An Ecological Ammensalism with Multifarious restraints - A Numerical Study

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Abstract

The paper purports to examine a mathematical model of An Ecological Ammensalism with multifarious restraints with the aid of classical method of Rk method of fourth order. The mathematical model consists of Ammensal-enemy species pair with cover for Ammensal, alternative resources for enemy and migrating for both the species.The model is characterized by a couple of first order non linear ordinary differential equations.The realtion betwenn the carrying capacity of Ammensal species and the dominance reversal time is identified.Some results are obtained from the relationship between cover protected constant of Ammensal species and the dominance reversal time.

Key words: *Non-linear system, Ammensal species, Enemy species, Carrying capacity, Dominance reversal time, Stability*.

AMS classificates: 92D25, 92D40

1. Introduction

Kapur J.N [15,16] investigated various mathematical models in Biology and Medicine with detailed information. Later N.C.Srinivas [18] discovered few competitive models to dissolve the complicated real life situations. Lakshmi Narayan with N.Ch.pattbahi Ramacharulu [17] enriched the competitive mathematical models with computational techniques. K.V.L.N.Acharyuly and N.Ch.Pattabhiramacharyulu [1-14] looked into multifaceted mathematical models in ecological system of Ammensal and Enemy species.

The authors employed the classical RK method of fourth order to this model for identifying the relations among Carrying capacity, Cover protected constant and Dominance reversal time. The present paper is a numerical study to examine a mathematical model of Ecological Ammensalism with multifarious restraints with the help of classical method of RK- method of fourth order. The mathematical model comprises Ammensal-enemy species pair with cover for Ammensal, alternative resources for enemy and migrating for both the species. The model is characterized by a couple of first order non linear ordinary differential equations. The relation between the carrying capacity of Ammensal species and the dominance reversal time is identified. Few results are obtained from the relationship between cover protected constant of Ammensal species and the dominance reversal time. The interactions are observed by changing the value of one variable while fixing all other parameters. The dominance reversal time is found in all possible cases. The figures are illustrated with the help of Matlab wherever needed. The results are explained along with the conclusions.

Notation Adopted:

N_1 (t) : The population of the species S_1 at time t
N_2 (t) : The population of the species S_2 at time t
a_i : The natural growth rates of S_i, i = 1, 2.
a_{ii} : The rate of decrease of S_i; due to its own insufficient resources ,i=1,2.
a_{12} : The inhibition coefficient of S_1 due to S_2 i.e The Ammensal coefficient.
a_{21} : The inhibition coefficient of S_2 due to S_1
H_1(t) : The replenishment or renewal of S_1 per unit time
H_2(t) : The replenishment or renewal of S_2 per unit time
K_i : a_i/a_{ii} are the carrying capacity of N_i, i = 1, 2.
α : a_{12}/a_{11} is the coefficient of Ammensalism.
h_1 : a_{11} H_1 is the rate of harvest of the Ammensal
h_2 : a_{22} H_2 is the rate of harvest of the enemy.
m_1 : Rate of decrease of the Ammensal due to harvesting.
m_2 : Rate of decrease of the enemy due to harvesting.
m : a constant characterized by the cover provided for the Ammensal species.

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{22}, K_1, K_2, α , h_1, h_2, m_1, m_2 and m are assumed to be non-negative constants.

2. The Basic Model Equations:

The model equations for a two species ecological Ammensalism is constucted by the following system of non-linear ordinary different equations.

$$\frac{dN_1(t)}{dt} = (1-m_1)a_1N_1(t) - a_{11}N_1^2(t) - (1-m)a_{12}N_1(t)N_2(t) - h_1(t) \qquad (1)$$

$$\frac{dN_2(t)}{dt} = (1-m_2)a_2N_2(t) - a_{22}N_2^2(t) - h_2(t) \qquad (2)$$

with the conditions $N_i(0) = N_{i0} \geq 0, i = 1,2$;

3. Numerical Solutions of the Growth Rate Equations:

The obtained numerical solutions of the mathematical model by using the fourth order Runge-Kutta method are tabulated in Table-1

