Information Fusion Filter for Multi-rate Multi-sensor Systems

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Abstract

This paper is concerned with the information fusion filtering problem for a class of multi-rate multi-sensor systems, where the system is described at the highest sampling rate and different sensors may have different lower sampling rates. Firstly, the local filters (LFs) at state updating points are proposed by using the LFs at measurement sampling points. Then, the distributed suboptimal fusion filter is obtained by the well-known covariance intersection fusion (CIF) algorithm. The filtering error variance matrices are derived to obtain the fusion weights. The computational cost is reduced since the cross-covariance matrices between any two local filters are avoided. Simulation example verifies the correctness and feasibility of the proposed algorithm.

Keywords: Multi-rate; Multi-sensor; Covariance Intersection Fusion; Filter

1. Introduction

In recent years, the information fusion filtering problem for systems with multiple sensors has gained lots of attention due to the widely applications such as target tracking, single processing and robot navigation [1].

When the stochastic system is measured by multiple sensors, there are two approaches to process the multiple measurements from different sensors. One is the centralized filter, the other is distributed fusion filter [2]. The centralized filter can give the global optimal estimation. However, it can result in high computational cost due to the high dimension augmented measurement. Recently, many researchers are focus on the distributed filter since it is easily for fault detection and isolation. There are many popular distributed fusion algorithms such as federated square-root filter [3], maximum likelihood fusion algorithm [4], weighting fusion algorithms in the linear minimum variance sense [5] and CIF algorithm [6]. However, the above algorithms are only suitable for single rate systems.

For multi-rate systems, the first important study goes back to the switch decomposition technique proposed by Kranc [7]. Generally, there are two methods for the state estimation problem for multi-rate systems. One is based on multiscale system theory and the other is based on Kalman filtering theory. On the basis of multiscale system theory, many famous fusion strategies are proposed for multirate systems with the sampling rate ratio being one or positive integer power to two. However, the state estimators are very complex and high computational burden. On the basis of Kalman filtering theory, many useful filtering strategies are proposed such as optimal signal reconstruction method [8], asynchronous centralized fusion algorithm [9], sequential filtering algorithm [10], left synchronously lifting technology [11], and measurement augmented approach [12]. But, the computational cost of the above filtering strategies is high since they are given by

state/measurement augmentation. In order to avoid the state augmentation, the multi-rate fusion problem is transformed into an equivalent single rate fusion problem. For non-uniform sampling systems, a distributed fusion filter is given [13], for uniform sampling systems, the corresponding distributed fusion filters are also given in [14-15]. However, the cross-covariance matrices are needed to obtain the fusion weights. Furthermore, the multi-rate fusion filters for systems with network constrains are studied in [16-18]. However, the state/measurement augmentation is not avoided.

In this article, a new LF is obtained by using the LF for the measurement sampling points. Then, the well-known CIF algorithm is used to fuse all the LFs. The proposed new LF can reduce the computational cost since the state augmentation is avoided. The proposed CIF filter can further reduce the computational burden since the cross-covariance matrices are avoided.

2. Problem Formulation

Consider the following linear discrete-time stochastic multi-rate systems measured by Q sensors

$$x(tb + b) = \Phi x(tb) + \Gamma w(tb) \tag{1}$$

$$z_r(tb_r) = H_r x(tb_r) + v_r(tb_r) \ , \ b_r = c_r b \ , \ r \in \{1, \cdots, Q\} \tag{2}$$

where $x(tb) \in \Box^n$ is the state vector, $z_r(tb_r) \in \Box^{n_r}$, $r \in \{1, \cdots, Q\}$ are the measured outputs, Φ, Γ and H_r, $r \in \{1, \cdots, Q\}$ are the constant matrices with suitable dimensions. $w(tb) \in \Box^q$ and $v_r(tb_r) \in \Box^{n_r}$, $r \in \{1, \cdots, Q\}$ are white noises. The state $x(tb)$ is updated at the highest rate with a period b and the r th sensor measurement $z_r(tb_r) \in \Box^{n_r}$ is sampled at a lower rate with a period $b_r = c_r b$ where c_r is a positive integer. Superscript r stands for the r th sensor.

Assumption 1 $w(tb)$ and $v_r(tb_r)$ are uncorrelated white noises with zero means and variance matrices R^w and R_r^v, respectively.

