BLOOD FLOW THROUGH STENOSED ARTERIES: NEW FORMULA FOR COMPUTING PERIPHERAL PLASMA LAYER THICKNESS

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Abstract

The present paper deals wth a mathematical model for blood flow through stenosed arteries with axially variable peripheral layer thickness and variable slip at the wall. The model consists of a core region of suspension of all the erythrocytes assumed to be a Casson fluid and a peripheral layer of plasma as a Newtonian fluid. For such models, in literature, the peripheral layer thickness is assumed a priori based on experimental observations. In the present analysis, new analytic expression for the thickness of the peripheral layer has been obtained in terms of measurable quantities (flow rate (Q), centerline velocity (U), pressure gradient (-dp/dz), plasma viscosity (μ_p), and yield stress (θ)). Using the experimental values of Q, U, (-dp/dz), μ_p and θ, the value of the peripheral layer thickness has been computed. The theoretically obtained peripheral layer thickness is compared with its experimental value. It is found that the agreement between the two is very good (error < 1.4%). This information of blood could be useful in the development of new diagnosis tools for many diseases.

Keywords: *Blood flow, Axially variable slip velocity, Stenosed arteries, A two-fluid model, Casson fluid, Different shapes of stenoses.*

1. Introduction

Circulatory disorders are well known to be reasonable in most cases of death, and stenosis or arteriosclerosis is one such case. Stenosis, a medical term which means narrowing of an artery, tube or orifice, is the abnormal and unnatural growth in arterial wall thickness that develops at various locations of the cardiovascular systems under diseased conditions. The actual causes of stenosis are not well known but it has been suggested that the deposits of cholesterol on the arterial wall and proliferation of connective tissues may be reasonable for the same (Chaturani and Ponalagusamy, 1986; Young, 1968; Shukla et al. 1980b). The presence of stenosis in the cardiovascular system can cause circulatory disorders by reducing or occluding the blood supply which may result in serious consequences (myocardial infarction, cerebral strokes).

Many investigators (Chaturani and Kaloni,1976; Chaturani and Upadhya,1979; Shukla et al.,1980b; Majhi and Usha,1984; Chaturani and Biswas,1983; Philip and Chandran,1996) have theoretically studied the flow of blood through uniform and stenosed tubes and analyzed the influence of slip velocity or peripheral plasma layer thickness on the flow variables such as velocity, wall shear stress and flow resistance. In these models, the peripheral layer thickness and slip velocity are assumed a priori based upon the experimental observations. To understand the flow patterns in stenosed arteries, Young(1968), Macdonald(1979), Deshpande et al.(1979), Shankar and Hemalatha(2006) etc., have analyzed the flow of blood through an arterial stenosis. Lee and Fung(1970) have obtained the numerical results for the streamlines and distribution of velocity, pressure, vorticity and the shear stress for different Reynolds number in blood flow through locally constricted tubes. In these models, the flow of blood is represented by one-layered model. Bugliarello and Sevilla(1970) and Bugliarello and Hayden(1963) have experimentally observed that when blood flows through narrow tubes there exists a cell free plasma layer near the wall. In view of their experiments, it is preferable to represent the flow of blood through narrow tubes by a two-layered model instead of one-layered model.

Shukla et al (1980a,b) have taken two-layered models and analysed the influence of peripheral plasma viscosity on flow characteristics. Chaturani and Kaloni(1976), Chaturani and Ponalagusamy (1982), Sankar and Lee(2007), Sankar and Ismail(2009), Sankar and Lee(2009) and Ponalagusamy(1986) have considered the flow of blood represented by a two-layered model. In all these models, the peripheral layer thickness is assumed a priori. It would be of interest to obtain the analytic expression for peripheral layer thickness in terms of the measurable flow variables (flow rates, pressure gradient, etc.).

The focus of this investigation is to obtain, for the first time, analytical expression for peripheral layer thickness in terms of measurable flow variables. (pressure gradient tube radius, flow rate etc.,)

2. Formulation of the problem

Consider an axially symmetric, steady, laminar and fully developed flow of blood through an arterial stenosis as shown in Figure 1. Here the flow of blood is represented by a two-layered model (a core of red blood cell suspension surrounded by a peripheral layer of plasma (Figure 1)). It is assumed that the rheology of blood in the core region has been characterized as a non-Newtonian fluid obeying the law of Casson model and the peripheral layer of plasma as a Newtonian fluid.

