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The Input-output's Control Strategy of the Fashion Company

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Abstract

This paper deals with the problems of control strategy of the fashion company. The input-output model is founded based on the singular stochastic dynamic Leontief input-output model. A new mathematic method is applied to study the singular systems without converting them into general linear systems. The parameter uncertainties are considered and are assumed to be time-invariant and Markovian jumping. A new stability criterion for singular system is given to ensure the robust stability of singular input-output model in terms of linear matrix inequality. Finally, the corresponding computer control algorithm is provided.

Keywords: *Stochastic Dynamical Leontief input-output model; Control Strategy; Singular linear system*

1. Introduction

Recently Chinese fashion companies faces the severe situation. The production cycle of Chinese fashion companies is often more than 90 days. This brings many risks to companies' run. So much attention has been paid for the control of the input-output problem. Firstly, economists studied the static economic systems to find how the fashion company arranges the production plan. They hope that the exact plan can help the company to control the risk. However, most companies' run are dynamic and always changed. So the economists want to study the current production state of a fashion company and research how various policies can be used to move the system from its present status to a future more desirable state when they deal with those dynamic systems. A number of fundamental notions and methods based on the theory of economical cybernetics have been extended to the area of economics [1]. The work is mainly characterized by the study of increasingly larger deterministic models and by movements into control theory. These pioneers' work were followed by many new area of studies. Robust control is applied to economical cybernetics lately [2]. Many methods and results on robust economic cybernetics have been reported [3]. Reference [4], [5] investigated the problem of consumption and investment decision with a higher interest rate. The input-output model of a company is established and the problem of the control strategy is studied in [6].

When a more detailed description of the production side of a fashion company is desired with the development of macroeconomics, this leads to a so-called input-output analysis (i.e. input-output economics). Harvard Professor Leontief, who opened the door to the input-output economics, put forward the Leontief "production" (or "input-output") model in 1949. Then in the region of input-output economics, many models were established to describe the real economics [7, 8]. Firstly, the general linear system models were well investigated. However, many models are singular linear systems, which are much harder to solve than general linear systems. Singular systems are also referred to as implicit systems, descriptor systems, generalized systems, generalized state-space systems, differential-algebraic systems or semi-state systems [9]. In the area of cybernetics there was a rapid progress in the control of singular systems because such

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systems can describe many real systems such as economic systems [10]. Many classic results of regular systems were extended to descriptor systems [11-13]. Recently, an algebraic method via a linear matrix inequality has been applied to the singular systems [14-18]. On the other hand, singular systems in economics are generally converted into general linear systems by means of the selection of state vector, control vector and output vector. Little research is about the direct disposal method to singular linear economic models. In this paper we will directly deal with a kind of singular linear systems in input-output economics instead of converting them into general systems. The dynamic Leontief input-output model supposes that all of coefficients of the equation are static. But in fact, economics system usually shows its complexity and this makes the coefficients of equation variable with the time. To reflect this kind of change, Markovian jumping system is utilized to describe the shift of coefficients of dynamic Leontief input-output equation.

The purpose of this paper is the research of the input-output problem of fashion companies. A new input-output model, which is an extended economic discrete-time singular dynamic input-output model with Markovian jumping parameters, is founded. Then the stability problem of the new input-output model, which belongs to singular nonlinear systems, is investigated. This paper will directly treat with the singular system via the linear matrix inequality approach. We will consider a sufficient condition under which the discrete-time Leontief dynamic input-output system is admissible. It is hoped that this condition can be easily tested. Then we will design the corresponding state feedback controller.

2. Input-output Model

In this section, the stochastic dynamic Leontief input-output model will be detailed described. Consider the economic dynamic input-output model described by

$$x(k) = Ax(k) + B[x(k+1) - x(k)] + Y(k) .$$

The vector $x(k) = [x_1(k) \cdots x_n(k)]^T \in R^n$ is the total output vector and $x_i(k)$ is the total output from sector i . $Y(k) = [y_1(k) \cdots y_n(k)]^T$ is the final net product vector and $y_i(k)$ denotes the final net product of sector i . The matrix $A = [c_{ij}]$ is the direct consumption coefficient matrix, $B = [d_{ij}]$ is the capital coefficient matrix.

