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Multi-Futures Hedging Strategy Based on Markov Regime-Switching Panel GARCH Model

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Abstract

This paper first develops a new Markov regime-switching panel GARCH model (MSPG) for multi-futures hedging, which has two merits: First, the panel GARCH model is more parsimonious than multivariate GARCH model. Secondly, the MSPG model allowing for regime shifts which voids the spurious volatility persistence problem. In this article, two-state MSPG model is applied to study the multi-futures hedging, and the comparison of hedging performance with pure panel GARCH is made, which indicates that MSPG model outperforms pure panel GARCH model by superior hedging effectiveness.

Keywords*: Hedging; Panel GARCH; Markov regime switching model; Persistence in variance; Structural break*

1. Introduction

Hedging is one of the main functions of a futures market, which is widely applied and studied for hedging the market risk. Recent years, many researchers have been showing great interest in hedging with futures. This is witnessed by the large amount of papers emerged in this field. The most common issue involved in hedging is the estimation of the optimal hedge ratio (OHR).

Several distinct methods have been proposed to estimate the OHR. The estimation of OHR depends on the specified objective function. Actually various objective functions are currently being used. As we all know that the initial purpose of hedging is just for avoiding risk (even though nowadays people may use it for profiting), so the most widely-used hedging strategies are the ones on the basis of minimization of the variance of hedged portfolio, see [1]. We also choose this most common and easy understandable objective function for studying in this paper.

The benchmark approach is estimate the slope coefficient from the spot return on the futures return using ordinary least squares (OLS). An improved approach proposed is based on the error-correction (EC) model for avoiding spurious regression. While the third and also the most widely used approach is based on the generalized autoregressive conditional heteroscedasticity (GARCH) model to estimate time-varying OHRs. Among the dynamic hedge models such as Bivariate-GARCH model [2], Kalman filter [3] and so on. Instead of searching for possible information variables, GARCH models cover its own history of spot and futures prices to explain variations in variances and covariance. [4] also uses SV model to study the issue of multiperiod hedging.

[1] Proposed to measure hedging effectiveness by the percentage reduction in the (unconditional) variance of the hedged portfolio relative to the unhedged spot position. This measure has been widely adopted to compare different hedge strategies in their usefulness to reduce risk ever since proposed.

However, under certain circumstances it is difficult to find the exactly corresponding futures for many commodities. Then we can turn to look for some futures relevant to the chosen spot to replace the corresponding future and subsequently hedging portfolio can be

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constructed to hedge. Therefore, the research on multi-futures hedging makes more sense than the ideal one to one futures hedging.

[5] Built a multi-futures hedging stochastic model stochastic process with the restriction of wealth of asset portfolios being non-negative. [6] Set up an optimal hedging model under the assumption that the price of the futures and actual subject to Ito stochastic process. [7] Showed that multiple-future hedging is more efficient than single future hedging.

The multi-future hedging should consider the risk correlation between futures and spots, consider both the risk correlation between futures and the risk correlation between spots. There are only few literatures on multi-future hedging. [8] Build up a multi-futures hedging model on the basis of the Variance - covariance matrix. [9] Built two futures hedging model. The model can hedge raw and processed materials, although it can hedge the products. [10] Developed a multi-future model with capital constraint.

All of the mentioned multi-futures hedging researches above unavoidably chose multivariate GARCH model to solve for the OHR. Though the sophisticated multivariate GARCH models could also characterize the heteroskedasticity and cross-sectional dependence, even the unrestricted Vector GARCH (p, q) model involves $[2N(N+1)+N(N+1)(p+q)]/4$ individual parameters [11], which is so-called the curse of dimensionality. The applied literatures on the vector GARCH model often constrain the dimension N to be no more than 3, which greatly limits the study of constructing hedging portfolio. This means if more than 3 relevant futures are chosen for hedging, the VGARCH model may not be able to deal with it. Panel model could solve this tough thing. [12] First proposed Pooled Panel-GARCH (PP-GARCH) model and study the inflation uncertainty in the G7 countries. [13] Examined the empirical relationship between output growth and volatility based on panel data of G7 countries over the period 1965–2007. [11] Showed the persistence of European stock markets with structural break by panel GARCH model.

However, there may exist regime shifts in the market for different time periods, which would account for the persistence to some extent. [14] And [15] proposed the Markov regime-switching model with ARCH volatility, and [16] applied this model recently. And [17] first proposed the GRS model. [18] developed Multivariate Regime–Switching GARCH and empirically study international Stock Markets.

Based on these researches, we put forward Markov regime-switching panel GARCH model. And MSPG model we developed is different from the model proposed by [18]. They allow the panel GARCH to be subject to regime shift modelled simply by adding dummy variables. But it is more reasonable that the time point of regime changes is unobservable. By comparison, our model takes regime shifts as a latent variable which obeys a first-order Markov process.

The rest of the paper is organized as follows: The next section introduces Markov switching panel GARCH model. Section 2 reviews the definition of panel GARCH model. Section 3 introduces MSPG model. Section 4 empirically studies multiple-future hedging. Section 5 concludes.