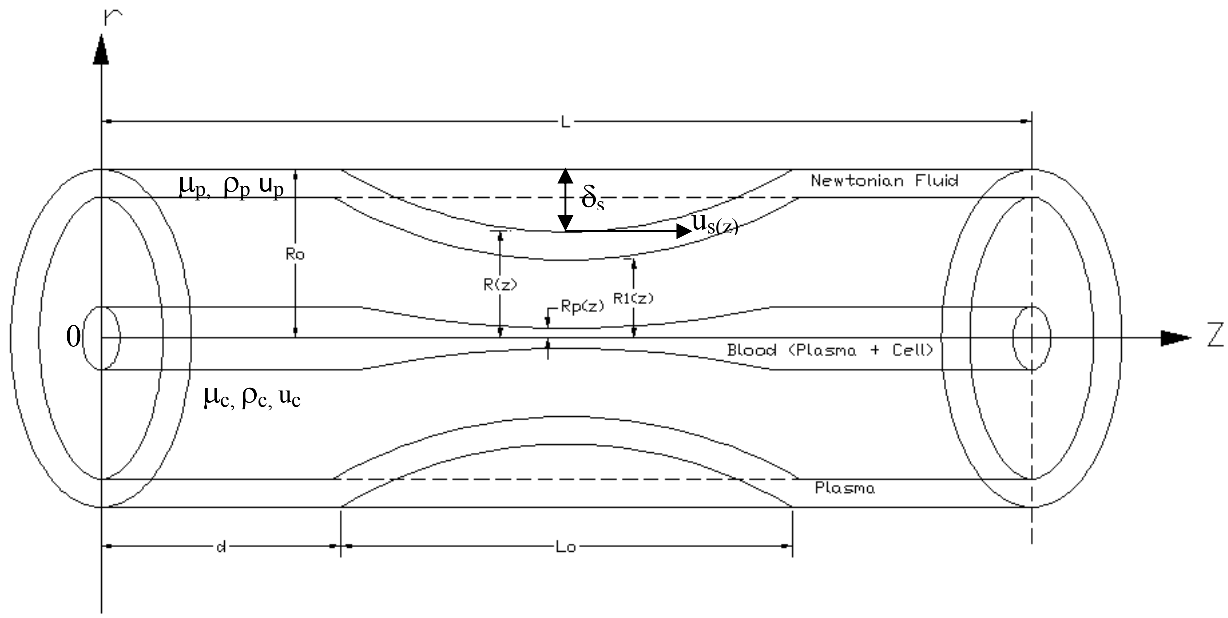

Figure 1. Geometry of Stenosed Artery(in dimensionless form)

We shall take the cylindrical coordinate system ($\bar{z},\bar{r},\bar{\phi}$) whose origin is located on the vessel (stenosed artery) axis. The problem is investigated under the following assumptions (Philip and Chandra,1996):

i) the motion is slow, so the inertia effects can be neglected
ii) the variation of cross-section of the artery(tube) is considered to be very small
iii) no body forces act on fluid
iv) flow, which is due to the pressure gradient, is one- dimensional and fluid is incompressible.

The consistency function $\bar{\mu}(\bar{r})$ may be written as

$$\bar{\mu}(\bar{r}) = \bar{\mu}_c \quad \text{for } 0 \le \bar{r} \le \bar{R}_1(\bar{z}) \qquad \ldots (1)$$

$$= \bar{\mu}_p \quad \text{for } \bar{R}_1(\bar{z}) \le \bar{r} \le \bar{R}(\bar{z}) \qquad \ldots (2)$$

where $\bar{\mu}_c$ and $\bar{\mu}_p$ are the viscosities of the central core fluid(Casson fluid) and the plasma(Newtonian fluid) respectively and $\bar{R}_1(\bar{z})$ and $\bar{R}(\bar{z})$ are the radii of the central core region and the artery in the stenotic region. The peripheral layer is of axially variable

thickness $\bar{\delta}(\bar{z})$. Thus, the core region is given by $0 \le \bar{r} \le \bar{R}_1(\bar{z}) = \bar{R}(\bar{z})$ - $\bar{\delta}(\bar{z})$
and the

Peripheral region is given by $\overline{R}_1(\overline{z}) \le \overline{r} \le \overline{R}(\overline{z})$.

