

## The New Decoding Method of Rate Compatible LDPC Codes Based on the IR- HARQ schemes

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### Abstract

A new decoding method based on IR\_HARQ scheme that uses rate-compatible LDPC codes as the FEC codes is presented in this paper. This method can use the former decoding result, which can improve the performance of decoding scheme. This paper also evaluates the new decoding method by simulation. The results show that better performance both on system throughput and iterative numbers can be obtained.

### 1. Introduction

The Incremental Redundancy Hybrid ARQ (IR-HARQ)<sup>[1]</sup> scheme which can get better performance than that of other HARQ requires that the FEC is Rate Compatible (RC) codes. So many researches has mainly been focused on how to build the RC codes. However, the decoding scheme is the area to be ignored. With the IR-HARQ scheme, there are always parts of bits has been decoded correctly during the former decoding process. To make the best use of the information provided by the correctly decoding bits will improve the performance of decoding schemes greatly.

This paper proposes a new decoding method based on IR\_HARQ scheme that uses rate-compatible LDPC<sup>[2,3,4]</sup> codes as the FEC codes. Instead of the initial reliabilities of the previous decoding process, the final reliabilities could be selected as the initial reliabilities of the previously received bits during the latest decoding process. And the new decoding method calculates the privilege factor by the properties of the channel. The proposed scheme will not increase the complexity of the decoder, but it will enhance the performance of the retransmission decoding process and reduce iterative numbers.

### 2. Incremental Redundancy Hybrid ARQ Schemes

The matrix of RC-LDPC code which we proposed here can be decomposed as:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{inf_1} & \mathbf{H}_{inf_2} & \mathbf{L} & \mathbf{H}_{inf_s} & \mathbf{H}_{ch} \end{bmatrix} \quad (1)$$

The principle of IR\_HARQ based on this LDPC codes works as follows: Initially, data bits  $\mathbf{u}_1 \ \mathbf{u}_2 \ \mathbf{L} \ \mathbf{u}_s$  and the parity vectors  $\mathbf{v}$  are transmitted in the channel. If decoding is unsuccessful, the receiver requests additional redundancy. Then the punctured data word  $\mathbf{u}_2 \ \mathbf{u}_3 \ \mathbf{L} \ \mathbf{u}_s$  is encoded using the same LDPC code to produce the new parity vectors  $\mathbf{v}$ . Then the transmitter sends the parity vectors  $\mathbf{v}$ . At the receiver, the received sequence is combined with the previously received sequence data bits  $\mathbf{u}_2 \ \mathbf{u}_3 \ \mathbf{L} \ \mathbf{u}_s$  for decoding. The process is repeated until the decoding is successful or maximal repeat transmitted number  $\mathbf{S}$  is reached.

With this technique we gradually improve the bit-error-rate (BER) of specific data parts. As a result, the average BER of the entire codeword is also lowered so that finally the original parity information is sufficient for successful decoding of the entire codeword.

### 3. Modified Decoding Schemes

Moreover one of the most attractive points of IR\_HARQ scheme based on proposed LDPC code is that retransmission decoding can use the former decoding result, which can improve the performance of decoding scheme.

In the following, we assume BPSK modulation, which maps a codeword  $C = [c_1 \ c_2 \ \dots \ c_N]$  into a transmitted sequence  $X = \{x_1, x_2, \dots, x_N\}$ , according to  $x_n = 2c_n - 1$ . Then  $X$  is transmitted over a channel corrupted by AWGN. The received value corresponding to  $x_n$  after the demodulator is  $r_n = x_n + v_n$ , where  $v_n$  is a random variable with zero mean and variances  $N_0/2$ ,  $N_0 = 2\sigma_n^2$ .