Table-1

Case	a_1	a_{11}	a_{12}	a_2	a_{22}	m_1	m_2	h_1	h_2	N_{10}	N_{20}	m	t*
1	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	0	0.225
2	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	0.4	0.235
3	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	0.8	0.247
4	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	1.2	0.255
5	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	1.6	0.274
6	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	2.0	0.294
7	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	2.4	0.317
8	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	2.8	0.348
9	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	3.2	0.398
10	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	3.6	0.546
11	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	4.0	*
12	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	4.4	*
13	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	4.8	*
14	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	5.0	*
15	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	10	*
16	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	15	*
17	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	20	*
18	1.592739	3.30158	1.773829	3.110636	1.959451	0.6	0.7	0.5	0.5	3.762642	0.429724	25	*

The fixed parameters are taken as $\mathbf{a_1=1.592739, a_{11}=3.30158, a_{12}=1.773829, a_2=3.110636, a_{22}= 1.959451, N_{10}=3.762642, N_{20}=0.429724, m_1 = 0.6, m_2 = 0.7, h_1 = 0.5; h_2 = 0.5.}$ (3)
The varying variable is m i.e. **m= .0.4,0.8,1.2,1.6,2.0,2.4,2.8,3.2,3.6,4.0,4.4,4.8,5,10,15,20,25** and then t* is traced. The derived solutions are tabled as in **Table-1** and illustrated from Fig.(1) to Fig.(17)