In fact, the vector $Y(k)$ can be considered as the economic discrete-time singular dynamic Leontief input-output model's control vector, because we can indirectly affect the quantity of final net product by controlling the quantity of investment. Then $x(k)$ can be treated as state vector. Thus the dynamic input-output model can be turned into state space model. In economics, capital coefficient matrix B is not always invertible, because the product of some sectors can not be treat as capital product and applied to invest. So the dynamic input-output model can be rewritten as

$$Bx(k+1) = (I - A + B)x(k) - Y(k) \tag{1}$$

where $rankB = r < n$. This system is a singular linear system. So model (1) is an economic discrete-time singular dynamic input-output system. In fact, the direct consumption coefficient matrix A is not unchangeable and we suppose that A is uncertain. If A and $Y(k)$ are uncertain and have the forms of $A - \Delta A$ and $(I + \Delta D)Y(k)$, the discrete-time singular dynamic Leontief input-output system can be as follows:

$$Bx(k+1) = (I - A + B + \Delta C)x(k) - (I + \Delta D)Y(k) \tag{2}$$

If there are metrices $E = B$, $C = I - A + B$, $u(k) = -Y(k)$, the system (1) above can turn into

$$Ex(k+1) = Cx(k) + u(k) \tag{3}$$

where $rankE = rankB = r \leq n$. This dynamic input-output model is not a precise model, but it describes the real economics system more correctly because of the existence of uncertain factors in society. When parameter uncertainties are considered, the dynamic input-output model(2) can be

$$Ex(k+1) = (C + \Delta C)x(k) + (I + \Delta D)u(k) \tag{4}$$

where ΔC and ΔD are unknown constant matrices.

Obviously, the parameter uncertainties are not very big because of real economics system's limitation. So, we can assume that

$$[\Delta C \quad \Delta D] = \mathrm{M}F(\sigma)[N_1 \quad N_2] \tag{5}$$

where the matrices M , N_1 and N_2 are determinate real constant matrices. Function $F(\sigma)$ is unknown uncertain matrix satisfying

$$F(\sigma)F(\sigma)^T \leq I \tag{6}$$

with $\sigma \in \Omega$.

Remark 1: The definitions above are widely used in the area of the robust control problem of singular uncertain systems [15], [19], [20].

In this paper, the parameter uncertainness is introduced and the parameters are considered as the Markovian jumping parameters. That is the coefficient matrices C and E will be considered to Markovian jumping parameters. Then the system (3) turns into

$$E(r(t))x(k+1) = (C(r(t)) + \Delta C)x(k) + (I + \Delta D)u(k) \; . \tag{7}$$

And, the function $r(t)$ is a random function representing a continuous-time discrete-state Markovian process which takes value in a finite set $M = \{1, 2, 3 \cdots, s\}$ and has the transition probability matrix $\Pi = [\pi_{ij}]_{i,j \in M}$. And $\pi_{ij} > 0$ is the transition probability from mode i to mode j . Then the system (7) is a stochastic dynamic input-output system.

3. Main Result

In this section, we will give a computer control algorithm to the computer control problem of dynamic input-output system. Our aim of designing this strategy is that the dynamical input-output model can move to the desired state even though the system is perturbed by some uncertain factors. Most economic problems are stochastic. The input-output model is also uncertain. Some time series in this system are known to contain much noise. So, the control in this case has uncertainty and risk. Some methods need be adopted to let the economic system with the effect of noise run correctly.

For the dynamical input-output model and its equivalent form (2), reference [19] provides a new method, which can be used to design a desired state feedback controller. Using the similar method of [19], we bring forward the following definition for dynamical input-output model.

Definition 1: Dynamical uncertain input-output model (2) is said to be generalized quadratic stable if there are matrices $P > 0$ and Q, S s. t.

$$\omega_1 + \omega_1^T < 0$$

$$B^T S = 0 ,$$

where

$$\omega_1 = \frac{1}{2}(I - A + B)^T P(I - A + B) + (I - A + \Delta A)^T PB + (I - A + B + \Delta A)^T SQ^T .$$

Then, one LMI-based method will be established for solving generalized quadratic stability of (2). We conclude this result by presenting two preliminary theorems.