The constitutive equation for a Casson fluid (blood) is given by

$$\sqrt{|\overline{\tau}|} = \sqrt{|\overline{\tau}_0|} + \sqrt{\overline{\mu}_c}\sqrt{\left|\frac{\partial \overline{u}_c}{\partial \overline{r}}\right|}, \text{ if } |\overline{\tau}| \ge |\overline{\tau}_0|$$

$$\frac{\partial \overline{u}_c}{\partial \overline{r}} = 0, \quad \text{f } |\overline{\tau}| \le |\overline{\tau}_0| \qquad \text{....(3)}$$

The non-dimensional variables are

$$r = \frac{\overline{r}}{\overline{R}_0} \quad z = \frac{\overline{z}}{\overline{z}_0} \quad R = \frac{\overline{R}}{\overline{R}_0} \quad R_1 = \frac{\overline{R}_1}{\overline{R}_0} \quad p = \frac{\overline{p}}{\overline{\rho}_p \overline{U}_0^{\,2}} \quad u_c = \frac{\overline{u}_c}{\overline{U}_0}$$

$$u_p = \frac{\overline{u}_p}{\overline{U}_0} \quad \delta_s = \frac{\overline{\delta}_s}{R_0} \quad \theta = \frac{\overline{\tau}_0}{\overline{\rho}_p \overline{U}_0^2}$$

where $\overline{u}$ is the velocity component in the axial direction($\overline{z}$), $\overline{p}$ the pressure, $\overline{\rho}_p$ is the density, $\overline{R}_0$ is the radius of the normal artery, $\overline{z}_0$ the one-fourth length of the stenosis $\overline{L}_0, \overline{U}_0$ the average velocity in the normal artery region, $\overline{\tau}_0$ the yield stress of Casson's fluid and $\overline{\delta}_s$ is the maximum height of the stenosis. The quantities in the peripheral layer and in the central core are denoted by subscripts p and c respectively. '-'over a letter denotes the corresponding dimensional quantity.

Under the assumptions made in the present analysis, the momentum equations can be approximated as (Young, 1968; Chaturani and Ponnalagarsamy, 1986):

for region $0 \le r \le R_1(z)$,

$$\frac{-\partial u_c}{\partial r} = \mu R_{ep}[\frac{1}{\sqrt{\beta}}\{-\frac{r}{2}\frac{\partial p}{\partial z}\}^{1/2} - \theta^{1/2}]^2 \qquad \text{... (4)}$$

$$0 = -\frac{\partial p}{\partial r} \qquad \text{... (5)}$$

for region $R_1(z) \le r \le R(z)$

$$0 = -\frac{\partial p}{\partial z} + \frac{\beta}{R_{ep}}\left[\frac{\partial^2 u_p}{\partial r^2} + \frac{1}{r}\frac{\partial u_p}{\partial r}\right] \qquad \text{...(6)}$$

$$0 = -\frac{\partial p}{\partial r} \qquad \text{... (7)}$$

where $\mu = \frac{\bar{\mu}_p}{\bar{\mu}_c}$, $\beta = \frac{\bar{z}_0}{\bar{R}_0}$ and $R_{ep} = \frac{\bar{U}_0 \bar{R}_0 \bar{\rho}_p}{\bar{\mu}_p}$.

The boundary conditions are

$$\left.\begin{array}{llll} \text{(i)} & u_p = u_s(z) & \text{at} & r = R(z) \\ \text{(ii)} & u_p = u_c & \text{at} & r = R_1(z) \\ \text{(iii} & \tau_p = \tau_c & \text{at} & r = R_1(z) \\ \text{(iv)} & \frac{\partial u_c}{\partial r} = 0 & \text{at} & r = R_p \end{array}\right\} \quad \ldots(8)$$

where $u_s(z)(= \frac{\bar{u}_s}{\bar{U}_0}$)is the non-dimensional axially variable slip velocity, τ is the shear stress and R_p (= R_p/R_0)is the plug core radius. It may be remarked that $u_s(z)$ is a function of z. The geometry of the stenosis(in non-dimensional form-Figure (1)) is given by (Ponalagusamy,1986),

$$R(z) = 1 - A[L_0^{\,n-1}(z-d) - (z-d)^n], \text{for} \quad d \le z \le d + L_0 \quad \ldots (9)$$

$$= 1, \text{ otherwise}$$

Where n (≥ 2) is a parameter determining the shape of the stenosis, $R(z)$ is the radius of the artery in the stenotic region, L_0 is the length of the stenosis, d indicates its location and A is given by

$$A = \frac{\delta_s}{R_0 L_0^{\,n}} \frac{n^{n/(n-1)}}{(n-1)}$$

Here δ_s denotes the maximum height of the stenosis at

$$z = d + \frac{L_0}{n^{1/(n-1)}}$$

such that the ratio of the stenotic height to the radius of the normal artery is much less than unity. It is of interest to note that an increase in the value of n leads to the change of stenosis shape. When n = 2, the geometry of stenosis becomes symmetric at $z = d + \frac{L_0}{2}$.