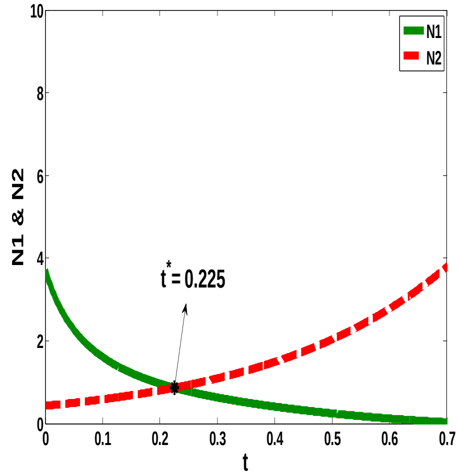


Fig.1; Case-1 in Table-1

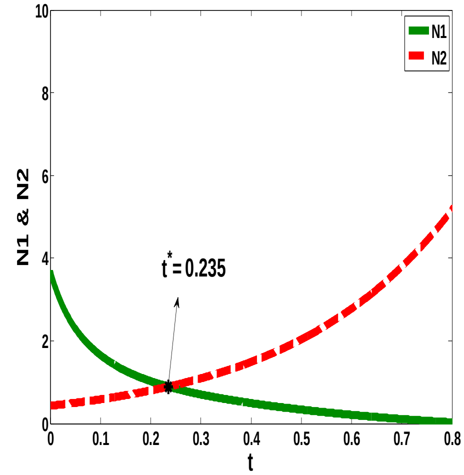


Fig.2; Case-2 in Table-1

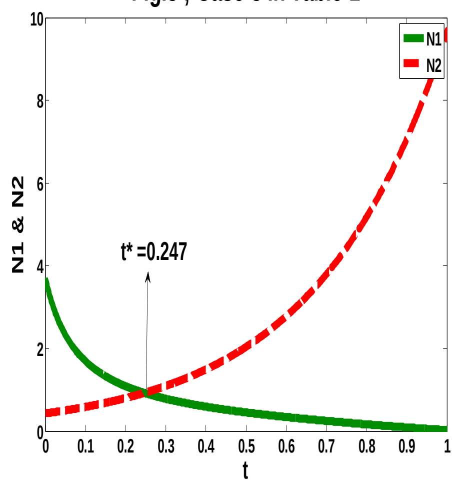


Fig.3 ; Case-3 in Table-1

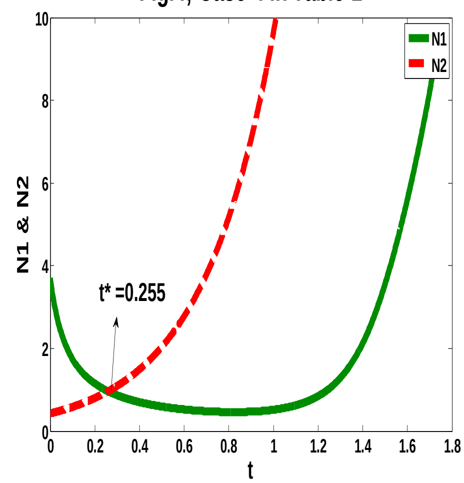


Fig.4; Case-4 in Table-1

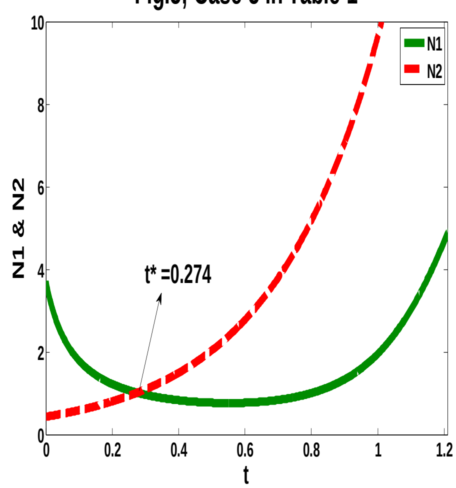


Fig.5; Case-5 in Table-1

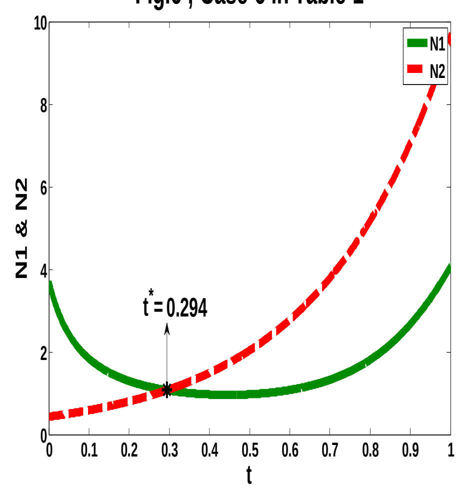


Fig.6 ; Case-6 in Table-1

Fig.7 ; Case-7 in Table-1

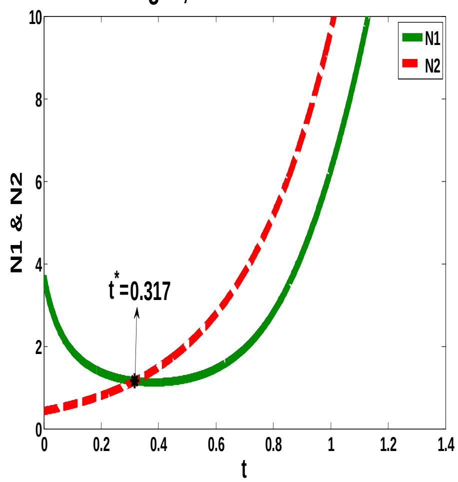


Fig.8 ; Case-8 in Table-1

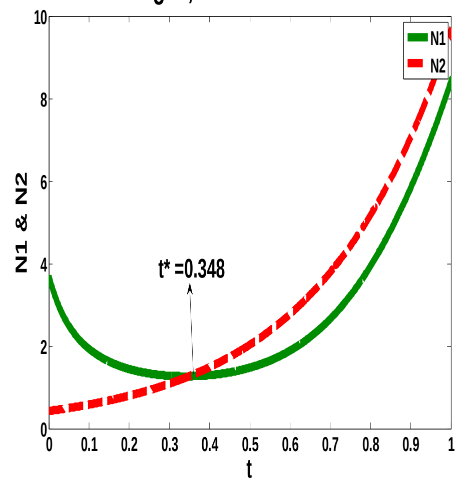


Fig.9 ; Case-9 in Table-1

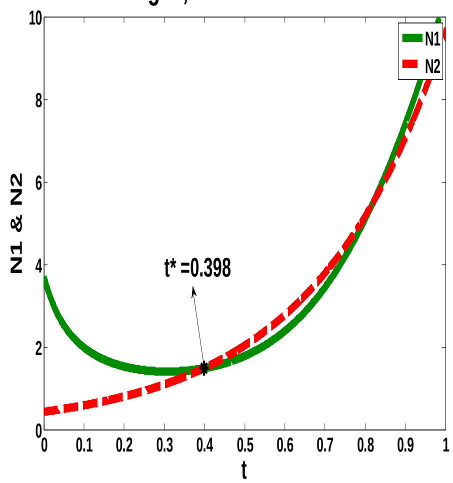


Fig.10 ; Case-10 in Table-1

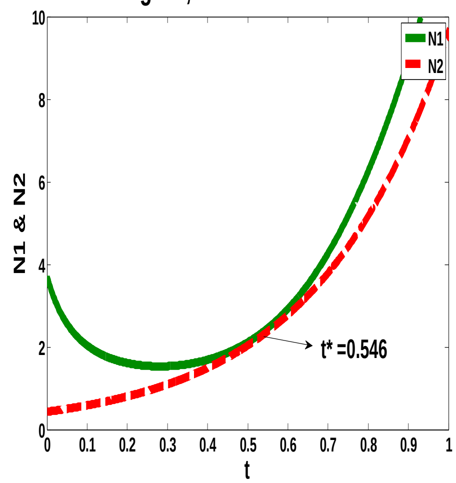


Fig.11 ; Case-11 in Table-1

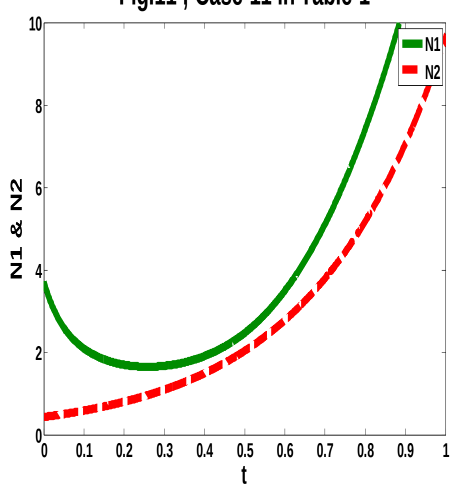


Fig.12; Case-12 in Table-1

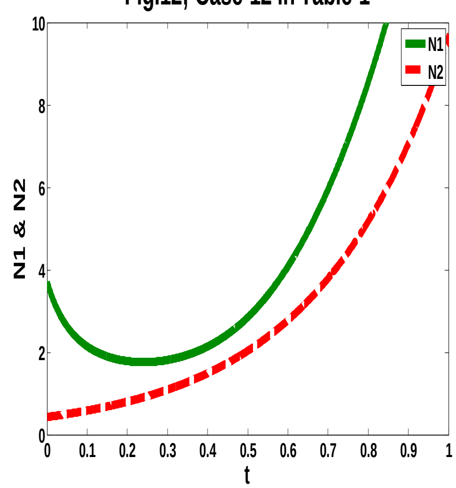