2. Panel Garch Model

Consider the following general pooled regression model.

$$y_{it} = \mu_i + \varepsilon_{it}, \quad i = 1, \ldots, N;\ t = 1, \ldots, T \tag{1}$$

For a cross-section of N time series and T time periods, the conditional mean equation for time series y_{it} can be expressed as a panel with fixed effects. Where μ_i represents possible individual effects, and ε_{it} is a disturbance term with a zero mean and normal distribution along with the following conditional moments:

$$E[\varepsilon_{it}\varepsilon_{js} \mid I_{t-1}] = 0 \qquad for\ i \neq j\ and\ t \neq s \tag{2}$$

$$E\ [\varepsilon_{i\ t}\ \varepsilon_{\ j}|_{s}\ I_{-1\ t}\ ] = 0 \qquad = \quad f\ o\ r\ \not{i} \tag{3}$$

$$E[\varepsilon_{it}\varepsilon_{js} \mid I_{t-1}] = \sigma^{2}_{ij,t} \qquad for\ i \neq j\ and\ t = s \tag{4}$$

$$E[\varepsilon_{it}\varepsilon_{js} \mid I_{t-1}] = \sigma^{2}_{it} \qquad for\ i = j\ and\ t = s \tag{5}$$

Where I_{t-1} represents the information set at time $t-1$.

The first condition implies there is no non-contemporaneous cross-sectional correlation, and the second condition implies there is no autocorrelation. The third and fourth assumptions gives the general conditions of the conditional variance–covariance process. The conditional variance and covariance processes of time series follow a GARCH (1, 1):

$$\begin{aligned} \sigma^{2}_{it} &= \phi_i + \delta\varepsilon^{2}_{it-1} + \gamma\sigma^{2}_{it-1}, \qquad i = 1, \ldots, N \\ \sigma_{ij,t} &= \varphi_{ij} + \rho\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \lambda\sigma_{ij,t-1}, \quad i \neq j \end{aligned} \tag{6}$$

Where $\phi_i > 0,\ \gamma + \delta < 1,\ \lambda + \rho < 1$. ϕ_i And φ_{ij} represent the corresponding individual specific effects. In this model, the full parameter vector has $\frac{1}{2}N(N+1)+4$ elements which means it's more parsimonious than unrestricted vector GARCH model is. In matrix notation, Equation (1) can simply be expressed as:

$$y_t = \mu + \varepsilon_t, \quad t = 1, \ldots, T \tag{7}$$

Where the disturbance term has a multivariate normal distribution $N(0, \Omega_t)$. The log-likelihood function can be written as:

$$L = -\frac{1}{2}NT\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln|\Omega_t| - \frac{1}{2}\sum_{t=1}^{T}[(y_t - \mu)' \times \Omega_t^{-1} \times (y_t - \mu)] \tag{8}$$

3. Markov Regime Switching Panel GARCH Model

3.1. Definition of Model

$$r_t = \mu + \varepsilon_t, \quad t = 1, \ldots, T \tag{9}$$

Where r_t represents a M-dimensional time series, is usually of asset return series, μ represents the mean of time series r_t and the residual ε_t subject to

$$\varepsilon_t \mid \Psi_{t-1} = H^{1/2}_{t|s_t}\xi_t \tag{10}$$

Where Ψ_{t-1} represents the information set at time $t-1$. $\xi_t \sim N(O_{M\times 1}, I_M)$, I_M denotes the identity matrix of dimension M. And $\{s_t\}$ is a Markov chain with finite state space $\{1, 2, \ldots, k\}$ and a primitive $k \times k$ transition matrix P.

$$P=\begin{bmatrix} p_{11} & \cdots & p_{k1} \\ \vdots & \cdots & \vdots \\ p_{1k} & \cdots & p_{kk} \end{bmatrix} \tag{11}$$

Where $p_{ij}=p(s_t=j\mid s_{t-1}=i), i, j=1,\ldots,k$. If $i\neq j$, p_{ij} gives the probability that state i will be followed by state j, if $i=j$, p_{ii} gives the probability that there will be no change in the state of the market in the following period *etc*. These transition probabilities are assumed to remain constant between successive periods. $\{\xi_t\}$ And $\{s_t\}$ are assumed to be independent. Denote $\pi_t=[\pi_{`t},\cdots,\pi_{kt}]'$ and $\pi_\infty=[\pi_{1\infty},\cdots,\pi_{k\infty}]'$ the distribution at time t and the stationary distribution of the Markov chain respectively. $H_{s_t,t}$ represents the conditional covariance matrix could be modelled as Markov regime switching panel GARCH model as the following:

$$\sigma^2_{i,t|s_t}=\phi_{s_t,i}+\delta_{s_t}\varepsilon^2_{i,t-1}+\gamma_{s_t}\sigma^2_{i,t} \qquad for\ i=1,\ldots M \tag{12}$$

$$\sigma^2_{ij,t|s_t}=\varphi_{s_t,ij}+\rho_{s_t}\varepsilon_{i,t-1}\varepsilon'_{j,t-1}+\lambda_{s_t}\sigma_{ij,t} \qquad for\ i\neq j \tag{13}$$