3. Solution

Using boundary conditions (8), the solutions of Eqs. (4) and (6) can be obtained as

$$u_c = \frac{q(z)R_{ep}}{4\beta}\left[R^2 - R_1^2(1-\mu) - \mu r^2\right] + \mu R_{ep}\ \theta(R_1 - r) - \frac{2\sqrt{2}\mu R_{ep}}{3}\left(\frac{\theta q(z)}{\beta}\right)^{1/2}\left(R_1^{3/2} - r^{3/2}\right) + u_s(z) \quad \ldots (10)$$

$$u_p = \frac{q(z)R_{ep}}{4\beta}\left[R^2 - r^2\right] + u_s(z) \quad \ldots (11)$$

where $q(z) = -\frac{\partial p}{\partial z}$

The velocity in the plug core region u_{pl} is

$$u_{pl} = \frac{q(z)R_{ep}}{4\beta}\left[R^2 - R_1^2(1-\mu) - \mu R_p^2\right] + \mu R_{ep}\ \theta(R_1 - R_p) - \frac{2\sqrt{2}\mu R_{ep}}{3}\left(\frac{\theta q(z)}{\beta}\right)^{1/2}\left(R_1^{3/2} - R_p^{3/2}\right) + u_s(z), \quad \ldots (12)$$

where $R_p = 2\beta\ \theta / q(z)$.

The flow rate Q may be obtained as

$$Q = \frac{q(z)R_{ep}}{8\beta}\left[R^4 - R_1^4(1-\mu)\right] + R^2 u_s(z) + \frac{\mu R_{ep}\theta}{3}\{R_1^3 - R_p^3\} - \frac{2\sqrt{2}\mu R_{ep}}{7\sqrt{\beta}}\{\theta q(z)\}^{1/2}\{R_1^{7/2} - R_p^{7/2}\} \quad \ldots (13)$$

4. Analytic Expressions For Slip Velocity, Core Viscosity And Peripheral Layer Thickness

Btrunn(1975) has indicated that the introduction of a thin solvent layer near the wall produces the same effect as that of the slip at the wall. In the case of one layered model ($R = R_1$) with slip at the wall, the flow rate Q_{1L} (from Eq. (13)) can be obtained as

$$Q_{1L} = \frac{\rho^* q(z)\mathrm{Re}}{8\beta}R^4 + R^2 u_s(z) + \frac{\mathrm{Re}\,\theta\rho^*}{3}\{R^3 - R_p^3\} - \frac{2\sqrt{2}\rho^*\mathrm{Re}}{7\sqrt{\beta}}\{\theta q(z)\}^{1/2}\{R^{7/2} - R_p^{7/2}\}, \quad \ldots (14)$$

where $\rho^* = \frac{\overline{\rho}p}{\overline{\rho}}$, $\mathrm{Re} = \frac{\overline{\rho}\,\overline{U}_0\overline{R}_0}{\overline{\mu}^*}$ and $\overline{\rho}$ and $\overline{\mu}^*$ are the density and viscosity of the fluid when the flow is one-layered. For the two-layered model without slip at the wall ($u_s = 0$), the flow rate Q_{2L} (from Eq.(13)) can be obtained as

Q_{2L} =

$$\frac{q(z)R_{ep}R^4}{8\beta}\left[1-(1-\frac{\delta(z)}{R})^4(1-\mu)\right]+\frac{\mu R_{ep}\theta R^3}{3}\{(1-\frac{\delta(z)}{R})^3-(\frac{R_p}{R})^3\}$$
$$-\frac{2\sqrt{2}\mu R_{ep}}{7\sqrt{\beta}}\{\theta q(z)R^7\}^{1/2}[\{1-\frac{\delta(z)}{R}\}^{7/2}-(\frac{R_p}{R})^{7/2}] \quad \text{.... (15)}$$

where $\delta(z)=\bar{\delta}(\bar{z})/\bar{R}_0$ is the non-dimensional peripheral layer thickness which is a function of axial distance z. Since the two models (one-layered with slip and two-layered without slip) represent the same phenomena, the flow rates can be equated as