#### 3.1. BP algorithm

We assume an LDPC code with parity check matrix  $H = (h_{ij})_{M \times N}$  is used for error correction. We denote the set of information bits  $i$  that participate in check by  $N(i) = \{j : h_{ij} = 1\}$ . Similarly, we defined the set of checks in which information bits  $j$  participates as  $M(j) = \{i : h_{ij} = 1\}$ . We also denote  $N(i) \setminus j$  as the set  $N(i)$  with bit  $j$  excluded, and  $M(j) \setminus i$  as the set  $M(j)$  with check  $i$  excluded. The LLR  $u_{i,j}$  is the information which is sent from check node  $i$  to bit node  $j$ . The LLR  $v_{i,j}$  is the information of which is sent from bit check  $j$  to check node  $i$ . We can write the LLR BP algorithm as follows:

**Step1:** Initialization: For each  $i, j$ , set:

$$v_{i,j} = 2r_j / \sigma_n^2 \quad (2)$$

**Step2:** Processing in check nodes: For each  $i, j$ ,

$$u_{i,j} = 2 \tanh^{-1} \left[ \prod_{k \in N(i) \setminus j} \tanh(v_{i,k} / 2) \right] \quad (3)$$

Where function  $\tanh^{-1}$  is the inverse function of  $\tanh$ .

**Step3:** Processing in information bit nodes: For each  $i, j$ ,

$$v_{i,j} = 2r_j / \sigma_n^2 + \sum_{k \in M(j) \setminus i} u_{k,j} \quad (4)$$

**Step4:** Hard decision and stopping criterion test:

$$v_j = 2r_j / \sigma_n^2 + \sum_{k \in M(j)} u_{k,j} \quad (5)$$

$$\begin{cases} \hat{x}_j = 0, & v_j > 0 \\ \hat{x}_j = 1, & v_j \leq 0 \end{cases} \quad (6)$$

If  $H\hat{x} = \mathbf{0}$  then  $\hat{x}$  is considered as a valid decoded word and the decoding process ends; if the number of iterations exceeds some maximum number and  $\hat{x}$  is not a valid codeword, a failure is declared and the decoding process ends; otherwise the decoding repeats.

### 3.2 Modified BP Decoding Schemes

In the first decoding cycle, we decode codeword according to eq. (2) to eq. (6). In case of decoding failure, the packet is retransmitted. The Initialization step is as follows:

We assume an LDPC code with parity check matrix  $\mathbf{H} = (h_{ij})_{M_t \times N_t}$ ,  $t = 2, 3, 4, \dots$ ,  $M_t = M, N_t < N$ . The codeword is systemic form, and information bits are located in  $0 \sim N_t - M_t$ .

The Initialization step is as follows:

For each  $i, j$ ,

$$v_{i,j} = \begin{cases} \alpha v'_{i,j}, & i \leq M, j \leq N_t - M_t \\ 2r_j / \sigma_n^2, & \text{other} \end{cases} \quad (7)$$

Where  $v'_{i,j}$  the result of the former decoding result is  $\alpha$  is determined as:

$$\alpha = f\left(\sqrt{\frac{4}{\sigma_n^2}}\right), \quad f(x) = \int_{-\infty}^{+\infty} \frac{e^{-\left(\frac{t-x^2/2}{2\delta^2}\right)^2}}{\sqrt{2\pi x^2}} \log[1 + e^{-t}] dt \quad (8)$$

Where the function  $f(x)$  is given as:

$$f(x) = \begin{cases} a_{1,1}x^3 + b_{1,1}x^2 + c_{1,1}x, & 0 \leq x \leq 1.6363 \\ 1 - e^{a_{1,2}x^3 + b_{1,2}x^2 + c_{1,2}x + d}, & 1.6363 \leq x \leq 10 \\ 1, & x \geq 10 \end{cases} \quad (9)$$

$$a_{1,1} = -0.0421061, b_{1,1} = 0.29252,$$

$$c_{1,1} = -0.00640081, a_{1,2} = 0.00181491,$$

$$b_{1,2} = -0.142675, c_{1,2} = -0.0822054, d = 0.0549608$$

Step 2 to step 4 according to eq. (3) to eq. (6).