This could be expressed as a compact form

$$H_{t|s_t}=\Phi_{s_t}+A_{s_t}\varepsilon_{t-1}\varepsilon'_{t-1}+B_{s_t}H_{t-1} \tag{14}$$

where $\Phi_{s_t}=\begin{bmatrix} \phi_{s_t,1} & \cdots & \varphi_{s_t,1i} & \cdots & \varphi_{s_t,1M} \\ \vdots & \ddots & \vdots & & \vdots \\ \varphi_{s_t,i1} & \cdots & \phi_{s_t,i} & \cdots & \varphi_{s_t,iM} \\ \vdots & & \vdots & \ddots & \vdots \\ \varphi_{s_t,M1} & \cdots & \varphi_{s_t,Mi} & \cdots & \phi_{s_t,M} \end{bmatrix}, A_{s_t}=\begin{bmatrix} \delta_{s_t} & \cdots & \rho_{s_t} & \cdots & \rho_{s_t} \\ \vdots & \ddots & \vdots & & \vdots \\ \rho_{s_t} & \cdots & \delta_{s_t} & \cdots & \rho_{s_t} \\ \vdots & & \vdots & \ddots & \vdots \\ \rho_{s_t} & \cdots & \rho_{s_t} & \cdots & \rho_{s_t} \end{bmatrix},$

$$B_{s_t}=\begin{bmatrix} \gamma_{s_t} & \cdots & \lambda_{s_t} & \cdots & \lambda_{s_t} \\ \vdots & \ddots & \vdots & & \vdots \\ \lambda_{s_t} & \cdots & \gamma_{s_t} & \cdots & \lambda_{s_t} \\ \vdots & & \vdots & \ddots & \vdots \\ \lambda_{s_t} & \cdots & \lambda_{s_t} & \cdots & \gamma_{s_t} \end{bmatrix}.$$

Subject to $\phi_{s_t,i}>0,\ \gamma_{s_t}+\delta_{s_t}<1,\ \lambda_{s_t}+\rho_{s_t}<1$. $\phi_{s_t,i}$ and $\varphi_{s_t,ij}$ represents the corresponding individual specific effects. Both the unconditional correlation matrix and the parameters driving the system dynamics can be regime dependent. The Markov chain is governed by transition matrix P in (11). We name the model defined by (9)-(14) by Markov switching panel GARCH model, or briefly, MSPG (1, 1; k). In the single-regime case, *i.e.*, MSPG (p, q; 1) is just the most general conditional heteroskedastic specification panel $GARCH(p,q)$ process.

In this article we estimate two-regime MRS model, i.e., MSPG (p, q; 2). As a matter of fact, we found some scholars also estimate three-regime MRS models, say [19] study the relationship between spot and futures in stock indices; nonetheless, in their study the third regime seems to capture only jumps in the futures prices when switching between contracts of different maturities but it does not reflect fundamental changes in market. So we choose a two-regime MRS model because this model captures the dynamics of the spot and futures returns in a more parsimonious and efficient way and is totally reasonable because two regimes imply the periods of less volatile and more volatile. Additionally, for the popularity of GARCH (1, 1) model, and its well-characterizing performance, we build up MSPG (1, 1, 2) model in this paper.

3.2. Estimating Log-Likelihood Function

In the following passage we illustrate clearly how the likelihood function for MSPG model is constructed. We need to estimate log-likelihood function

$$\begin{aligned} LL &= \sum_{t=1}^{T} Log(f_t(r_t \mid \Phi_{t-1}) \\ &= \sum_{t=1}^{T} Log[\sum_{k=1}^{2} \Pr(s_t = k \mid \Phi_{t-1}) f(r_t \mid s_t = k, \Phi_{t-1})] \\ &= \sum_{t=1}^{T} Log[\mathrm{p}_{1t} f_{1,t} + (1 - p_{1t}) f_{2,t}] \end{aligned} \tag{15}$$

Where

$$\begin{aligned} f_{k,t} &= f(r_t \mid s_t = k, \Psi_{t-1}) \\ &= \frac{1}{\sqrt{(2\pi)^M \mid H_{t|s_t} \mid}} \exp\{-\frac{1}{2}\varepsilon_t' H_{t|s_t}^{-1} \varepsilon_t\}, \quad k = 1, 2 \end{aligned} \tag{16}$$

Which is the state-contingent conditional probability density function of return in which k denotes the state.

This requires us to obtain $p_{1,t}$, $f_{1,t}$ and $f_{2,t}$.

1. Let's get the expression of $p_{1,t}$ first. Similar to the case of univariate Markov regime-switching GARCH model proposed by [17, 20], the recursive expression of the state probability $p_{1,t}$ of being in regime 1 at time t up to time $t-1$ is as following:

$$\begin{aligned} p_{1,t} &= \Pr(s_t = 1 \mid \Psi_{t-1}) \\ &= \sum_{i=1}^{2} \Pr(s_t = 1 \mid s_{t-1} = i, \Psi_{t-1}) \Pr(s_{t-1} = i \mid \Psi_{t-1}) \\ &= P \times \Pr(\mathrm{s}_t = 1 \mid \Psi_{t-1}) + (1 - \mathrm{Q})[1 - \Pr(\mathrm{s}_t = 1 \mid \Psi_{t-1})] \end{aligned} \tag{17}$$

Where P and Q are transition probabilities, which are the probabilities that the regime 1 and 2 at time $t-1$ followed by regime 1 and 2 at time t respectively.

$$P = \Pr[s_t = 1 \mid s_{t-1} = 1] \tag{18}$$

$$Q = \Pr[s_t = 2 \mid s_{t-1} = 2] \tag{19}$$

Next by Bayes' Rule, we have

$$\begin{aligned}\Pr(s_t \mid \Psi_{t-1}) &= \Pr(s_{t-1} = 1 \mid r_{t-1}, \Psi_{t-2}) \\ &= \frac{f(r_{t-1} \mid s_{t-1} = 1, \Psi_{t-2}) \Pr(s_{t-1} = 1 \mid r_{t-1}, \Psi_{t-2})}{f(r_{t-1} \mid s_{t-1} = 1, \Psi_{t-2}) \Pr(s_{t-1} = 1 \mid r_{t-1}, \Psi_{t-2}) + f(r_{t-1} \mid s_{t-1} = 2, \Psi_{t-2}) \Pr(s_{t-1} = 2 \mid r_{t-1}, \Psi_{t-2})}\end{aligned} \tag{20}$$

