33.1299	30.1836	19.3994	17.4704	17.4810
(Inoised,soft invariant) T=3/2*sigma	27.9316	25.1489	15.1528	13.3606	13.4196
T=	0.09	0.12	0.75	7.5	45

Figure 10. Comparaison between Original Image, Noised Image, Denoised by Wavelet (Hard and Soft Thresholding, Hard and Soft Invariant Thresholding) for T=3/2*Sigma

Table 3. Comparaison between Original Image, Noised Image, Denoised by Wavelet (Hard and Soft Thresholding, Hard and Soft Invariant Thresholding) for T=3.5*Sigma

Images	PSNR Sigma=0.06	PSNR Sigma=0.08	PSNR Sigma=0.5	PSNR Sigma=5	PSNR Sigma=30
(Ioriginal,Inoised)	26.8297	24.8828	14.8728	12.4559	13.0827
(Inoised,hard thresholding) T=3.5*sigma	26.2843	24.2029	14.8325	12.4717	13.1069
(Inoised, soft thresholding) T=3.5*sigma	25.0694	23.5698	14.7811	12.4771	13.1022
(Inoised,hard invariant) T=3.5*sigma	26.4672	24.5747	14.9014	12.5009	13.1503
(Ibruitée,invariante douce) T=3.5*sigma	25.0928	23.5197	14.5937	12.4335	13.0885
T=	0.21	0.28	1.75	17.5	105
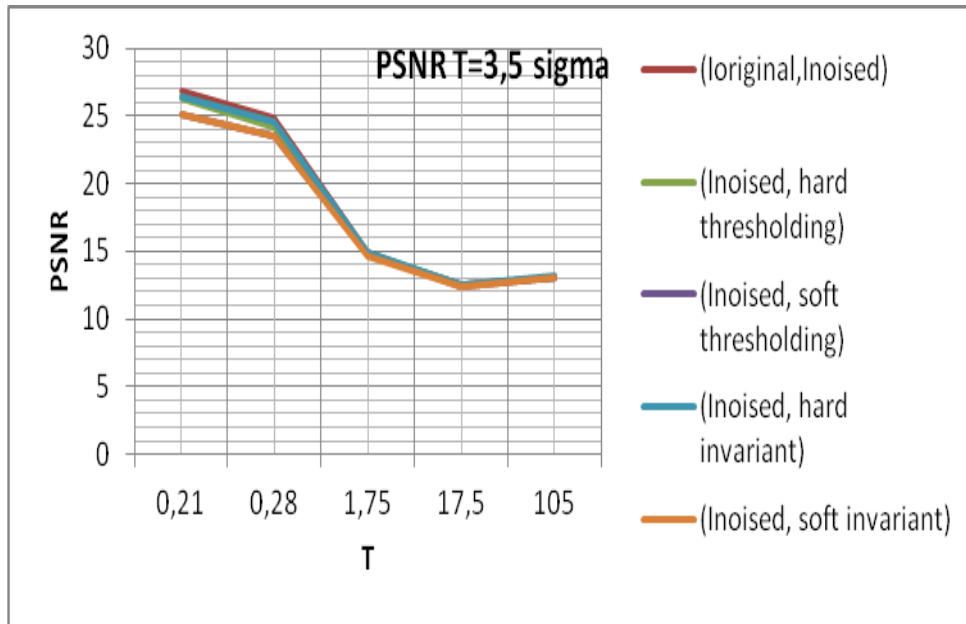
Figure 11. Comparaison between Original Image, Noised Image, Denoised by Wavelet (Hard and Soft Thresholding, Hard and Soft Invariant Thresholding) for T=3.5*Sigma

After the comparison between the results obtained when we have used different values of sigma, It is clear that the value of PSNR is high for low values of T, this is for sigma=0.06 in the case of the comparaison between (noised Image and hard Invariant Denoising) whereas high values of T this technique will not be efficient, so we apply other filters such as median filter. This means that image quality is better using the technique of invariant translation denoising than other thresholding.

6. Conclusion

We have discussed in this paper the use of wavelet thresholding techniques in reducing noise which has a bad effect on medical imaging thus the degradation of its quality.

This manuscript is organized in five sections. Section 1 presents an introduction of the medical imaging and the methods used in reducing the noise. Section 2 describes the basics of wavelets when used as filter. The thresholding operators for denoising are studied in Section 3 and the selections of threshold value are represented in Section 4. In Section 5 we have applied the dwt2d and thresholdine wavelet techniques on the medical image and were completed by comparing the results obtained. Section 6 draws the conclusion of this study.

In this paper we have demonstrated that the use of translation invariant wavelet could offer better results than hrd hard and soft thresholding.

References

[1]   T. Aksenova, A. Barachant, and S. Bonnet, "Spatial Filtering Optimisation in Motor Imagery EEG-Based BCI", Neurocomp, **(2008)**.
[2]   W. Yijun, G. Shangkai, and G. Xiaorong, "Common Spatial Pattern Method for Channel selelction in Motor Imagery Based Brain-Computer Interface", Engineering in Medicine and Biology Society, **(2005)**.
[3]   H. Attouch, G. Buttazzo and G. Michaille, "Variational Analysis in Sobolev and BV Spaces", MPS-SIAM Series on Optimization, **(2006)**.
[4]   D. Azé, Dominique et J.B. Hiriart-Urruty, "Analyse variationnelle et optimisation: éléments de cours, exercices et problèmes corrigés", Cépaduès-Éd., **(2010)**.

[5]   A. Chambolle, "An Algorithm for Total Variation Minimization and Applications", JMIV, 20, **(2004)**, pp 89-97.

[6]   L. Boubchir, "Approches Bayésiennes pour le Débruitage des Images dans le Domaine des Transformées Multi-échelles Parcimonieuses Orientées et Non Orientées", Thèse en Doctorat, Université de Caen, **(2007)**.

[7]   A. Pizurica, W. Philips, I. Lemahieu. "A versatile wavelet domain noise filtration technique for medical imaging", Medical Imaging, IEEE Transactions on, vol. 22, no 3, **(2003)**, p. 323-331.

[8]   A. Aldroubi and M. Unser eds, "Wavelets in Medicine and Biology", CRC Press, **(1996)**.

[9]   M. Unser, A. Aldroubi and A. Laine eds, "Wavelets in Medical Imaging", Special Issue on Wavelets in Medical Imaging, vol.22, no. 3, **(2013)**, pp. 285-288.

[10]   P. Valérie, A. Bilgot, O. Le Cadet, and L. Desbat, "Transformée en Ondelettes Continue Directionnelle : applications en Imagerie Médicale", ESAIM Proceeding, Orsay, France, vol. 18, **(2007)**, pp.16–228.

[11]   Y.Jin, E.Angelini, and A.Laine, "Wavelets in Medical Image Processing: De-noising, Segmentation, and Registration", Department of Biomedical Engineering, Columbia University, **(2005)**.

Author

R. BOUCHOUAREB received the Engineering studies from the Department of electrical engineering, BATNA University, ALGERIA in 2002.The Master degree in Telecommunication from Department of Electrical Engineering, BATNA University, ALGERIA, in 2010. Currently, she is a student Ph.D. in Electrical Engineering, BATNA University, ALGERIA. rachidabouchouareb@gmail.com.