Fig.13 ; Case-13 in Table-1

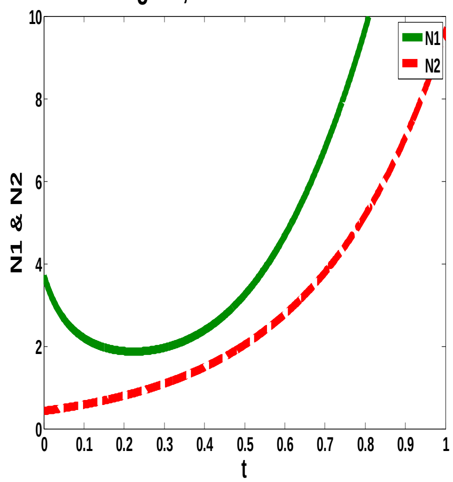


Fig.14 ; Case-14 in Table-1

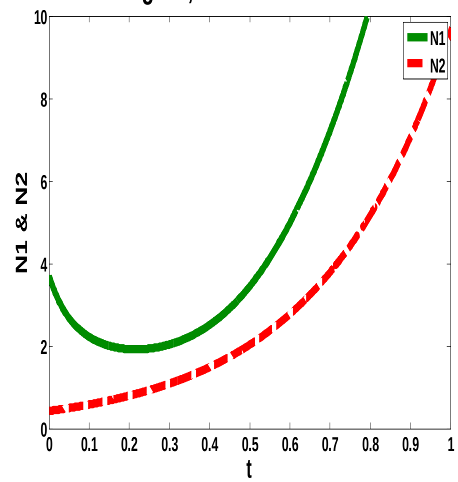


Fig.15 ; Case-15 in Table-1

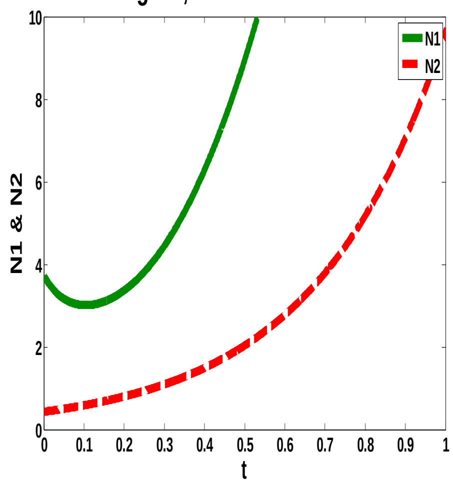


Fig.16 ; Case-16 in Table-1

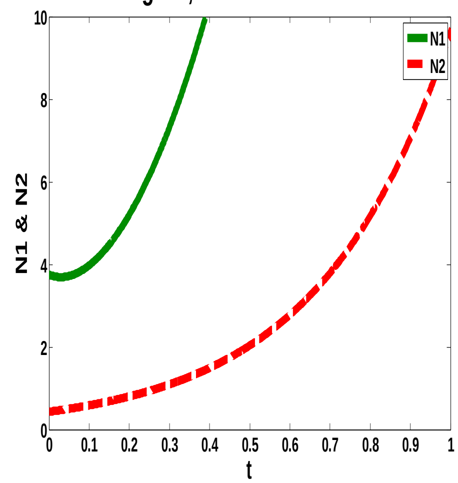
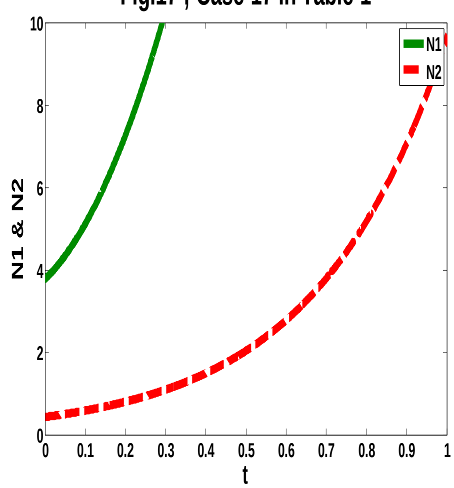


Fig.17 ; Case-17 in Table-1



4. Conclusion

i) Case (1) to Case (3): Initially the Ammensal species reigns over the enemy species. The dominance is reversed after dominance reversal time (t*).Ammensal species diminishes steadily and subsists at a very low growth rate .The enemy species has a steady increase through out the interval.

ii) Case (4) to Case (6): The Ammensal species outnumbers the enemy species till dominance reversal time (t*) .Further it is noticed that both the species flourish and later maintain the declining variation with significant growth rates.

iii)Case (7) to Case (10) :The Ammensal species prevails the enemy species upto the dominance reversal time (t*) after which the enemy species predominates Ammensal species slightly for some period of time. Further it is observed that both the species survive side by side with considerable exponential growth rates.

iv)Case(11) to case(17) : This is the situation where both the species fly high steadily and Ammensal species is the dominant one as it reigns over the other species throughout the interval .It is identified that both the species exert a gradual deviation with flourishing growth rates . Further the Ammensal species will not be effected by enemy species, because of its sufficient cover protection.