Assumption 2 The initial state vector $x(0)$ is uncorrelated with $w(tb)$ and $v_r(tb_r)$, and satisfies

$$\mathrm{E}\{x(0)\} = \sigma \ \text{ and } \ \mathrm{E}\{(x(0) - \sigma)(x(0) - \sigma)^{\mathrm{T}}\} = \Omega \tag{3}$$

Our objective is to find the distributed CIF filter $\hat{x}_o(tb)$ based on the measurement information $(z_r(tb_r), z_r(tb_r - b_r), \cdots, z_r(0))$, $r \in \{1, \cdots, Q\}$.

3. Distributed Information Filter

3.1. Local Filter

In this subsection, we first establish the state space model at the measurement sampling points of the r th sensor[2].

$$x(tb_r + b_r) = \Phi_r x(tb_r) + \Gamma_r w_r(tb_r) \tag{4}$$

$$z_r(tb_r) = H_r x(tb_r) + v_r(tb_r) \tag{5}$$

where

$\Phi_r = \Phi^{c_r}, \Gamma_r = [\Phi^{c_r-1}\Gamma \quad \Phi^{c_r-2}\Gamma \quad \cdots \quad \Gamma], w_r(tb_r) = [w^{\mathrm{T}}(tb_r) \quad w^{\mathrm{T}}(tb_r + b) \quad \cdots \quad w^{\mathrm{T}}(tb_r + b_r - b)]^{\mathrm{T}}$.
Further we have the following statistical property

$$\mathrm{E}[w_r(tb_r)] = 0 \ , \ R_r^w = \mathrm{E}[w_r(tb_r)w_r^{\mathrm{T}}(tb_r)] = \mathrm{diag}(R^w, \cdots, R^w)_{c_r q \times c_r q} \tag{6}$$

Observe that systems (4) and (5) are transformed into the single rate systems. In the following, we will give the LFs $\hat{x}_r(tb_r | tb_r)$ and the corresponding estimation error

variance matrices $\underline{P}_r(tb_r \mid tb_r)$ at the measurement sampling points for the r th sensor subsystem by applying the classical Kalman filter.

Lemma 1[2] Under Assumptions 1 and 2, the LFs for systems (4) and (5) are computed by

$$\underline{\hat{x}}_r(tb_r \mid tb_r) = \underline{\hat{x}}_r(tb_r \mid tb_r - b_r) + K_r(tb_r)\underline{\varepsilon}_r(tb_r) \tag{7}$$

$$\underline{\hat{x}}_r(tb_r + b_r \mid tb_r) = \Phi_r \underline{\hat{x}}_r(tb_r \mid tb_r - b_r) + L_r(tb_r)\underline{\varepsilon}_r(tb_r) \tag{8}$$

$$\underline{\varepsilon}_r(tb_r) = z_r(tb_r) - H_r \underline{\hat{x}}_r(tb_r \mid tb_r - b_r) \tag{9}$$

$$K_r(tb_r) = \underline{P}_r(tb_r \mid tb_r - b_r) H_r^{\mathrm{T}} \underline{Q}_{\varepsilon_r}^{-1}(tb_r) \tag{10}$$

$$L_r(tb_r) = \Phi_r K_r(tb_r) \tag{11}$$

$$\underline{Q}_{\varepsilon_r}(tb_r) = H_r \underline{P}_r(tb_r \mid tb_r - b_r) H_r^{\mathrm{T}} + R_r^v \tag{12}$$

$$\underline{P}_r(tb_r \mid tb_r) = \underline{P}_r(tb_r \mid tb_r - b_r) - K_r(tb_r)\underline{Q}_{\varepsilon_r}(tb_r)K_r^{\mathrm{T}}(tb_r) \tag{13}$$

$$\underline{P}_r(tb_r + b_r \mid tb_r) = (\Phi_r - L_r(tb_r)H_r)\underline{P}_r(tb_r \mid tb_r - b_r)(\Phi_r - L_r(tb_r)H_r)^{\mathrm{T}}$$
$$+ \Gamma_r R_r^w \Gamma_r^{\mathrm{T}} + L_r(tb_r)R_r^v L_r^{\mathrm{T}}(tb_r) \tag{14}$$

where $\underline{\hat{x}}_r(tb_r \mid tb_r)$ is the filter, $\underline{\hat{x}}_r(tb_r \mid tb_r - b_r)$ is the one-step predictor at the measurement sampling points, $\underline{\varepsilon}_r(tb_r)$ is the innovation sequence with variance $\underline{Q}_{\varepsilon_r}(tb_r)$, $K_r(tb_r)$ is the filtering gain, $L_r(tb_r)$ is the one-step prediction gain, $\underline{P}_r(tb_r \mid tb_r)$ is the filtering error variance matrix, $\underline{P}_r(tb_r \mid tb_r - b_r)$ is the one-step prediction error variance matrix. The initial values are $\underline{\hat{x}}_r(0 \mid -1) = \sigma$ and $\underline{P}_r(0 \mid -1) = \Omega$.

