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Joule Heating and Mass Transfer on MHD Peristaltic Hemodynamic Jeffery Fluid with Porous Medium in a Tapered Vertical Channel-Blood Flow Analysis Model

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Abstract

The aim of the present attempt was simultaneous effects of joule heating and mass transfer on hydromagnetic peristaltic hemodynamic Jeffery fluid model filled with porous medium in a tapered vertical channel. The Mathematical modeling is investigated by utilizing long wavelength and low Reynolds number assumptions. The result indicates an appreciable increase in the temperature with increase in β, Br, Pr, Da and M whereas the temperature of the fluid diminished with an increase in Jeffery fluid (λ_1) and we notice that the temperature profile is found almost parabolic in nature. The concentration distribution decreases when increase in β, Pr, Sc, Br and Sr in an entire asymmetric vertical tapered channel.

***Keywords**: Mass transfer, Joule heating, MHD, Hemodynamic fluid, porous medium and vertical tapered channel*

1. Introduction

Peristalsis consists of contraction and expansion of tract performing the progressive waves which propel the contents forward along the tract by Latham [1]. The stimulus for these waves is the distension of the tract with fluid material such as food, blood, and secretions from glands, urine, embryo and others. This distension of tract at any cross section irritates the inner layer of the tract (mucosa in case of gastrointestinal tract), at the same time the nerve plexus (network of intersecting nerves) connected with the central nervous system via fibers initiates the peristaltic waves along the walls of the tract. This mechanism regulates the flow from the area of lower pressure to area of higher pressure. Nowadays, peristalsis has exploited its significance in industry, like in sanitary fluid transport, artificial blood pumps in heart-lung machine and transport of corrosive fluids where the contact of fluid with the boundary is prohibited. Mathematical studies of peristalsis were initiated by Fung & Yih [2], Shapiro *et al*. [3] and others.

Peristaltic flow of blood under effect of a magnetic field in a non-uniform channels has been studied by Kh.Mekheimer [4]. In another attempt, T.Hayat et.al. [5] discussed on peristaltic transport of nanofluid in a compliant wall channel with convective conditions and thermal radiation. In another paper, A.M. Abd-Alla *et al*. [6] investigated on the peristaltic flow of a Jeffrey fluid under the effect of radially varying magnetic field in a tube with an endoscope. Effect of an endoscope and rotation on the peristaltic flow involving a Jeffrey fluid with magnetic field has studied by A.M. Abd-Alla *et al*. [7]. Peristaltic transport of Jeffrey fluid in a channel with compliant walls and porous space was discussed by T. Hayat et.al. [8]. Jeffrey fluid model for blood flow through a tapered

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artery with a stenosis was studied by N.S. Akbar et.al. [9].The literature on the topic is now available and we can only mention a few relevant interesting investigations in References, R.H. Reddy *et al*. [10], Srinivas, S. and Pushparaj, V [11], S. V. H. N. Krishna Kumari *et al*. [12], Lika Hummady *et al*. [13], Ravikumar [14, 15, 16, 17&18], T.Hayat, M.U. Qureshi and N.Ali [19], M. Mishra, A. R. Rao [20].

Heat transfer analysis is prevalent in the study of peristaltic flows due to its large number of applications in processes like hemodialysis (method used for removing waste products from blood in the case of renal failure of kidney) and oxygenation. In peristaltic flows when the fluid is forced to flow due to the sinusoidal displacements of the tract boundaries, the fluid gains some velocity as well as kinetic energy. The viscosity of the fluid takes that kinetic energy and converts it into internal or thermal energy of the fluid. Consequently, the fluid is heated up and heat transfer occurs. This phenomenon is modeled by the energy equation with dissipation effects. For two dimensional flows the energy equation reduces to a second order partial differential equation that is parabolic in nature. Peristaltic flow and heat transfer in a vertical porous annulus with long wave approximation was studied by K. Vajravelu *et al*. [21]. Mass Transfer Effects on Unsteady MHD Blood Flow through Parallel Plate Channel with Heat Source and Radiation analyzed by R. Latha and B. Rushi Kumar [22]. S. Nadeem and N.S. Akbar [23] investigated on an Influence of radially varying MHD on the peristaltic flow in an annulus with heat and mass transfer. Influence of induced magnetic field and heat transfer on the peristaltic motion of a Jeffrey fluid in an asymmetric channel discussed by Safia Akram and S. Nadeem [24]. K. Nirmala *et al*. [25] pointed out on combined effects of hall current, wall slip, viscous dissipation and Soret effect on MHD Jeffrey fluid flow in a vertical channel with Peristalsis. Influence of Joule heating on MHD peristaltic flow of a nanofluid with compliant walls was investigated by M. Gnaneswara Reddy *et al*. [26].The influence of heat and mass transfer on MHD peristaltic flow through a porous space with compliant walls have studied by S. Srinivas and M. Kothandapani [27].Some pertinent studies on the present topic can be found from the list of Refs. Such as K. Venugopal Reddy *et al*.[28], Sk Abzal [29] and G. Ravindranath reddy *et al*.[30], Khilap Singh *et al*.[31] and Bala Siddulu Malga *et al*. [32].