Lemma 1 [21]: If there are matrices A, B, C and A is symmetrical, we can know that:

$$A + BFC + (BFC)^T < 0$$

for any F satisfying $F^T F \le 0$ if and only if there exists a scalar $\varepsilon > 0$ s. t.

$$A + \varepsilon BB^T + \varepsilon^{-1} C^T C < 0$$

Lemma 2 [22]: For positive-definite matrix Q and matrices P , R there is the following conclusion:

$$P^T R + R^T P \le R^T QR + P^T Q^{-1} P .$$

Now, we will design the computer control algorithm.

Theorem 1: The dynamical input-output model (2) is generalized quadratic stabilizable if there exist scalars $\varepsilon > 0 , \eta > 0$, matrices $P > 0$ and Q s. t.

$$P^{-1} - \varepsilon^{-1} MM^T > 0 \tag{8}$$

and

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} \\ * & -\Xi_{22} \end{bmatrix} < 0 \tag{9}$$

where

$$\Gamma = P^{-1} - \varepsilon^{-1} MM^T \tag{10}$$

$$\begin{aligned}\Xi_{11} = {} & QS^T (I - A + B) + \left[QS^T (I - A + B)\right]^T - B^T PB + \varepsilon N_1^T N_1 + \varepsilon^{-1} QS^T MM^T SQ^T \\ & + (I - A^T + B^T + \varepsilon^{-1} QS^T MM^T)\Gamma^{-1} \times (I - A + B + \varepsilon^{-1} MM^T SQ^T)\end{aligned} \tag{11}$$

$$\Xi_{12} = QS^T + (I - A^T + B^T + \varepsilon^{-1} QS^T MM^T)\Gamma^{-1} + \varepsilon N_1^T N_2 \tag{12}$$

$$\Xi_{22} = \Gamma^{-1} + \varepsilon N_2^T N_2 + \eta I . \tag{13}$$

In this case, a robust stable state feedback control criteria can be

$$Y(k) = \Xi_{22}^{-1} \Xi_{12}^T x(k) . \tag{14}$$

Proof: Applying controller (14), formular (2) turns into the following closed-loop system:

$$Bx(k + 1) = [I - A + B - \Xi_{22}^{-1} \Xi_{12}^T + MF(\sigma)(N_1 - N_2 \Xi_{22}^{-1} \Xi_{12}^T)]x(k) . \tag{15}$$

It is easy to prove that $\Xi_{22} > 0$. By the Schur complement formula, formula (9) is equivalent with:

$$\Xi_{11} - \Xi_{12}\Xi_{22}^{-1}\Xi_{12}^{T} < 0 \quad . \tag{16}$$

It is easy to show $\Gamma^{-1} + \varepsilon N_2^T N_2 < \Xi_{22}$. Then, there is the following inequality:

$$\begin{aligned}
& QS^T(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T) + (I - A + B - \Xi_{22}^{-1}\Xi_{12}^T)^T SQ^T \\
& + \varepsilon(N_1 - N_2\Xi_{22}^{-1}\Xi_{12}^T)(N_1 - N_2\Xi_{22}^{-1}\Xi_{12}^T)^T \\
& + [(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T)^T + \varepsilon^{-1}QS^T MM^T] \\
& \times \Gamma^{-1}(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T + \varepsilon^{-1}MM^T SQ^T) \\
& \le QS^T(I - A + B) + (I - A^T + B^T)SQ^T + \varepsilon N_1^T N_1 \\
& + (I - A^T + B^T + \varepsilon^{-1}QS^T MM^T) \\
& \times \Gamma^{-1}(I - A + B + \varepsilon^{-1}MM^T SQ^T) \\
& - \Xi_{12}\Xi_{22}^{-1}\Xi_{21}
\end{aligned} \tag{17}$$