5. Relation Between Carrying Capacity and Dominance Reversal Time:

The fixed parameters are considered as a_{11}**=3.30158,a_{12}=1.773829, a_2=3.110636, a_{22}= 1.959451, N_{10}=3.762642,N_{20}=0.429724, m= 0.4,m_1 = 0.6,m_2 = 0.7,h_1 = 0.5;h_2 = 0.5.** The varying variable is **$a_{1,}$ i.e a_1= 0.592739, 1.592739, 2.592739, 3.592739, 4.592739, 5.592739, 6.592739, 7.592739, 8.592739, 9.592739, 10.592739** and then t* is derived. The obtained solutions are tabled as in **Table-2** and illustrated from Fig. (18) to Fig.(28)

Table-2

S.No	a_1	a_{11}	a_{12}	a_2	a_{22}	m	m_1	m_2	h_1	h_2	N_{10}	N_{20}	t*
1	0.592739	3.30158	1.773829	3.110636	1.959451	0.5	0.6	0.7	0.8	0.8	3.762642	0.429724	0.224
2	1.592739	3.30158	1.773829	3.110636	1.959451	0.5	0.6	0.7	0.8	0.8	3.762642	0.429724	0.232
3	2.592739	3.30158	1.773829	3.110636	1.959451	0.5	0.6	0.7	0.8	0.8	3.762642	0.429724	0.242
4	3.592739	3.30158	1.773829	3.110636	1.959451	0.5	0.6	0.7	0.8	0.8	3.762642	0.429724	0.252
5	4592739	3.30158	1.773829	3.110636	1.959451	0.5	0.6	0.7	0.8	0.8	3.762642	0.429724	0.261
6	5.592739	3.30158	1.773829	3.110636	1.959451	0.5	0.6	0.7	0.8	0.8	3.762642	0.429724	0.274
7	6.592739	3.30158	1.773829	3.110636	1.959451	0.5	0.6	0.7	0.8	0.8	3.762642	0.429724	0.287
8	7.592739	3.30158	1.773829	3.110636	1.959451	0.5	0.6	0.7	0.8	0.8	3.762642	0.429724	0.298
9	8.592739	3.30158	1.773829	3.110636	1.959451	0.5	0.6	0.7	0.8	0.8	3.762642	0.429724	0.309
10	9.592739	3.30158	1.773829	3.110636	1.959451	0.5	0.6	0.7	0.8	0.8	3.762642	0.429724	0.325
11	10.592739	3.30158	1.773829	3.110636	1.959451	0.5	0.6	0.7	0.8	0.8	3.762642	0.429724	0.340

The solution curves are depicted from Fig.(18) to Fig.(28) as below.