2. Formulation of the Problem

The model simulates the peristaltic transport of a viscous fluid through an infinite two-dimensional asymmetric vertical tapered channel through the porous medium. Asymmetry in the flow is due to the propagation of peristaltic waves of different amplitudes and phase on the channel walls. We assume that the fluid is subject to a constant transverse magnetic field B_0. The flow is generated by sinusoidal wave trains propagating with steady speed c along the tapered asymmetric channel walls.
The geometry of the wall surface is defined as

$$Y = \overline{H_2} = b + m'\overline{X} + d\sin\left[\frac{2\pi}{\lambda}\left(\overline{X} - c\bar{t}\right)\right] \tag{2.1}$$

$$Y = \overline{H_1} = -b - m'\overline{X} - d\sin\left[\frac{2\pi}{\lambda}\left(\overline{X} - c\bar{t}\right) + \phi\right] \tag{2.2}$$

Where b is the half-width of the channel, d is the wave amplitude, c is the phase speed of the wave and m' $(m' << 1)$ is the non-uniform parameter, λ is the wavelength, t is the time and X is the direction of wave propagation. The phase difference ϕ varies in the range $0 \le \phi \le \pi$, $\phi = 0$ corresponds to symmetric channel with waves out of phase and further b, d and ϕ satisfy the following conditions for the divergent channel at the inlet $d\cos(\frac{\phi}{2}) \le b$.

It is assumed that the left wall of the channel is maintained at temperature T_0 while the right wall has temperature T_1.

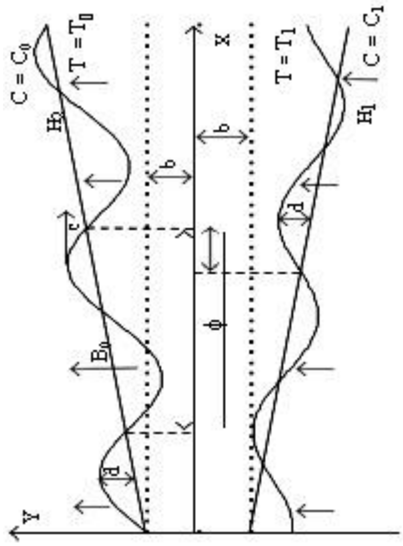

Figure 1. Schematic Diagram of the Physical Model

The constitutive equations for an incompressible Jeffrey fluid are

$$\overline{T} = -\overline{p}\,\overline{I} + \overline{S} \tag{2.3}$$

$$\overline{S} = \frac{\mu}{1+\lambda_1\left(\dot{\bar r} + \lambda_2 \ddot{\bar r}\right)} \tag{2.4}$$

where $\overline{T}$ and $\overline{S}$ are Cauchy stress tensor and extra stress tensor, respectively, $\overline{p}$ is the pressure, $\bar{I}$ is the identity tensor, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time $\dot{r}$ is the shear rate and dots over the quantities indicate differentiation with respect to time.

In laboratory frame, the equations of continuity, momentum, energy and concentration are described as follows

$$\frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \overline{Y}} = 0 \tag{2.5}$$

$$\rho\left[\overline{U}\frac{\partial \overline{U}}{\partial \overline{X}} + \overline{V}\frac{\partial \overline{U}}{\partial \overline{y}}\right] = -\frac{\partial \overline{P}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{X}\,\overline{X}}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{X}\,\overline{Y}}}{\partial \overline{Y}} - \left[\sigma B_0^{\,2}\right]\overline{U} - \left[\frac{\mu}{k_1}\right]\overline{U} \tag{2.6}$$

$$\rho\left[\overline{U}\frac{\partial \overline{V}}{\partial \overline{X}} + \overline{V}\frac{\partial \overline{V}}{\partial \overline{Y}}\right] = -\frac{\partial \overline{p}}{\partial \overline{Y}} + \frac{\partial \overline{S}_{\overline{X}\,\overline{Y}}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{Y}\overline{Y}}}{\partial \overline{Y}} - \left[\sigma B_0^{\,2}\right]\overline{V} \tag{2.7}$$