Where $r_t = [r_{1,t} \quad r_{2,t} \quad \cdots \quad r_{M,t}]$ is a vector of returns at time t, and $f(r_{t-1} \mid s_{t-1} = 1, \Psi_{t-2})$ and $f(r_{t-1} \mid s_{t-1} = 2, \Psi_{t-2})$ are defined by (16) denoted by $f_{1,t-1}$ and $f_{2,t-1}$. Substituting (20) into (17) yields the recursive expression of the regime probability $p_{1,t}$.

$$\begin{aligned}p_{1,t} &= \Pr(s_t = 1 \mid \Psi_{t-1}) \\ &= P\left[\frac{f_{1,t-1} p_{1,t-1}}{f_{1,t-1} p_{1,t-1} + f_{2,t-1}(1 - p_{1,t-1})}\right] + (1 - Q)\left[\frac{f_{2,t-1}(1 - p_{1,t-1})}{f_{1,t-1} p_{1,t-1} + f_{2,t-1}(1 - p_{1,t-1})}\right]\end{aligned} \tag{21}$$

And the steady-state probabilities of s_t is taken as the initial value for recursive expression of the regime probability $p_{1,t}$ is

$$\Pr(s_1 = 1 \mid \Psi_0) = \frac{1 - Q}{2 - P - Q} \tag{22}$$

Where P and Q are transition probabilities defined in (18) and (19).

2. Then we turn to get $f_{k,t}$ $(k = 1, 2)$ defined in (16). We can see the conditional covariance matrix depends on the whole history information up to time $t - 1$. This is a well-known problem known as "path-dependency problem" [14, 15] just de the general regime-switching model faces. Referring to the method Gray (1996) introduces for the univariate case, we recombine the conditional variance:

$$\begin{aligned}\sigma_{i,t}^2 &= E[r_{i,t}^2 \mid \Psi_{t-1}] - E[r_{i,t} \mid \Psi_{t-1}]^2 \\ &= p_{1,t}(\mu_{1i}^2 + \sigma_{i,t|s_t=1}^2) + (1 - p_{1,t})(\mu_{2i}^2 + \sigma_{i,t|s_t=2}^2) - \mu_i^2, \quad i = 1, \ldots, M \\ &= p_{1,t}\sigma_{i,t|s_t=1}^2 + (1 - p_{1,t})\sigma_{i,t|s_t=2}^2\end{aligned} \tag{23}$$

It shows clearly that the conditional variance depends only on the current regime, not on all the history information, so the path-dependency problem is solved. Similarly, the recombining method for the residual is given by

$$\begin{aligned}\varepsilon_{i,t} &= r_{i,t} - E[r_{i,t} \mid \Psi_{t-1}] \\ &= r_{i,t} - \mu_i \quad (i = 1, \ldots, M)\end{aligned} \tag{24}$$

Where $p_{1,t} = \Pr(s_t = 1 \mid \Psi_{t-1})$ is the regime probability of being in state 1 given all information up to time $t-1$. In order to make the likelihood function easily estimated. The calculation of regime probabilities was illustrated by [17, 20] with a nonlinear recursive expression of the regime probability as a function of transition probabilities and conditional distributions.

Furthermore, MSPG is a multivariate model, which includes not only variances and residuals, but also the covariance of spot and futures returns to be recombined.

$$\begin{aligned}\sigma_{ij,t} &= \mathrm{cov}(r_{i,t}, r_{j,t} \mid \Psi_{t-1}) \\ &= E[r_{i,t} r_{j,t} \mid \Psi_{t-1}] - E[r_{i,t} \mid \Psi_{t-1}]E[r_{j,t} \mid \Psi_{t-1}] \\ &= p_{1,t}(\mu_i\mu_j + \sigma_{ij,t|s_t=1}) + (1-p_{1,t})(\mu_i\mu_j + \sigma_{ij,t|s_t=2}) - \mu_i\mu_j \\ &= p_{1,t}\sigma_{ij,t|s_t=1} + (1-p_{1,t})\sigma_{ij,t|s_t=2}\end{aligned} \tag{25}$$

Figure 1 illustrates the evolution of conditional covariance matrix in our path-independent MSPG model. Each conditional covariance matrix depends only on the current regime, not on the entire history of regimes. After recombining at time 1, $\sigma^2_{i,1}$ represents the conditional variance at time 2, given the process is in regime 1 and $\varepsilon_{i,1}$ represents the residual in the mean equation (9). $\sigma_{ij,1}$ is the covariance of time series i and j after recombining at time 1. H's and ε's are covariance matrix and residual matrix defined in (14). Φ's, A's and B's are corresponding coefficient matrix defined in (14). Since MSPG model is multivariate model, we also similarly collapse the conditional covariance in possible regime into a single conditional variance and residual at each point in time.