Q_{1L} = Q_{2L} = Q*,

$$\frac{\rho^* q(z)\mathrm{Re}}{8\beta}R^4+R^2u_s(z)+\frac{\mathrm{Re}\,\theta\rho^*}{3}\{R^3-R_p{}^3\}-\frac{2\sqrt{2}\rho^*\mathrm{Re}}{7\sqrt{\beta}}\{\theta q(z)\}^{1/2}\{R^{7/2}-R_p{}^{7/2}\}=$$

$$\frac{q(z)R_{ep}R^4}{8\beta}\left[1-(1-\frac{\delta(z)}{R})^4(1-\mu)\right]+\frac{\mu R_{ep}\theta R^3}{3}\{(1-\frac{\delta(z)}{R})^3-(\frac{R_p}{R})^3\}$$
$$-\frac{2\sqrt{2}\mu R_{ep}}{7\sqrt{\beta}}\{\theta q(z)R^7\}^{1/2}[\{1-\frac{\delta(z)}{R}\}^{7/2}-(\frac{R_p}{R})^{7/2}] \quad \text{....(16)}$$

For a two-layered model without slip at the wall (u_s = 0), the expression for velocity in the core region is obtained from Eq.(10) as

$$u_c=\frac{q(z)R_{ep}R^2}{4\beta}[1-(1-\frac{\delta(z)}{R})^2(1-\mu)-\mu\left(\frac{r}{R}\right)^2]$$
$$+\theta\mu RR_{ep}\{(1-\frac{\delta(z)}{R})-\frac{r}{R}\}-\frac{2\mu RR_{ep}}{3}\frac{\{2q(z)\theta R^3\}^{1/2}}{\beta}[\{1-\frac{\delta(z)}{R}\}^{3/2}-(\frac{r}{R})^{3/2}] \quad \text{... (17)}$$

The centerline velocity U (at r = R_p) from Eq.(17) can be obtained as

$$U=\frac{q(z)R_{ep}R^2}{4\beta}\left[1-1-\left(\frac{\delta(z)}{R}\right)^2(1-\mu)-\mu\left(\frac{R_p}{R}\right)^2\right]$$

$$+\theta\mu RR_{ep}\left[\left\{1-\frac{\delta(z)}{R}\right\}-\left(\frac{R_p}{R}\right)\right]-\frac{2\mu R_{ep}}{3}\left\{\frac{2q(z)\theta R^3}{\beta}\right\}^{1/2}\left[\left\{1-\frac{\delta(z)}{R}\right\}^{3/2}-\left(\frac{R_p}{R}\right)^{3/2}\right]$$

,… (18)

Elimination of μ from Eqs. (16) and (18) gives

$$[\frac{16q(z)}{7}\{\theta Q^* R^7 R_{ep}^3\}^{1/2}-\frac{2R_{ep}^2}{3}\{\frac{2q^3(z)\theta R^{11}}{\beta}\}^{1/2}](1-\frac{\delta}{R})^{11/2}+\frac{q(z)\theta R^5 R_{ep}^2}{3}(1-\frac{\delta}{R})^5$$

$$+[\frac{q^2(z)R^4R_{ep}^2}{4\beta}\{R^2-R_p^2\}+q(z)R^4R_{ep}\{\frac{2R_{ep}}{3}(\frac{2q(z)\theta R_p^3}{\beta})^{1/2}-U-\theta R_p R_{ep}\}](1-\frac{\delta}{R})^4$$

$$+\frac{16}{7}\{\theta Q^* R^3 R_{ep}\}^{1/2}[4U\beta-q(z)R^2R_{ep}]((1-\frac{\delta}{R})^{7/2}+\{\frac{2\theta R^3 R_{ep}}{3}\}[q(z)R^2R_{ep}-4U\beta](1-\frac{\delta}{R})^3$$

$$+\{\frac{q(z)R^2R_{ep}}{4\beta}\}[8\beta Q^*-q(z)R^4R_{ep}+\frac{8\theta\beta R_{ep}R_p{}^3}{3}-\frac{64\beta}{7R^2}\{\theta Q^* R_{ep}R_p{}^7\}^{1/2}](1-\frac{\delta}{R})^2+$$

$$[\frac{2R_{ep}^2}{3}\{\frac{2q^3(z)\theta R^{11}}{\beta}\}^{1/2}-\frac{16R_{ep}Q^*}{3}\{2\beta q(z)\theta R^3\}^{1/2}](1-\frac{\delta}{R})^{3/2}+[8\beta Q^*\theta RR_{ep}-q(z)\theta R^5R_{ep}^2](1-\frac{\delta}{R})$$

$$=2Q^*q(z)R_p^2R_{ep}-\frac{q^2(z)R_p^2R_{ep}^2R^4}{4\beta}+8\beta Q^*\theta RpR_{ep}-q(z)\theta RpR^4R_{ep}^2+\frac{2\theta R_{ep}R_p{}^3}{3}\{R^2q(z)R_{ep}-4U\beta\}$$

$$+\{\frac{2q(z)\theta R_p^3}{9\beta}\}^{1/2}[16Q^*\beta R_{ep}-2R^4R_{ep}^2q(z)]-\frac{16}{7}\{\theta Q^* R_{ep}R_p{}^7\}^{1/2}\{R_{ep}q(z)-4U\beta\}$$

…..(19)

Since all the quantities on right hand side of Eq. (19) are measurable experimentally, the peripheral layer thickness $\delta(z)$ can be computed.