## 4. Simulation Results

We chose (2304, 1920) LDPC code<sup>[5]</sup> with 5/6 rate, from which we can obtain LDPC codes with 1/2 rate, 2/3 rate, 3/4 rate, 4/5 rate by puncturing some of the data bits. We set Stop-and-Wait retransmission protocol, while the maximum number of repeat transmission is 4. The maximum decoding iterative numbers are 20. The simulations were performed on AWGN channel. The results are shown from Figure 1 to Figure 2, in which we could see iterative numbers and throughput performance comparisons of modified BP algorithm and general BP algorithm. The new decoding scheme obtains higher throughput performance, especially between -3dB and 0dB. Furthermore, the iterative times of the new decoding scheme is less than its counterpart, which implies the reducing of transmission delay.

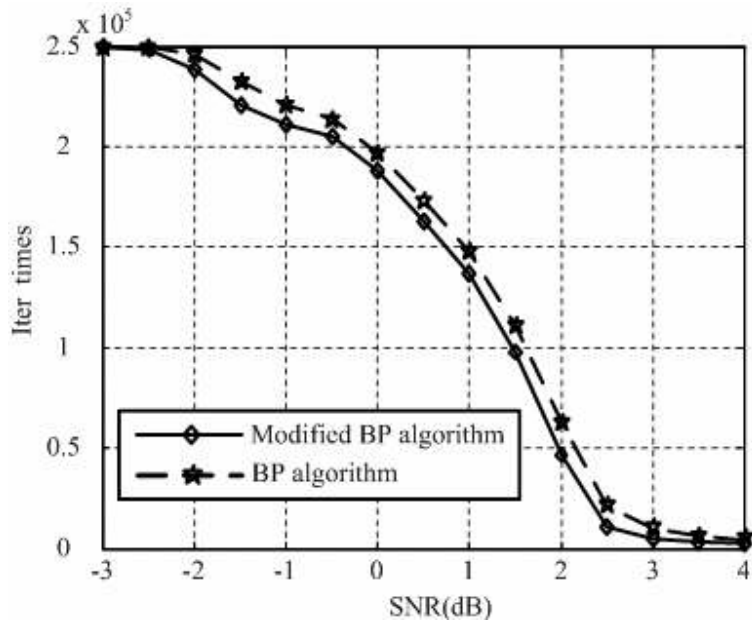


Figure 1 Iterative times performance comparison of decoding schemes

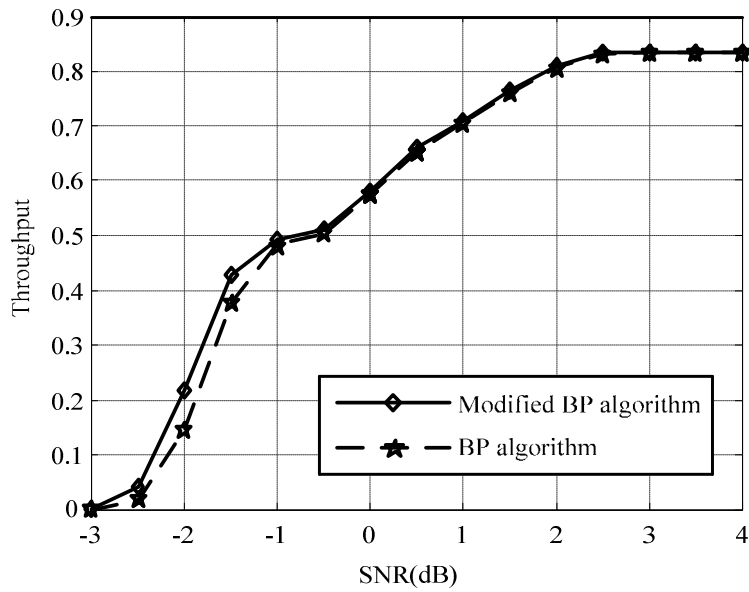


Figure 2 Throughput performance comparison of decoding schemes

## 5. Conclusions

A modified BP algorithm based on RC-LDPC code has been proposed. Instead of the initial reliabilities of the previous decoding process, the final reliabilities could be selected as the initial reliabilities of the previously received bits during the latest decoding process. This makes the best use of the results of previous decoding. And the results of simulation show that better performance both on system throughput and iterative times can be obtained.

## 6. References

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