$$\rho C_p\left[\overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right]\overline{T} = k\left[\frac{\partial^2}{\partial \overline{X}^2} + \frac{\partial^2}{\partial \overline{Y}^2}\right]\overline{T} + Q_0 + \sigma B_0^{\,2}\overline{U}^2 \tag{2.8}$$

$$\left[\overline{U}\frac{\partial \overline{C}}{\partial \overline{X}} + \overline{V}\frac{\partial \overline{C}}{\partial \overline{Y}}\right] = D_m\left[\frac{\partial^2 \overline{C}}{\partial \overline{X}^2} + \frac{\partial^2 \overline{C}}{\partial \overline{Y}^2}\right] + \frac{D_m k_T}{T_m}\left[\frac{\partial^2 \overline{T}}{\partial \overline{X}^2} + \frac{\partial^2 \overline{T}}{\partial \overline{Y}^2}\right] \tag{2.9}$$

Where

$$\overline{S}_{\overline{X}\overline{X}} = \frac{2\mu}{1+\lambda_1}\left(1+\lambda_2\left(\overline{U}\frac{\partial}{\partial\overline{X}}+\overline{V}\frac{\partial}{\partial\overline{Y}}\right)\right)\frac{\partial\overline{U}}{\partial\overline{X}}$$

$$\overline{S}_{\overline{X}\overline{Y}} = \frac{\mu}{1+\lambda_1}\left(1+\lambda_2\left(\overline{U}\frac{\partial}{\partial\overline{X}}+\overline{V}\frac{\partial}{\partial\overline{Y}}\right)\right)\left(\frac{\partial\overline{U}}{\partial\overline{Y}}+\frac{\partial\overline{V}}{\partial\overline{X}}\right)$$

$$\overline{S}_{\overline{Y}\overline{Y}} = \frac{2\mu}{1+\lambda_1}\left(1+\lambda_2\left(\overline{U}\frac{\partial}{\partial\overline{X}}+\overline{V}\frac{\partial}{\partial\overline{Y}}\right)\right)\frac{\partial\overline{U}}{\partial\overline{Y}}$$

$\overline{U}$ and $\overline{V}$ are the velocity components in the laboratory frame ($\overline{X}$, $\overline{Y}$), k_1 is the permeability of the porous medium, ρ is the density of the fluid, p is the fluid pressure, k is the thermal conductivity, μ is the coefficient of the viscosity, Q_0 is the constant heat addition/absorption, C_p is the specific heat at constant pressure, σ is the electrical conductivity, g is the acceleration due to gravity $\overline{T}$ is the temperature of the fluid, $\overline{C}$ is the concentration of the fluid, T_m is the mean temperature, D_m is the coefficient of mass diffusivity, and K_T is the thermal diffusion ratio.

The relative boundary conditions are

$$\overline{U}=0,\overline{T}=T_0,\ \overline{C}=C_0 \text{ at } \overline{Y}=\overline{H}_1$$

$$\overline{U}=0,\overline{T}=T_1,\ \overline{C}=C_1 \text{ at } \overline{Y}=\overline{H}_2$$

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

$$x=\overline{X}-c\bar{t},\ y=\overline{Y},\overline{u}=\overline{U}-c,\bar{v}=\overline{V},\ \overline{p}(x)=\overline{P}\left(\overline{X},\bar{t}\right) \tag{2.10}$$

Where $\overline{u}$, $\overline{v}$ are the velocity components in the wave frame ($\bar{x}$, $\bar{y}$), $\overline{p}$ is pressures and $\overline{P}$ fixed frame of references. We introduce the following non-dimensional variables and parameters for the flow:

$$x=\frac{\overline{x}}{\lambda}\quad y=\frac{\overline{y}}{b}\ \bar{t}=\frac{ct}{\lambda}\quad u=\frac{\overline{u}}{c}\quad v=\frac{\bar{v}}{c\delta}\quad S=\frac{b\overline{S}}{\mu c}\quad h_1=\frac{\overline{H_1}}{b}\quad h_2=\frac{\overline{H_2}}{b}\quad p=\frac{b^2\overline{p}}{c\lambda\mu}$$

$$\theta=\frac{\overline{T}-T_0}{T_1-T_0}\quad \Theta=\frac{\overline{C}-C_0}{C_1-C_0}\ \delta=\frac{b}{\lambda}\quad Da=\frac{k_1}{b^2}\ \mathrm{Re}=\frac{\rho cb}{\mu}\quad S_c=\frac{\mu}{D_m\rho}$$