Next, we will derive the LFs $\hat{x}_r(tb_r - lb \mid tb_r)$, $l \in \{1, \cdots, c_r\}$ at the state updating points by applying the LFs $\underline{\hat{x}}_r(tb_r \mid tb_r)$ at the measurement sampling points and the state update equation.

Theorem 1 Under Assumptions 1 and 2, for systems (1) and (2), we have the LFs $\hat{x}_r(tb_r - lb \mid tb_r)$, $l = 1, 2, \cdots, c_r$ at the state updating points as follows

$$\hat{x}_r(tb_r - lb \mid tb_r) = \Phi^{-l}[\underline{\hat{x}}_r(tb_r \mid tb_r) - \sum_{k=1}^{l} \Phi^{k-1} \Gamma \hat{w}_r(tb_r - kb \mid tb_r)] \tag{15}$$

$$\hat{w}_r(tb_r - kb \mid tb_r) = M_r(tb_r - kb \mid tb_r)\underline{\varepsilon}_r(tb_r), \, k \in \{1, \cdots, l\} \tag{16}$$

$$M_r(tb_r - kb \mid tb_r) = R^w \Gamma^{\mathrm{T}} (\Phi^{k-1})^{\mathrm{T}} H_r^{\mathrm{T}} \underline{Q}_{\varepsilon_r}^{-1}(tb_r), \, k \in \{1, \cdots, l\} \tag{17}$$

where $\underline{\varepsilon}_r(tb_r)$, $\underline{Q}_{\varepsilon_r}(tb_r)$, $\underline{\hat{x}}_r(tb_r \mid tb_r)$ are given by Lemma 1.

Proof: From the iteration of (1)

$$x(tb_r) = \Phi^l x(tb_r - lb) + \sum_{k=1}^{l} \Phi^{k-1} \Gamma w(tb_r - kb), \, l \in \{1, \cdots, c_r\} \tag{18}$$

By arranging (18)

$$x(tb_r - lb) = \Phi^{-l}[x(tb_r) - \sum_{k=1}^{l} \Phi^{k-1} \Gamma w(tb_r - kb)] \tag{19}$$

Taking projection of both sides of (19) on the linear space $L(z_r(tb_r), z_r(tb_r - b_r), \cdots, z_r(0))$, we have (15). The white noise smoother $\hat{w}_r(tb_r - kb \mid tb_r)$ is obtained by the following recursive projection equation

$$\hat{w}_r(tb_r - kb \mid tb_r) = \hat{w}_r(tb_r - kb \mid tb_r - b_r) + M_r(tb_r - kb \mid tb_r)\underline{\varepsilon}_r(tb_r), \, k \in \{1, \cdots, l\} \tag{20}$$

From the uncorrelation of $w(tb_r - kb)$ and $L(z_r(tb_r - b_r), \cdots, z_r(0))$, we have the white noise predictors

$$\hat{w}_r(tb_r - kb \mid tb_r - b_r) = 0, \, k \in \{1, \cdots, l\} \tag{21}$$

Substituting (21) into (20), we can obtain (16). The white noise smoothing gain matrices $M_r(tb_r - kb \mid tb_r)$ are given by

$$M_r(tb_r - kb \mid tb_r) = \mathrm{E}[w(tb_r - kb)\underline{\varepsilon}_r^{\mathrm{T}}(tb_r)]\underline{Q}_{\varepsilon_r}^{-1}(tb_r) \tag{22}$$

Substituting (2) into (9), the innovation sequences can be rewritten as

$$\underline{\varepsilon}_r(tb_r) = H_r\underline{\tilde{x}}_r(tb_r \mid tb_r - b_r) + v_r(tb_r) \tag{23}$$

From $\underline{\tilde{x}}_r(tb_r \mid tb_r - b_r) = x(tb_r) - \underline{\hat{x}}_r(tb_r \mid tb_r - b_r)$, $w(tb_r - kb) \perp \underline{\hat{x}}_r(tb_r \mid tb_r - b_r)$, (18) and Assumption 1, we have

$$\mathrm{E}[w(tb_r - kb)\underline{\varepsilon}_r^{\mathrm{T}}(tb_r)] = \mathrm{E}[w(tb_r - kb)x^{\mathrm{T}}(tb_r)]H_r^{\mathrm{T}} = R^w\Gamma^{\mathrm{T}}(\Phi^{k-1})^{\mathrm{T}}H_r^{\mathrm{T}} \tag{24}$$

Substituting (24) into (22), (17) is obtained.