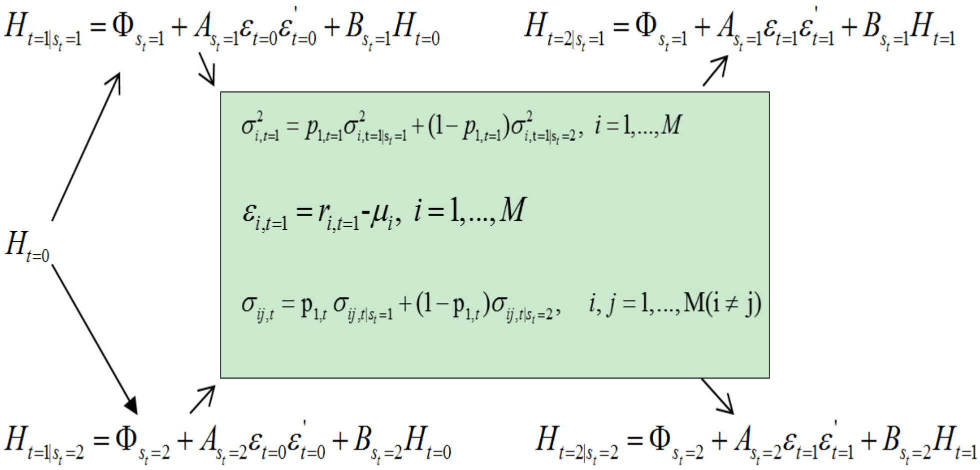

Figure 1. Evolution of Conditional Covariance Matrix

Now we can see from above equations that the conditional covariance depends only on current regime, not on the whole past history of information up to time t.

Having specified the conditional variance $\sigma^2_{i,t}$ and conditional covariance $\sigma_{ij,t}$ and the dynamics of the switching between regimes, then the conditional covariance matrix $H_{t|s_t}$ depends only on current regime instead of all the past information. Then the log-likelihood function (15) can be estimated without "path-dependency problem". And then we can finally estimate the parameters

$$\{P, Q; \mu_i (i = 1, \cdots, M); \phi_{s_t, i} (i = 1, \cdots, M); \varphi_{s_t, ij} (i \neq j, i, j = 1, \cdots, M), \gamma_{s_t}, \delta_{s_t}, \lambda_{s_t}, \rho_{s_t}\}$$

4. Multiple-Futures Hedging Based On Minimizing Variance Method

4.1. Minimum Variance Hedge

Denote $\beta_{i,t} (i = 1, \cdots, n)$ the ratio of the $i-th$ future to spot at time t and then stack them into a $1 \times n$ row vector

$$\mathrm{B}_t = (\beta_{1,t}, \beta_{2,t}, \cdots, \beta_{n,t}) \tag{26}$$

The return of hedging portfolio at time t is as following:

$$r_{p,t} = r_{s,t} - \mathrm{B}_t^{'} r_{f,t} \tag{27}$$

Where $r_{p,t}$ represents the return of hedging portfolio at time t, $r_{s,t}$ represents return of spot, $r_{f,t}$ represents the vector of all the future returns. And B_t represents the hedging ratio vector $r_{f,t}$. Then we can set up our objective function:

$$\min Var(r_{p,t}) = Var(r_{s,t}) + B\,Cov(r_{fi}, r_{fj})_t B' - 2B\,C \tag{28}$$

$\sigma^2_{p,t}$ represents the variance of the portfolios' return at time t. $\sigma^2_{s,t}$ represents the variance of the spot's return. $\sigma(r_{fi}, r_{fj})_t$ represents the covariance matrix of future returns at time t. $\sigma(r_s, r_f)_t$ represents the $n \times 1$ covariance vector of futures return and spot at time t. Hedge ratios are calculated with variances and covariance estimated from (26) and (28) by time-varying minimum variance:

$$\hat{B}^{'} = \sigma(r_{fi}, r_{fj})_t^{-1} \sigma(r_s, r_f)_t \tag{29}$$

Where $Cov(r_{fi}, r_{fj})$ and $Cov(r_s, r_f)$ could be obtained from estimating MSPG model we developed above.

4.2. Hedging Performance Comparison

The variance of the estimated optimal hedged portfolio can be expressed as

$$Var_{hedged} = Var(r_s - \hat{\mathrm{B}} r_f^{'}) \tag{30}$$

Where $\hat{B}$ is the optimal hedge ratio vector estimated by (29).

Following [1], the hedging effectiveness refers to the gain or loss in the variance of terminal revenue due to the price changes in an unhedged position relative to those in a hedged position defined as

$$H_e = \frac{Var_{unhedged} - Var_{hedged}}{Var_{unhedged}} \tag{31}$$

5. Data and Empirical Analysis

5.1. Data and Empirical Results

For performance comparisons, optimal hedging portfolios are generated with MSPG (1, 1, 2) and MSPG (1, 1, 1), *i.e.*, pure panel GARCH (1, 1) model. We choose three futures contracts, CSI300 and CSI300 future, Aluminum future, Rubber future.

The data are therefore sampled daily. The sample period ranges from December 25, 2012 to September 18, 2013 and the data include 171 observations all together. The data for the period December 25, 2012 to August 15, 2013 for estimation and in-sample forecasts, and the data for the period August 16, 2013 to September 18, 2013 are used for out-of-sample forecasts.

Figure 2 shows the returns of spot: CSI300 and the returns of futures: Aluminum, Rubber, CSI300 futures. And the in-sample and out-of-sample descriptive statistics are shown in Table 1 and Table 2 respectively. The parameters of MSPG (1, 1, and 1) and MSPG (1, 1, 2) are obtained by maximizing the log-likelihood functions shown in (15) with software WINRATS 7, and they are presented in Table 3 and Table 4.