$$S_r=\frac{D_m\rho k_T\left(T_1-T_0\right)}{\mu T_m\left(C_1-C_0\right)}\quad M=B_0\,b\sqrt{\frac{\sigma}{\mu}}\quad \mathrm{Pr}=\frac{\mu C_p}{k}\ E_c=\frac{c^2}{C_p\left(T_1-T_0\right)}\ \beta=\frac{Q_0b^2}{\mu C_p\left(T_1-T_0\right)}$$

$$\varepsilon=\frac{d}{b} \tag{2.11}$$

where $\varepsilon=\dfrac{d}{b}$ is the non-dimensional amplitude of channel , $\delta=\dfrac{b}{\lambda}$ is the wave number, $k_1=\dfrac{\lambda m'}{b}$ is the non - uniform parameter , Re is the Reynolds number, M is the Hartman number , $K=\dfrac{k}{b^2}$ Permeability parameter ,Pr is the Prandtl number, E_c is the Eckert number, β is the heat source/sink parameter, B_r (= E_cP_r) is the Brinkman number, S_c Schmidt number and S_r Soret number.

3. Solution of the Problem

In view of the above transformations (2.10) and non-dimensional variables (2.11), equations (2.5-2.9) are reduced to the following forms.

$$\delta\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=0 \tag{3.1}$$

$$\mathrm{Re}\,\delta\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=\left[-\frac{\partial p}{\partial x}+\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-M^2u-\frac{1}{Da}u\right] \tag{3.2}$$

$$\mathrm{Re}\,\delta^3\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=\left[-\frac{\partial p}{\partial y}+\delta^2\frac{\partial S_{xy}}{\partial x}+\delta\frac{\partial S_{yy}}{\partial y}-\delta^2\frac{1}{Da}v\right] \tag{3.3}$$

$$\mathrm{Re}\left[\delta u\frac{\partial \theta}{\partial x}+v\delta\frac{\partial \theta}{\partial y}\right]=\frac{1}{\mathrm{Pr}}\left[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right]+\beta+M^2Eu^2 \tag{3.4}$$

$$\mathrm{Re}\,\delta\left[u\frac{\partial \Theta}{\partial x}+v\frac{\partial \Theta}{\partial y}\right]=\frac{1}{S_c}\left[\delta^2\frac{\partial^2\Theta}{\partial x^2}+\frac{\partial^2\Theta}{\partial y^2}\right]+S_r\left[\delta^2\frac{\partial^2\Theta}{\partial x^2}+\frac{\partial^2\Theta}{\partial y^2}\right] \tag{3.5}$$

Where

$$S_{xx}=\frac{2\delta}{1+\lambda_1}\left(1+\frac{\lambda_2\,\delta\,c}{d}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right)\frac{\partial u}{\partial x}$$

$$S_{xy}=\frac{1}{1+\lambda_1}\left(1+\frac{\lambda_2\,\delta\,c}{d}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right)\left(\frac{\partial u}{\partial x}+\delta^2\frac{\partial v}{\partial x}\right)$$

$$S_{yy}=\frac{2}{1+\lambda_1}\left(1+\frac{\lambda_2\,\delta\,c}{d}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right)\frac{\partial v}{\partial y}$$

Applying long wave length approximation and neglecting the wave number along with low-Reynolds numbers. Equations (3.1-3.5) become

$$\frac{\partial^2 u}{\partial y^2}-(1+\lambda_1)\left(M^2+\frac{1}{Da}\right)u=(1+\lambda_1)\frac{\partial p}{\partial x} \tag{3.6}$$

$$\frac{\partial p}{\partial y}=0 \tag{3.7}$$

$$\frac{1}{\mathrm{Pr}}\left[\frac{\partial^2\theta}{\partial y^2}\right]+\beta+M^2E_c u^2=0 \tag{3.8}$$

$$\left[\frac{\partial^2\Theta}{\partial y^2}\right]+S_cS_r\frac{\partial^2\theta}{\partial y^2}=0 \tag{3.9}$$

The relative boundary conditions in dimensionless form are given by

$$u=-1,\ \theta=0,\ \Theta=0\ \ at\ \ y=h_1=-1-k_1x-\varepsilon\sin[2\pi(x-t)+\phi] \tag{3.10}$$

$$u=-1,\ \theta=1,\ \Theta=1\ \ \text{at}\ \ y=h_2=1+k_1x+\varepsilon\sin[2\pi(x-t)] \tag{3.11}$$