Remark 1: From Theorem 1, we see that the LFs at the state updating points are derived by applying the LFs at the measurement sampling points. Therefore, compared with the state augmented approach, the proposed filter can obviously reduce the computational burden for the r th sensor subsystem. Now, we give the computational cost at each measurement sampling points: our filter is $\mathrm{O}(n^3)$ and the augmented filter in [12] is $\mathrm{O}((c_r n)^3)$. On the other hand, from the point of view of using information, the two filters have the same estimation accuracy. We verify it in the simulation research.

Now, we have obtained the LFs at the state updating points. In the following text, we need to derive the filtering error variance matrices to obtain the fusion weights.

3.2. Fusion Filter

Theorem 2 Under Assumptions 1 and 2, for systems (1) and (2), we have the local filtering error variance matrices at the state updating points $P_r(tb_r - lb \mid tb_r)$, $l \in \{1, \cdots, c_r\}$ as follows

$$P_r(tb_r - lb \mid tb_r) = \Phi^{-l}\underline{P}_r(tb_r \mid tb_r)(\Phi^{-l})^{\mathrm{T}}$$
$$+ \sum_{k,m=1}^{l}\Phi^{k-1-l}\Gamma P_r^w(tb_r - kb, tb_r - mb \mid tb_r)\Gamma^{\mathrm{T}}(\Phi^{m-1-l})^{\mathrm{T}}$$
$$- \sum_{m=1}^{l}\Phi^{-l}P_r^{xw}(tb_r, tb_r - mb \mid tb_r)\Gamma^{\mathrm{T}}(\Phi^{m-1-l})^{\mathrm{T}} - \sum_{k=1}^{l}\Phi^{k-1-l}\Gamma P_r^{wx}(tb_r - kb, tb_r \mid tb_r)(\Phi^{-l})^{\mathrm{T}} \tag{25}$$

$$P_r^w(tb_r - kb, tb_r - mb \mid tb_r) = R^w\delta_{km} - R^w\Gamma^{\mathrm{T}}(\Phi^{k-1})^{\mathrm{T}}H_r^{\mathrm{T}}M_r^{\mathrm{T}}(tb_r - mb \mid tb_r)$$
$$- M_r(tb_r - kb \mid tb_r)H_r\Phi^{m-1}\Gamma R^w + M_r(tb_r - kb \mid tb_r)H_r\underline{P}_r(tb_r \mid tb_r - b_r)H_r^{\mathrm{T}}M_r^{\mathrm{T}}(tb_r - mb \mid tb_r)$$
$$+ M_r(tb_r - kb \mid tb_r)R_r^v M_r^{\mathrm{T}}(tb_r - mb \mid tb_r) , \quad k,m \in \{1, \cdots, l\} \tag{26}$$

$$P_r^{xw}(tb_r, tb_r - kb \mid tb_r) = (I_n - K_r(tb_r)H_r)\Phi^{k-1}\Gamma R^w, \quad k \in \{1, \cdots, l\} \tag{27}$$

where δ_{km} is Kronecker delta function, $P_r^w(tb_r - kb, tb_r - mb \mid tb_r)$ is the white noise smoothing error variance matrix, $P_r^{xw}(tb_r, tb_r - kb \mid tb_r)$ is the covariance matrix between state and system noise, also we have $P_r^{wx}(tb_r - kb, tb_r \mid tb_r) = (P_r^{xw}(tb_r, tb_r - kb \mid tb_r))^{\mathrm{T}}$. The initial values are $\underline{P}_r(tb_r \mid tb_r)$ and $\underline{P}_r(tb_r \mid tb_r - b_r)$ computed by Lemma 1.