5. Results And Discussion

It is pertinent to mention that we are only interested in computing the values of $\frac{\delta(z)}{R}$, which are real and less than or equal to unity.

Since the experimental values of pressure gradient, flow rate and centerline velocity for flow through an arterial with mild stenosis at different cross-sections for various values of stenotic height and shapes and red blood cells concentrations are not available, the variation of peripheral layer thickness with the axial distance cannot be obtained. However, to show the procedure and to see the accuracy of the method, we have used the experimental data of flow through a uniform tube. First, we write Eq.(19) in the dimensional form as

$$
\begin{aligned}
&[\frac{16}{7}\{\frac{\bar{\tau}_0\bar{q}^2{}_0\overline{Q}^*\overline{R}_0}{\pi\overline{U}_0^4\bar{\mu}_p^3}\}^{1/2}-\{\frac{8\bar{\tau}_0\bar{q}^3{}_0\overline{R}_0^5}{9\overline{U}_0^4\bar{\mu}_p^4}\}^{1/2}](1-\frac{\delta}{R})^{11/2}+\{\frac{\bar{\tau}_0\bar{q}_0\overline{R}_0^2}{3\overline{U}_0^2\bar{\mu}_p^2}\}(1-\frac{\delta}{R})^5\\
&[(\frac{\bar{q}^2{}_0\overline{R}_0}{4\overline{U}_0^2\bar{\mu}_p^2})(\overline{R}_0^2-\overline{R}_p^2)+\{\frac{8\bar{\tau}_0\bar{q}_0^3\overline{R}_0^2\overline{R}_p^3}{9\overline{U}_0^4\bar{\mu}_p^4}\}^{1/2}-(\frac{\bar{q}_0\overline{R}_0\overline{U}}{\overline{U}_0^2\bar{\mu}_p}\}-(\frac{\bar{q}_0\overline{R}_0\overline{R}_p\bar{\tau}_0}{\overline{U}_0^2\bar{\mu}_p^2}](1-\frac{\bar{\delta}}{\overline{R}_0})^4\\
&+\frac{64}{7}\{\frac{\bar{\tau}_0\overline{Q}^*\overline{U}^2}{\pi\overline{R}_0^3\overline{U}_0^4\bar{\mu}_p}\}^{1/2}-\frac{16}{7}\{\frac{\bar{\tau}_0\bar{q}^2{}_0\overline{Q}^*\overline{R}_0}{\pi\overline{U}_0^4\bar{\mu}_p^3}\}](1-\frac{\bar{\delta}}{\overline{R}_0})^{7/2}+[\{\frac{2}{3}\frac{\bar{\tau}_0\bar{q}_0\overline{Q}^*\overline{R}_0^2}{\pi\overline{U}_0^{24}\bar{\mu}_p^2}\}-\{\frac{8}{3}\frac{\bar{\tau}_0}{\bar{\mu}_p\overline{U}_0^2}\}](1-\frac{\bar{\delta}}{\overline{R}_0})^3\\
&+[\frac{2\bar{q}_0\overline{Q}^*}{\pi\overline{R}_0\overline{U}_0^2\bar{\mu}_p}-\frac{\bar{q}_0^2\overline{R}_0^3}{4\overline{U}_0^2\bar{\mu}_p^2}+\frac{2\bar{\tau}_0\bar{q}_0\overline{R}_p^3}{3\overline{R}_0\overline{U}_0^2\bar{\mu}_p{}^2}-\frac{16\bar{q}_0}{7\overline{U}_0^2}\{\frac{\bar{\tau}_0\overline{Q}^*\overline{R}_p{}^7}{\pi\overline{R}_0{}^6\bar{\mu}_p{}^3}\}^{1/2}](1-\frac{\bar{\delta}}{\overline{R}_0})^2\\
&[\frac{2}{3}\{\frac{2\bar{\tau}_0\bar{q}_0^3\overline{R}_0^5}{\overline{U}_0^4\bar{\mu}_p^4}\}^{1/2}-\frac{16}{3\pi}\{\frac{2\bar{\tau}_0\bar{q}_0\overline{Q}^*\overline{R}_0}{\overline{R}_0^3\overline{U}_0^4\bar{\mu}_p^2}\}^{1/2}](1-\frac{\delta}{R})^{3/2}+\{\frac{8\bar{\tau}_0\overline{Q}^*}{\pi\overline{R}_0^2\overline{U}_0^2\bar{\mu}_p}-\frac{\bar{\tau}_0\bar{q}_0\overline{R}_0^2}{\overline{U}_0^2\bar{\mu}_p^2}\}(1-\frac{\bar{\delta}}{\overline{R}_0})=\\
&\{\frac{2\bar{q}_0\overline{Q}^*\overline{R}_p^2}{\pi\overline{R}_0^3\overline{U}_0^2\bar{\mu}_p}\}-\frac{\overline{R}_0\bar{q}_0^2\overline{R}_p^2}{\overline{U}_0^2\bar{\mu}_p}+\frac{8\bar{\tau}_0\overline{Q}^*\overline{R}_p}{\pi\overline{R}_0\overline{U}_0^2\bar{\mu}_p}-\frac{\overline{R}_0\bar{q}_0\overline{Q}^*\overline{R}_p}{\overline{U}_0^2\bar{\mu}_p}+\frac{16\overline{Q}^*}{\pi\overline{R}_0^3\overline{U}_0^2\bar{\mu}_p}\{\frac{2}{9}\bar{\tau}_0\bar{q}_0\overline{R}_p^3\}^{1/2}-(\frac{8\bar{\tau}_0\bar{q}_0^3\overline{R}_p^7}{9\overline{R}_0^2\overline{U}_0^4\bar{\mu}_p^4})\\
&+2\{\frac{\bar{q}_0}{\bar{\mu}_p}-\frac{4\overline{U}}{\overline{R}_0{}^2}\}[\frac{\bar{\tau}_0\overline{R}_p^3}{3\overline{R}_0\overline{U}_0^2\bar{\mu}_p}-\frac{8}{7}\{\frac{\bar{\tau}_0\overline{Q}^*\overline{R}_p{}^7}{\pi\overline{R}_0{}^6\overline{U}_0^4\bar{\mu}_p}\}^{1/2}]
\end{aligned}
\qquad \ldots(20)
$$