Fig.18 ; S.N0-1 in Table-2

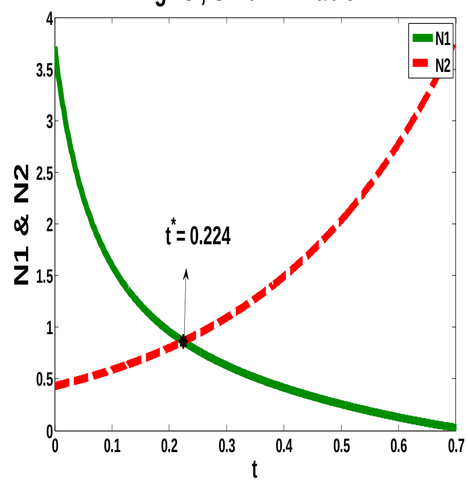


Fig.19 ; S.N0-2 in Table-2

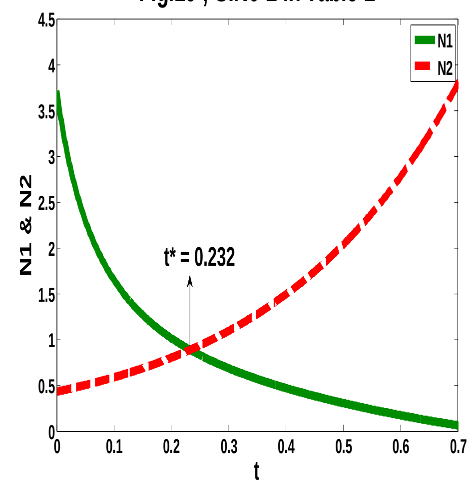


Fig.20 ; S.N0-3 in Table-2

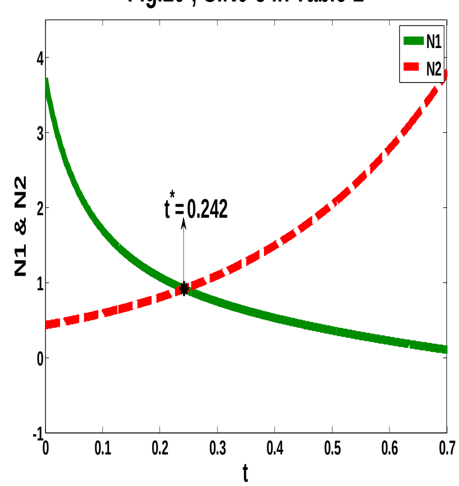


Fig.21 ; S.N0-4 in Table-2

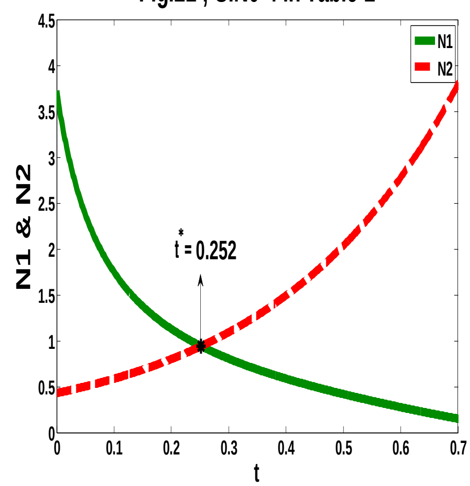


Fig.22; S.N0-5 in Table-2

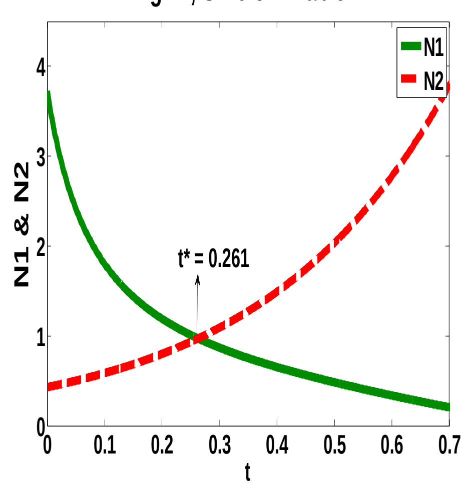


Fig.23 ; S.N0-6 in Table-2

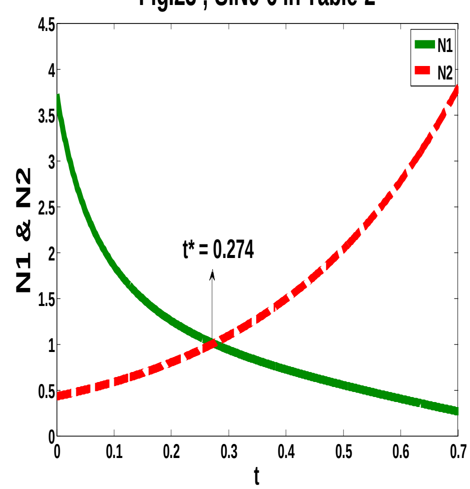


Fig.24; S.N0-7 in Table-2

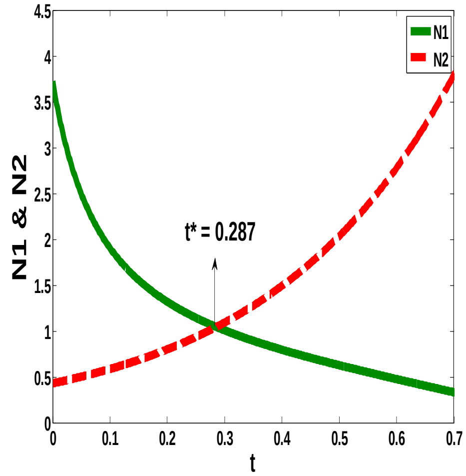


Fig.25 ; S.N0-8 in Table-2

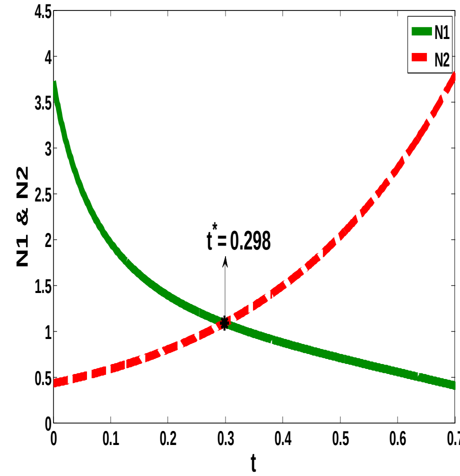


Fig.26 ; S.N0-9 in Table-2

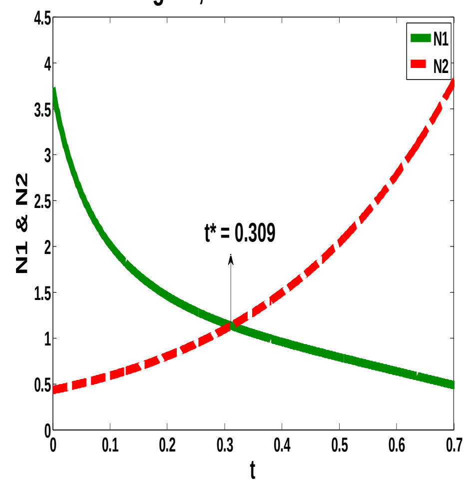


Fig.27 ; S.N0-10 in Table-2

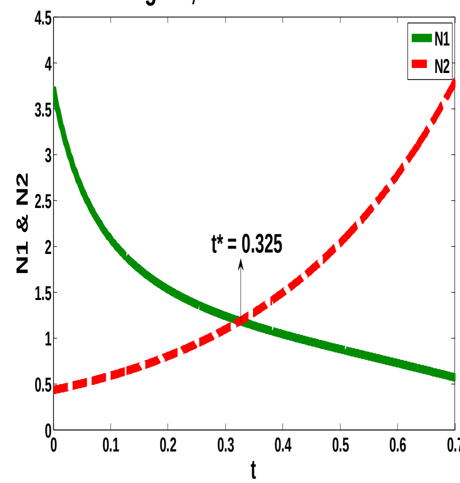
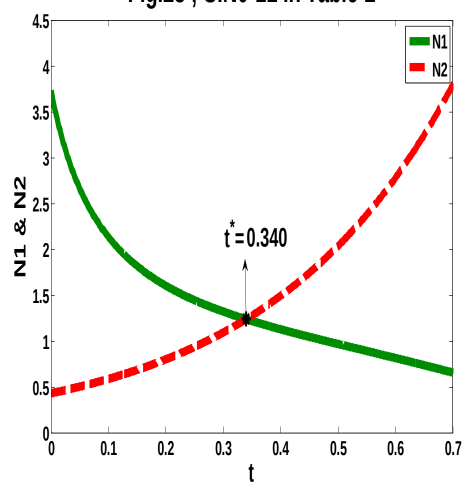


Fig.28 ; S.N0-11 in Table-2



The carrying capacity of Ammensal species is obtained by the ratio of the natural growth rate of Ammensal species and the rate of decrease of Ammensal spices (due to its own insufficient resources). The values of Carrying capacity of Ammensal species with respect to the derived numerical solutions are tabulated in Table-3 along with the corresponding values of dominance reversal time(t*).

Table-3

S.NO	K_1	t*
1	0.1795	0.224
2	0.4824	0.232
3	0.7853	0.242
4	1.0886	0.252
5	1.3910	0.261
6	1.6939	0.274
7	1.9968	0.287
8	2.2997	0.298
9	2.6026	0.309
10	2.9054	0.325
11	3.2083	0.340

6. Conclusion

The Ammensal species declines throughout the interval and dominates the enemy species up to dominance reversal time. It appears to be almost extinct with negligible growth rate. The enemy species flourishes throughout the interval and eclipses the Ammensal species after dominance reversal time.

The identified relationships can be classified as in Table-4 given below while keeping all the remaining parameters as constant.

Table-4

criterion	Conclusion
The carrying capacity of Ammensal species increases	The dominance reversal time (t*) gradually increases
The cover protection constant of Ammensal species increases	
The growth rate of Ammensal species increases	

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