Proof: Subtracting (15) from $x(tb_r - lb)$, we have the estimation error variance equation

$$\tilde{x}_r(tb_r - lb \mid tb_r) = x(tb_r - lb) - \hat{x}_r(tb_r - lb \mid tb_r)$$
$$= x(tb_r - lb) - \Phi^{-l}[\underline{\hat{x}}_r(tb_r \mid tb_r) - \sum_{k=1}^{l}\Phi^{k-1}\Gamma\hat{w}_r(tb_r - kb \mid tb_r)]$$
$$= \Phi^{-l}[x(tb_r) - \sum_{k=1}^{l}\Phi^{k-1}\Gamma w(tb_r - kb)] - \Phi^{-l}[\underline{\hat{x}}_r(tb_r \mid tb_r) - \sum_{k=1}^{l}\Phi^{k-1}\Gamma\hat{w}_r(tb_r - kb \mid tb_r)]$$

$$= \Phi^{-l} \tilde{\underline{x}}_r(tb_r \mid tb_r) - \sum_{k=1}^{l} \Phi^{k-1-l} \Gamma \tilde{w}_r(tb_r - kb \mid tb_r) \tag{28}$$

From(28), we have the estimation error variance matrix at the state updating points

$$P_r(tb_r - lb \mid tb_r) = \mathrm{E}[\tilde{x}_r(tb_r - lb \mid tb_r)\tilde{x}_r^{\mathrm{T}}(tb_r - lb \mid tb_r)]$$

$$= \Phi^{-l} \underline{P}_r(tb_r \mid tb_r)(\Phi^{-l})^{\mathrm{T}} + \sum_{k,m=1}^{l} \Phi^{k-1-l} \Gamma \, \mathrm{E}[\tilde{w}_r(tb_r - kb \mid tb_r)\tilde{w}_r^{\mathrm{T}}(tb_r - mb \mid tb_r)]\Gamma^{\mathrm{T}}(\Phi^{m-1-l})^{\mathrm{T}}$$

$$- \sum_{m=1}^{l} \Phi^{-l} \mathrm{E}[\tilde{\underline{x}}_r(tb_r \mid tb_r)\tilde{w}_r^{\mathrm{T}}(tb_r - mb \mid tb_r)]\Gamma^{\mathrm{T}}(\Phi^{m-1-l})^{\mathrm{T}}$$

$$- \sum_{k=1}^{l} \Phi^{k-1-l} \Gamma \, \mathrm{E}[\tilde{w}_r(tb_r - kb \mid tb_r)\tilde{\underline{x}}_r^{\mathrm{T}}(tb_r \mid tb_r)](\Phi^{-l})^{\mathrm{T}} \tag{29}$$

Subtracting (16) from $w(tb_r - kb)$, and using (23), we have the white noise estimation error equation

$$\tilde{w}_r(tb_r - kb \mid tb_r) = w(tb_r - kb) - \hat{w}_r(tb_r - kb \mid tb_r)$$

$$= w(tb_r - kb) - M_r(tb_r - kb \mid tb_r)H_r\tilde{\underline{x}}_r(tb_r \mid tb_r - b_r) - M_r(tb_r - kb \mid tb_r)v_r(tb_r) \tag{30}$$

From (30), the uncorrelation of $\tilde{\underline{x}}_r(tb_r \mid tb_r - b_r)$ and $v_r(tb_r)$, and Assumption 1, the white noise smoothing error variance matrix is computed by

$$P_r^w(tb_r - kb, tb_r - mb \mid tb_r) = \mathrm{E}[\tilde{w}_r(tb_r - kb \mid tb_r)\tilde{w}_r^{\mathrm{T}}(tb_r - mb \mid tb_r)], \, k,m \in \{1,\cdots,l\}$$

$$= \mathrm{E}[w(tb_r - kb)w^{\mathrm{T}}(tb_r - mb)] - \mathrm{E}[w(tb_r - kb)\tilde{\underline{x}}_r^{\mathrm{T}}(tb_r \mid tb_r - b_r)]H_r^{\mathrm{T}}M_r^{\mathrm{T}}(tb_r - mb \mid tb_r)$$

$$- M_r(tb_r - kb \mid tb_r)H_r\mathrm{E}[\tilde{\underline{x}}_r(tb_r \mid tb_r - b_r)w^{\mathrm{T}}(tb_r - mb)]$$

$$+ M_r(tb_r - kb \mid tb_r)H_r\mathrm{E}[\tilde{\underline{x}}_r(tb_r \mid tb_r - b_r)\tilde{\underline{x}}_r^{\mathrm{T}}(tb_r \mid tb_r - b_r)]H_r^{\mathrm{T}}M_r^{\mathrm{T}}(tb_r - mb \mid tb_r)$$