where $\bar{q}_0$ is the pressure gradient and $\bar{\delta}_0$ is the peripheral plasma layer thickness in the normal artery region. For blood with 40% and 6% red blood cell concentration, we have the following data from Bugliarello and Sevilla(1970) and Bugliarello and Hayden(1963).

For 40 μm Diameter

C = 40% and , $\overline{Q}^* = 19.2342*10^{-6}\, cm^3/\sec$,

$\bar{\delta}_0 = 3.2\ \mu m$, and , $\bar{q}_0 = 167.5*10^3\, dyne/cm^3$,

$\bar{\mu}_p = 0.0144P(at\ \ 25.5^0\text{C})$, $\overline{U} = 2.37 cm/\sec$ and $\bar{\tau}_0$=0.04 dyne/cm^2

For 66.6 μm Diameter

C = 6 % , $\overline{Q}^* = 45.6546 * 10^{-6} cm^3 / \sec$,

$\overline{\delta}_0$ = 12.876 μm, and $\overline{q}_0 = 14.2655 * 10^3 dyne / cm^3$,

$\overline{\mu}_p = 0.0143(at\ 25.5^0$C), $\overline{U} = 2.38 cm/\sec$ and $\overline{\tau}_0$=0.0064 dyne/cm^2

Using these value, the peripheral layer thickness is computed (Table 1) for blood flow in 40 μm and 66.6 μm tube diameter from Eq. (20). One can easily see from this table that the peripheral layer thickness, obtained from the present analysis , has a good agreement with the experimental observation(Bugliarello and Sevilla,1970; Bugliarello and Hayden,1963), the error is less than 1.40%.

Table 1. Comparison of Peripheral Layer Thickness $\overline{\delta}_o \mu m$

Tube diameter	$\overline{\delta}_o \mu m$		
	Present work	Experimental Results (C-N)	Difference %
	Casson – Newtonian (C-N)		Casson – Newtonian
40µm	3.1658	3.2000	1.0688
66.6µm	12.6973	12.8760	1.3878

6. Conclusion

It is of interest to mention that measuring the thickness of peripheral plasma layer experimentally is not so easy because its thickness is not constant even for the steady flow through uniform tubes, due to the random motion of the suspended particle (red blood cell); whereas the reliable values of pressure gradient, plasma viscosity and centerline velocity can be measured for a given flow rate, tube size and concentration of blood.. Therefore, it is preferable to use these reliable measurements for the computation of the value of peripheral layer thickness using newly developed equation (19).