Together with (16), it implies

$$\begin{aligned}
& QS^T(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T) + (I - A + B - \Xi_{22}^{-1}\Xi_{12}^T)^T SQ^T \\
& - B^T PB + \varepsilon(N_1 - N_2\Xi_{22}^{-1}\Xi_{12}^T)(N_1 - N_2\Xi_{22}^{-1}\Xi_{12}^T)^T + \varepsilon^{-1}QS^T MM^T SQ^T \\
& + [(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T)^T + \varepsilon^{-1}QS^T MM^T] \times \Gamma^{-1}(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T + \varepsilon^{-1}MM^T SQ^T) \\
& < 0
\end{aligned} \quad . \tag{18}$$

By the Schur complement formula, formula (18) implies

$$\begin{bmatrix} F_{11} & F_{12} & QS^T M \\ F_{21} & -P & PM \\ M^T SQ^T & M^T P & -\varepsilon I \end{bmatrix} < 0$$

where

$$F_{11} = QS^T(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T) + (I - A + B - \Xi_{22}^{-1}\Xi_{12}^T)^T SQ^T + \varepsilon(N_1 - N_2\Xi_{22}^{-1}\Xi_{12}^T)(N_1 - N_2\Xi_{22}^{-1}\Xi_{12}^T)^T$$

$$F_{12} = (I - A + B - \Xi_{22}^{-1}\Xi_{12}^T)^T P$$

$$F_{21} = P(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T) .$$

Then, we have

$$\begin{bmatrix} G_{11} & A^T P \\ PA & -P \end{bmatrix} + \varepsilon^{-1}\begin{bmatrix} QS^T M \\ PM \end{bmatrix}\begin{bmatrix} QS^T M \\ PM \end{bmatrix}^T + \varepsilon\begin{bmatrix} N_1^T \\ 0 \end{bmatrix}\begin{bmatrix} N_1^T \\ 0 \end{bmatrix}^T < 0 \tag{19}$$

where

$$G_{11} = QS^T(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T) + (I - A + B - \Xi_{22}^{-1}\Xi_{12}^T)^T SQ^T - B^T PB .$$

From lemma 1 and formula (5)-(6), we can know that

$$\begin{bmatrix} QS^T\Delta C + (QS^T\Delta C)^T & \Delta C^T P \\ P\Delta C & 0 \end{bmatrix} \le \varepsilon^{-1}\begin{bmatrix} QS^T M \\ PM \end{bmatrix}\begin{bmatrix} QS^T M \\ PM \end{bmatrix}^T + \varepsilon\begin{bmatrix} N_1^T \\ 0 \end{bmatrix}\begin{bmatrix} N_1^T \\ 0 \end{bmatrix}^T \quad . \tag{20}$$

This, together with (19) and lemma 2, will imply

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} < 0 \tag{21}$$

where

$$H_{11} = QS^T(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C) + (I - A + B - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C)^T SQ^T - B^T PB$$

$$H_{12} = (I - A + B - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C)^T P$$

$$H_{21} = P(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C)$$

$$H_{22} = -P .$$

Using the Schur complement formula, we can get the following formula:

$$(I - A - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C)^T P(I - A - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C)$$

$$+(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C)PB + (I - A + B - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C)^T SQ^T$$

$$+B^T P(I - A - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C)^T + QS^T(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C) = \tilde{W}_1 + \tilde{W}_1^T < 0$$

where

$$\tilde{W}_1 = 0.5 \times (I - A - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C)^T P(I - A - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C) + (I - A + B - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C)PB$$

$$+(I - A + B - \Xi_{22}^{-1}\Xi_{12}^T + \Delta C)^T SQ^T .$$

Then, according to Definition 1, we can prove that the system (2) is generalized quadratic stabilizable. This completes the proof.

Remark: According Theorem 1, we can design the computer control algorithm, because formula (8) and formula (9) are line matrix inequalities which can be easily solved by computers.

4. Conclusion

In this paper, the stabilization of the discrete singular dynamic Leontief input-output model of fashion companies has been studied. This singular linear Leontief model is investigated without being transformed into the general linear system. A sufficient condition has been proved and the design of state feedback controller has been completed. The analysis of the problem is solved via linear matrix inequality approach. Finally, the control algorithm is completed.

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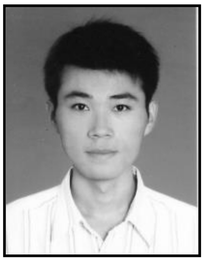

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