$$+ M_r(tb_r - kb \mid tb_r)\mathrm{E}[v_r(tb_r)v_r^{\mathrm{T}}(tb_r)]M_r^{\mathrm{T}}(tb_r - mb \mid tb_r) \tag{31}$$

where

$$\mathrm{E}[w(tb_r - kb)\tilde{\underline{x}}_r^{\mathrm{T}}(tb_r \mid tb_r - b_r)]$$

$$= \mathrm{E}[w(tb_r - kb)x^{\mathrm{T}}(tb_r)] = \mathrm{E}[\sum_{p=1}^{k} w(tb_r - kb)w^{\mathrm{T}}(tb_r - pb)\Gamma\Phi^{p-1}] = R^w\Gamma^{\mathrm{T}}(\Phi^{k-1})^{\mathrm{T}} \tag{32}$$

In (32), we use the fact that $\tilde{\underline{x}}_r(tb_r \mid tb_r - b_r) = x(tb_r) - \hat{\underline{x}}_r(tb_r \mid tb_r - b_r)$ and $w(tb_r - kb) \perp \hat{\underline{x}}_r(tb_r \mid tb_r - b_r)$. Substituting (32) into (31), we have (26). Using $\tilde{\underline{x}}_r(tb_r \mid tb_r) \perp \hat{w}_r(tb_r - kb \mid tb_r)$, we have

$$P_r^{xw}(tb_r, tb_r - kb \mid tb_r) = \mathrm{E}[\tilde{\underline{x}}_r(tb_r \mid tb_r)\tilde{w}_r^{\mathrm{T}}(tb_r - kb \mid tb_r)]$$

$$= \mathrm{E}[\tilde{\underline{x}}_r(tb_r \mid tb_r)(w(tb_r - kb) - \hat{w}_r(tb_r - kb \mid tb_r))^{\mathrm{T}}] = \mathrm{E}[\tilde{\underline{x}}_r(tb_r \mid tb_r)w^{\mathrm{T}}(tb_r - kb)] \tag{33}$$

From (7) and (23), we have the following filtering error equation

$$\tilde{\underline{x}}_r(tb_r \mid tb_r) = x(tb_r) - \hat{\underline{x}}_r(tb_r \mid tb_r) = \tilde{\underline{x}}_r(tb_r \mid tb_r - b_r) - K_r(tb_r)\varepsilon_r(tb_r)$$

$$= [I_n - K_r(tb_r)H_r]\tilde{\underline{x}}_r(tb_r \mid tb_r - b_r) - K_r(tb_r)v_r(tb_r) \tag{34}$$

Substituting (34) into (33), and using the Assumption 1, we have

$$P_r^{xw}(tb_r, tb_r - kb \mid tb_r) = [I_n - K_r(tb_r)H_r]\mathrm{E}[\tilde{\underline{x}}_r(tb_r \mid tb_r - b_r)w^{\mathrm{T}}(tb_r - kb)] \tag{35}$$

Using (32), we can obtain (27). Substituting (31) and (33) into (29), we can obtain (25).

Next, we shall give the following suboptimal CIF filter $\hat{x}_o(tb)$, $l \in \{1,\cdots,c_r\}$ by using the well known CIF algorithm[6]

$$\hat{x}_o(tb) = \sum_{r=1}^{Q} A_r(tb)\hat{x}_r(tb_r - lb \mid tb_r) \tag{36}$$

where LFs $\hat{x}_r(tb_r - lb \mid tb_r)$, $r \in \{1,\cdots,Q\}$ are computed by Theorem 1. The corresponding fusion weights $A_r(tb)$ are computed by

$$A_r(tb) = \omega_r(tb)(\sum_{r=1}^{Q} \omega_r(tb)P_r^{-1}(tb_r - lb \mid tb_r))^{-1}P_r^{-1}(tb_r - lb \mid tb_r) \tag{37}$$

where $\omega_r(tb)$ are computed by

$$\omega_r(tb) = \mathrm{tr}(P_r^{-1}(tb_r - lb \mid tb_r)) \Big/ \sum_{r=1}^{Q} \mathrm{tr}(P_r^{-1}(tb_r - lb \mid tb_r)) , \quad 0 \le \omega_r(tb) \le 1 , \quad \sum_{r=1}^{Q} \omega_r(tb) = 1 \quad (38)$$

where $P_r(tb_r - lb \mid tb_r)$ are given by Theorem 2.