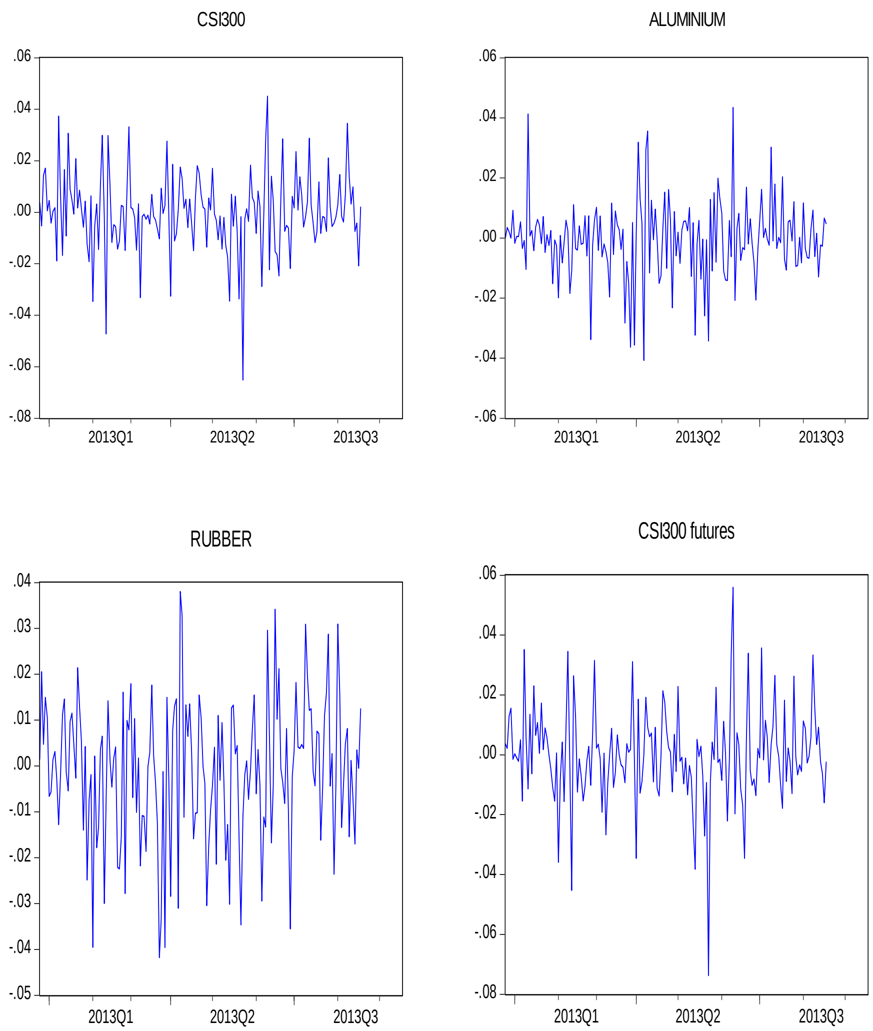

Figure 2. Returns of Spot and Futures

Table 1. In Sample Descriptive Statistics of Spots and Future Returns

	SPOT	FUTURES		
	CSI300	ALUMINIUM	RUBBER	CSI300 FUTURES
Mean	-0.000413	-0.000474	-0.001461	-0.000490
Median	0.000317	-0.000393	-0.000229	-0.000585
Std. Dev.	0.015639	0.013658	0.015522	0.016326
Skewness	-0.402220	-0.040391	-0.325934	-0.291728
Kurtosis	5.236373	4.780068	3.165583	6.240856
Jarque-Bera	34.59698	19.44789	2.770645	66.41685
Observations	147	147	147	147

Table 2. Out of Sample Descriptive Statistics of Spots and Future Returns

	SPOT	FUTURES		
	CSI300	ALUMINUIM	RUBBER	CSI300 FUTURES
Mean	0.002357	-0.001146	0.001383	0.002778
Median	-0.001615	-0.002340	0.001213	-0.000598
Std. Dev.	0.011540	0.007391	0.014023	0.012016
Skewness	0.832385	0.263429	0.314198	0.887688
Kurtosis	4.219960	1.933486	2.666543	3.389412
Jarque-Bera	4.082270	1.356072	0.484988	3.165955
Observations	23	23	23	23

Table 3. Parameter Estimates for MSPG (1, 1; 1) Model

$$r_t = \mu + \varepsilon_t, \ where\ \mu = \underset{(0.0031)}{[-0.00041} \quad \underset{(0.0045)}{-0.000474} \quad \underset{(0.0063)}{-0.001461} \quad \underset{(0.0008)}{-0.000490]}$$

$$\sigma^2_{i,t} = \phi_i + \gamma\sigma^2_{i,t-1} + \delta\varepsilon^2_{i,t-1} \quad for\ i = 1,2,3,4$$

$$\sigma_{ij,t} = \varphi_{ij} + \lambda\sigma_{ij,t-1} + \rho\varepsilon_{i,t-1}\varepsilon_{j,t-1}, \quad for\ i, j = 1,2,3,4;\ i \neq j$$

Where

$\phi_1 = 0.021$,	$\phi_2 = 0.034$,	$\phi_3 = 0.005$,	$\phi_4 = 0.046$,	$\gamma = 0.434$,	$\delta = .552$
(0.0023)	(0.0012)	(0.0042)	(0.0016)	(0.0067)	(0.0086)
$\varphi_{12} = 0.054$,	$\varphi_{13} = 0.032$,	$\varphi_{14} = 0.022$	$\lambda = 0.445$,	$\rho = 0.565$	
(0.0062)	(0.0071)	(0.0015)	(0.0023)	(0.0015)	
	$\varphi_{23} = 0.013$,	$\varphi_{24} = 0.031$			
	(0.0053)	(0.0039)			
		$\varphi_{34} = 0.049$			
		(0.0033)			