The present analysis could also serve as the check for the experimentally measured rheological values of blood. It may be mentioned at this stage that the variation of peripheral layer thickness with the axial distance in the stenotic region has not been analyzed due to the non-availability of the experimental values of pressure gradient and the centerline velocity at different cross-sections of the stenosed arteries for various values of stenotic heights, flow rates and concentrations. It would be of interest to conduct such experiments to provide this vital data which, in turn, could be useful in the understanding of the rheology of blood. This rheological information of blood in turn could be exploited for the development of new diagnostic tools for many diseases such as myocardial infarction, hypertension, renal, retinal, etc.

References

[1] Brunn.P.(1975) The velocity slip of polar fluids. Rheol Acta **14**, 1039-1054.

[2] Bugliarello.G and Sevilla.J.(1970) Velocity distribution and other characteristics of steady and pulsatile blood flow in fine glass tubes. Biorhelogy **l7**, 85-107.

[3] Bugliarello,G and Hayden,J.W.(1963) Detailed characteristics of the flow of blood in Vitro. Trans. Soc. Rheology **7**, 209-230.

[4] Chaturani,P. and Kaloni,P.N.(1976) Two –layered poiseuille flow model for blood flow through arteries of small diameter and arterioles. Biorheology **13**, 243-250.

[5] Chaturani,P. and Ponalagusamy,R (1982) A two-layered model for blood flow through stenosed arteries. Proc. of 11th National Conf. on fluid mechanics and fluid power, B.H.E.L(R & D),Hydrabad, India, 16-22.

[6] Chaturani,P. and Biswas.D.(1983) A theoretical study of blood flow through stenosed arteries with velocity slip at the wall. Proc .First Internat. Symposium on Physiological Fluid Dynamics, IIT Madras, India, 23-26.

[7] Chaturani,P. and Ponnalagarsamy,R.(1986) Pulsatile flow of Casson's fluid through stenosed arteries with applications to blood flow",Biorheology **23**, 499-511.

[8] Deshpamde,M.D, Giddens,D.P. and Mabon,R.F. (1979) Steady laminar flow through modeled vascular stenosis. Journal of Biomechanics **9**, 65-174.

[9] Lee,J.S. and Fung,Y.C.(1970) Flow in locally constricted tubes at low Reynolds Number. Journal of Applied Mechanics Transactions ASME **37**, 9-16.

[10] Macdonald,D.A.(1979) On steady flow through modeled vascular stenosis. J. Biomechanics **12**, 13-20.

[11] Majhi,S.N. and Usha.L.(1984) Fahraeus-Lindqvist effect and generalized poiseuille flow with or without wall slip. Proc. 13th Nat.Conf. Fluid Mech. and Fluid Power, REC, Tiruchirappalli, 125-128.

[12] Philip.D and Peeyush Chandra.(1996) Flow of Eringen Fluid (simple Micro Fluid) through an Artery with Mild Stenosis. Int. J. Engng Sci. **34**, 87-99.

[13] Ponalagusamy,R.(1986) Blood Flow Through Stenosed Tube. Ph.D. hesis, IIT, Bombay, India.

[14] Shukla,J.B.,Parihar,R.S. and Gupta,S.P.(1980a) Biorhelogical aspects of blood flow through artery with mild stenosis:Effects of peripheral layer. Biorheology **17**, 403 - 410.

[15] Shukla,J.B, Parihar, R.S. and Gupta,S.P.(1980b) Effects of peripheral layer viscosity on blood flow through the artery with mild stenosis. Bull.Math.Biology **42**, 797-805.

[16] Sankar, D.S. and Ismail, A.I.Md.(2009) Two-fluid mathematical models for blood flow in stenosed artery: A comparative study. Boundary Value Problems **2009**, 1-15.

[17] Sankar, D.S. and Lee, U.(2007) Two-phase non-linear model for the flow through stenosed blood vessels. J. Mechanical Science and Technology **21**, 678-689.

[18] Sankar, D.S. and Lee,U. (2009) Two-fluid Casson model for pulsatile blood flow through stenosed arteries: A theoretical model. Communications in Non-linear Science and Numerical Simulation, 2009(Article in press).

[19]Shankar, D.S. and Hemalatha,K.(2006) Pulsatile flow of Herschel-Bulkley fluid through stenosed arteries- A mathematical model. Int. J. Non-linear Mechanics, **41**, 979- 990.

[20] Young.D.F,(1968) Effects of a time-dependent stenosis on flow through a tube. Journal of Engng. Ind.Trans.AMSE **90**, 248-254.