Remark 2 From (36)-(38), we know that the proposed distributed fusion filter is suboptimal since the correlation between any two LFs is ignored. On the other hand, the computational cost is reduced since the computation of the cross-covariance matrices between any two sensor subsystems is avoided.

4. Simulation Results

Consider the following tracking system with three sensors

$$x(tb + b) = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} x(tb) + \sqrt{10} \begin{bmatrix} \dfrac{b^2}{2} \\ b \end{bmatrix} w(tb) \quad (39)$$

$$z_r(tb_r) = H_r x(tb_r) + v_r(tb_r) , \quad r = 1, 2, 3 \quad (40)$$

where $x(tb) = \begin{bmatrix} s(tb) & \dot{s}(tb) \end{bmatrix}^T$, $s(tb)$ and $\dot{s}(tb)$ are the position and velocity at tb time instant. b is the state update period. Our aim is to find the distributed suboptimal CIF filter $\hat{x}_o(tb)$.
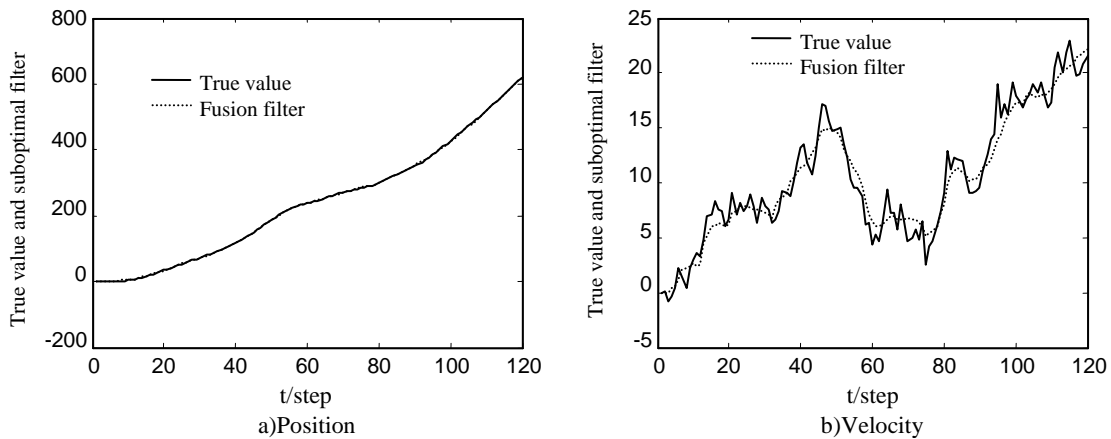

a)Position   b)Velocity

Figure 1. Distributed Suboptimal CIF Filter

a)Position   b)Velocity
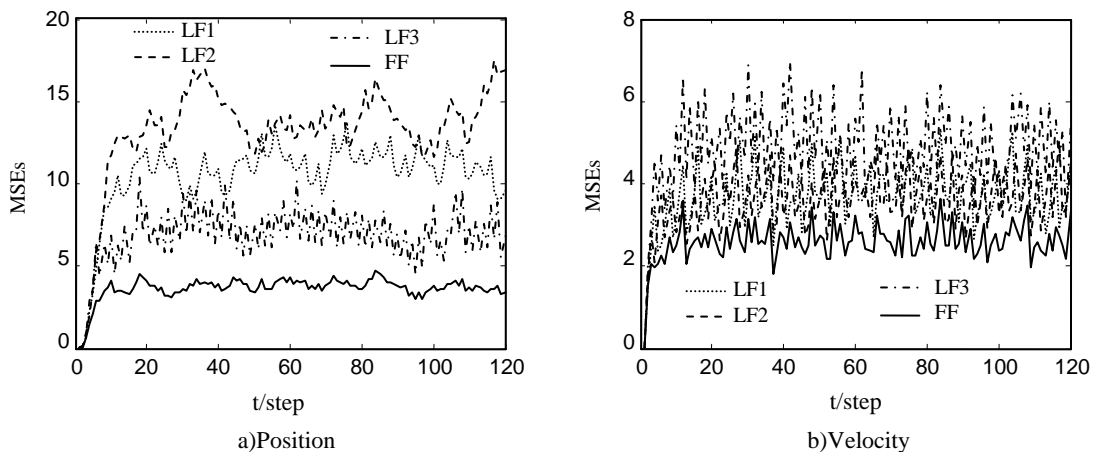
Figure 2. Comparison Curves of MSEs of LFs and the CIF Filter