Log-likelihood -4075.087

Table 4. Parameter Estimates for MSPG (1, 1; 2) Model

$$r_t = \mu + \varepsilon_t, \ where\ \mu = \underset{(0.0031)}{[-0.00041} \quad \underset{(0.0045)}{-0.000474} \quad \underset{(0.0063)}{-0.001461} \quad \underset{(0.0008)}{-0.000490]}$$

$$P = \begin{bmatrix} 0.821 & 0.659 \\ (0.0404) & (0.0214) \\ 0.179 & 0.341 \\ (0.0312) & (0.0452) \end{bmatrix} \qquad \pi_{1,\infty} = 0.7863, \quad \pi_{2,\infty} = 0.2137$$

$$\sigma^2_{i,t} = \phi_{s_t,i} + \gamma_{s_t}\sigma^2_{i,t-1} + \delta_{s_t}\varepsilon^2_{i,t-1} \quad for\ i = 1,2,3,4$$

$$\sigma_{ij,t} = \varphi_{s_t,ij} + \lambda_{s_t}\sigma_{ij,t-1} + \rho_{s_t}\varepsilon_{i,t-1}\varepsilon_{j,t-1}, \quad for\ i, j = 1,2,3,4\ \ and\ \ i \neq j$$

Where

For $s_t = 1$

$\phi_{1,1} = 0.011$,	$\phi_{1,2} = 0.021$,	$\phi_{1,3} = 0.002$,	$\phi_{1,4} = 0.021$,	$\gamma_1 = 0.015$,	$\delta_1 = 0.027$
(0.0030)	(0.0002)	(0.0012)	(0.0061)	(0.0057)	(0.0056)
$\varphi_{1,12} = 0.022$,	$\varphi_{1,13} = 0.017$,	$\varphi_{1,14} = 0.025$	$\lambda_1 = 0.019$,	$\rho_1 = 0.011$	
(0.0023)	(0.0011)	(0.0085)	(0.0071)	(0.0027)	
	$\varphi_{1,23} = 0.011$,	$\varphi_{1,24} = 0.016$			
	(0.0013)	(0.0019)			
		$\varphi_{1,34} = 0.030$			
		(0.0022)			

For $s_t = 2$					
$\phi_{2,1} = 0.034$,	$\phi_{2,2} = 0.057$,	$\phi_{2,3} = 0.052$,	$\phi_{2,4} = 0.096$,	$\gamma_2 = 0.089$,	$\delta_2 = 0.077$
(0.0032)	(0.0021)	(0.0023)	(0.0062)	(0.0017)	(0.0066)
$\varphi_{2,12} = 0.067$,	$\varphi_{2,13} = 0.058$,	$\varphi_{2,14} = 0.073$	$\lambda_2 = 0.078$	$\rho_2 = 0.074$	
(0.0042)	(0.0011)	(0.0025)	(0.0034)	(0.0041)	
	$\varphi_{2,23} = 0.043$,	$\varphi_{2,24} = 0.065$			
	(0.0043)	(0.0076)			
		$\varphi_{2,34} = 0.079$			
		(0.0073)			
Log-likelihood -3893.067					

Table 5 shows us the in- and out-of-sample hedging performance of both MSPG (1, 1; 1) model and MSPG (1, 1; 2) model. MSPG (1, 1; 1) has 62.9574% and 69.5235% in-sample variance reduction respectively. And MSPG (1, 1; 2) has 70.5355% and 76.2231% in-sample variance reduction respectively. Then we can conclude that during the period covered in the study, the existence of regime shifts has effect on hedging performance to some extent.

Table 5. In Sample and Out Of Sample Hedging Performance Comparison

	Variance		Percentage of variance reduction	
	In sample	Out-of-sample	In sample	Out-of-sample
Unhedged	0.00024457832	0.0001331716		
Panel GARCH(1,1)	0.00009059817	0.0000405860	62.9574%	69.5235%
MSPG(1,1,2)	0.00007206378	0.0000316641	70.5355%	76.2231%

Note:

1°.The in-sample data period starts from December 25, 2012 to August 15, 2013 and the out-of-sample data period starts from August 15, 2013 to September 18, 2013.

2°. Variance is calculated according to equation (26).

3°. Percentage of variance reductions are calculated according to (31).

Figure 3 and Figure 4 show the hedge ratios of futures to spot for MSPG (1, 1; 1) MSPG (1,1;2) respectively. And Figure 5 shows us the estimates of the probability of being in state 1 for

MSPG (1, 1; 2) model.

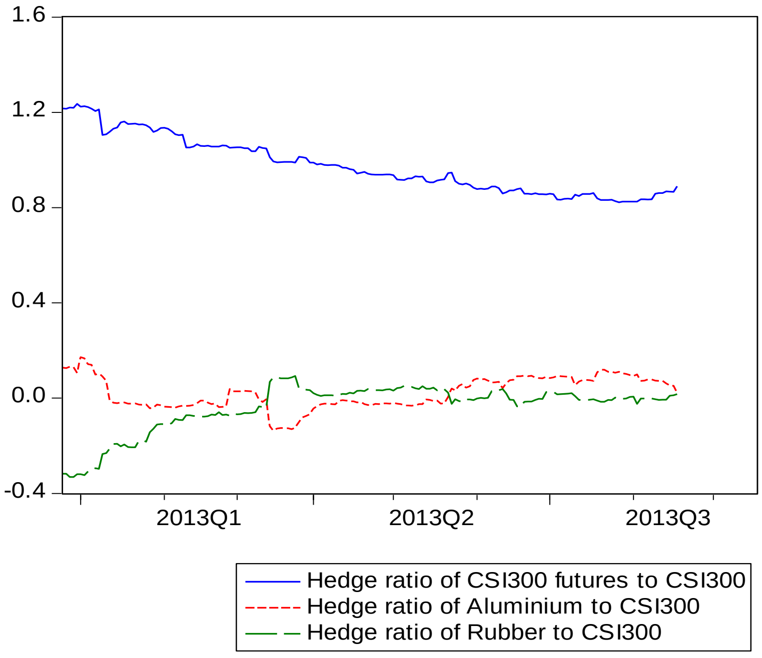

Figure 3. Hedge Ratios of Futures to Spot for MSPG (1, 1; 1) Model

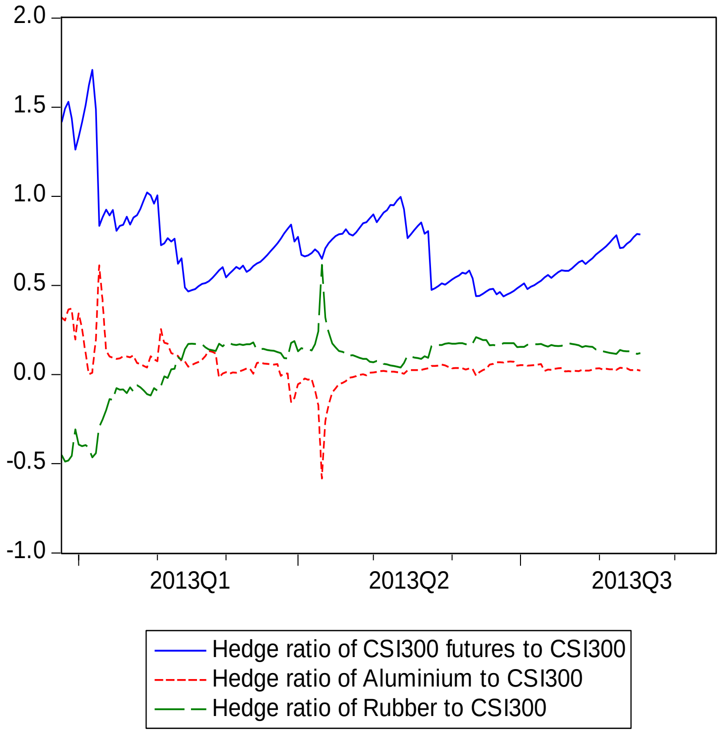

Figure 4. Hedge Ratio of Futures to Spot for MSPG (1, 1; 2) Model

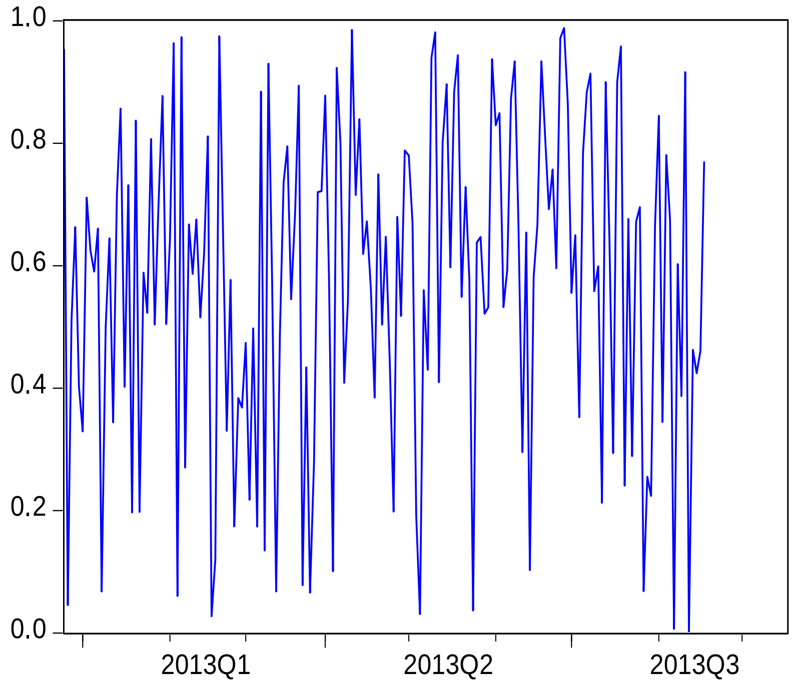

Figure 5. Smoothed Probabilities of Being in State 1 for MSPG Model

6. Conclusion

Base on the method that [17] proposed for solving the path-dependency problem for GRS model, we extend this method in the panel-GARCH framework to solve the path-dependency problem, and subsequently propose our MRSG model. After an application in multi-futures hedging based on minimum variance method, we find MSPG model have two merits: First, the panel GARCH model we introduce avoids the curse of dimensionality caused by estimating Multivariate GARCH model (more than 3 dimensions). Second, our MSPG model allows for regime shifts which improves the predictive ability of the model and hence improves the performance of hedging.

Finally, the comparison of hedging performance made between panel GARCH model and Markov regime-switching panel GARCH model (MSPG) shows that the latter exceeds the former both in-sample performance and out-of-sample, which predicts a promising application to multi-futures hedging.

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