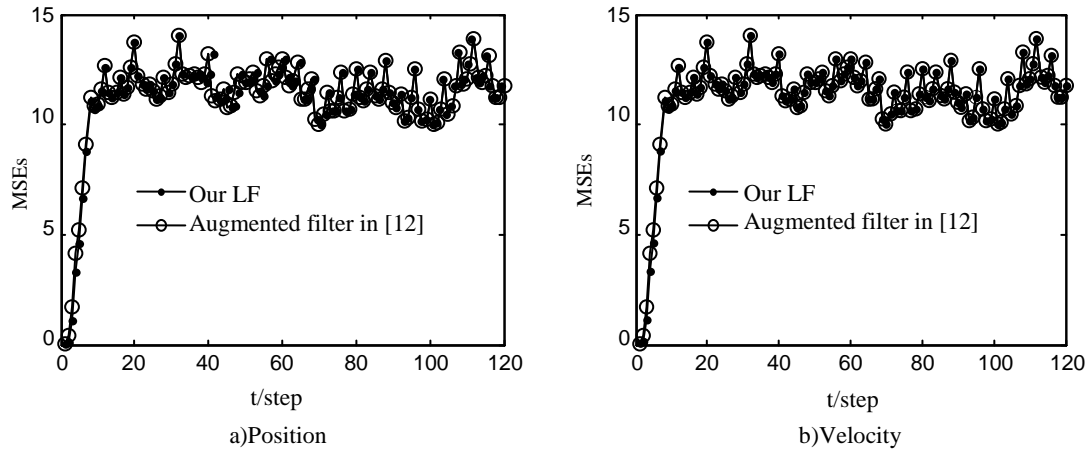a)Position   b)Velocity

Figure 3. Comparison Curves of MSEs of our LF and the Augmented Filter

In the simulation, we take 120 data and set $b = 0.5$ second, $c_1 = 4$, $c_2 = 3$, $c_3 = 2$, $H_1 = [1 \quad 1]$, $H_2 = [0.5 \quad 1]$, $H_3 = [1 \quad 0]$, $R^w = 1$, $R_1^v = 30$, $R_2^v = 15$, $R_3^v = 20$, $x(0) = [0 \quad 0]^T$ and $P_0 = 0.1 I_2$.

By using Lemma 1, we can obtain the LFs $\hat{\underline{x}}_r(tb_r | tb_r)$ and the corresponding filtering error variance matrices $\underline{P}_r(tb_r | tb_r)$ at the measurement sampling points. Further, the LFs $\hat{x}_r(tb_r - lb | tb_r)$, $l = 1, 2, \cdots, c_r$ at state updating points can be obtained by using Theorem 1. The corresponding error variance matrices are given by Theorem 2. Then, we can obtain the suboptimal CIF filter $\hat{x}_o(tb)$ by using (36)-(38).

The state tracking curves are given in Figure 1. The comparison curves of the mean square error (MSE) of the CIF and all LFs by 200 times Monte Carlo tests are given in Figure 2. From Figure 2, we see that proposed CIF has the higher accuracy than any LFs. In order to compare the computational cost and the accuracy of proposed LF and the filter in [12], we give comparison curves of the two filters for the first sensor subsystem. At the measurement sampling points, the computational cost of our LF is $O(n^3) = O(2^3) = O(8)$, the computational cost of the filter in [12] is $O((c_1 n)^3) = O(8^3) = O(512)$, $(n = 2, c_1 = 4)$. We see that our filter can obviously reduce the computational cost. The comparison curves of MSE by 200 times Monte Carlo tests are given in Figure 3. From Figure 3, we see that the two filters have the same estimation accuracy.

5. Conclusion

In this paper a multi-rate multi-sensor distributed suboptimal information fusion filtering problem for linear stochastic system is studied. Firstly, the LFs at the measurement sampling points are given by the classical Kalman filter. Then, the LFs at the state updating points are derived by the LFs at the measurement sampling points. The corresponding filtering error variance matrices are derived to obtain the fusion weights. Furthermore, a CIF algorithm is applied to fuse all the LFs. Simulation results show better performance than any LFs.

Acknowledgements

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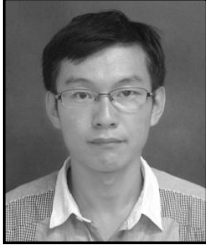
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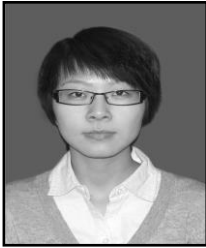
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