The solutions of velocity and temperature with subject to boundary conditions (3.10) and (3.11) are given by

$$u=a_1Sinh[\alpha_1\,y]+a_2\,Cosh[\alpha_1\,y]+A \tag{3.12}$$

Where

$$\alpha_1 = \sqrt{(1+\lambda_1)\left(M^2 + \frac{1}{Da}\right)} \quad A = -\frac{D}{M^2 D + 1}$$

$$a_1 = \left(\frac{(-1-A)}{\left(\frac{Cosh[\alpha_1 h_2] - Cosh[\alpha_1 h_1]}{Sinh[\alpha_1 h_1] - Sinh[\alpha_1 h_2]}\right) Sinh[\alpha_1 h_1] + Cosh[\alpha_1 h_1]} \right) \left(\frac{Cosh[\alpha_1 h_2] - Cosh[\alpha_1 h_1]}{Sinh[\alpha_1 h_1] - Sinh[\alpha_1 h_2]} \right)$$

$$a_2 = \left(\frac{(-1-A)}{\left(\frac{Cosh[\alpha_1 h_2] - Cosh[\alpha_1 h_1]}{Sinh[\alpha_1 h_1] - Sinh[\alpha_1 h_2]}\right) Sinh[\alpha_1 h_1] + Cosh[\alpha_1 h_1]} \right)$$

$$\theta = a_3 + a_4 y - \beta \Pr \frac{y^2}{2} - M^2 B_r \left[\frac{y^2}{4}\left(2A^2 - a_1^{\,2} + a_2^{\,2}\right) + \frac{2Aa_1 Sinh[\alpha_1 y]}{\alpha_1^{\,2}} + \frac{2Aa_2 Cosh[\alpha_1 y]}{\alpha_1^{\,2}} \right]$$

$$-M^2 B_r \left[\frac{a_1 a_2 Sinh[2\alpha_1 y]}{4\alpha_1^{\,2}} + \frac{\left(a_1^{\,2} + a_2^{\,2}\right) Cosh[2\alpha_1 y]}{8\alpha_1^{\,2}} \right] \qquad (3.13)$$

Where

$$a_3 = 1 - a_4 h_2 + \beta P_r \frac{h_2^{\,2}}{2} + M^2 B_r \frac{h_2^{\,2}}{4}\left[2A^2 - a_1^{\,2} + a_2^{\,2}\right] + M^2 B_r \frac{2Aa_1 Sinh[\alpha_1 h_2]}{\alpha_1^{\,2}} +$$
$$M^2 B_r \frac{2Aa_2 Cosh[\alpha_1 h_2]}{\alpha_1^{\,2}} + M^2 B_r \frac{a_1 a_2 Sinh[2\alpha_1 h_2]}{4\alpha_1^{\,2}} + M^2 B_r \frac{\left(a_1^{\,2} + a_2^{\,2}\right) Cosh[2\alpha_1 h_2]}{8\alpha_1^{\,2}}$$

$$a_4 = \left(\frac{-1}{(h_1 - h_2)} \right) \left[1 + \beta P_r \left[\frac{h_2^{\,2}}{2} - \frac{h_1^{\,2}}{2} \right] + M^2 B_r \left[\frac{h_2^{\,2}}{4} - \frac{h_1^{\,2}}{4} \right] \left[2A^2 - a_1^{\,2} + a_2^{\,2}\right] + \right.$$
$$M^2 B_r \left(\frac{2Aa_1 \left(Sinh[\alpha_1 h_2] - Sinh[\alpha_1 h_1]\right)}{\alpha_1^{\,2}} \right) + M^2 B_r \left(\frac{2Aa_2 \left[Cosh(\alpha_1 h_2) - Cosh(\alpha_1 h_1)\right]}{\alpha_1^{\,2}} \right) +$$
$$\left. M^2 B_r \left(\frac{a_1 a_2 \left(Sinh[2\alpha_1 h_2] - Sinh[2\alpha_1 h_1]\right)}{4\alpha_1^{\,2}} \right) + M^2 B_r \left(\frac{\left(a_1^{\,2} + a_2^{\,2}\right)\left(Cosh[2\alpha_1 h_2] - Cosh[2\alpha_1 h_1]\right)}{8\alpha_1^{\,2}} \right) \right]$$

$$\Theta = f_8 + f_9 y + f_1 \frac{y^2}{2} + f_2 f_3 y^2 + f_2 f_4 Sinh[\alpha_1 y] + f_2 f_5 Cosh[\alpha_1 y] + \qquad (3.14)$$

Where $f_1 = S_c S_r \beta P_r$ $\quad f_2 = S_c S_r M^2 B_r$ $\quad f_3 = \dfrac{\left(2A^2 - a_1^{\,2} + a_2^{\,2}\right)}{4}$ $\quad f_4 = \dfrac{(2Aa_1)}{\alpha_1^{\,2}}$

$$f_5 = \frac{(2Aa_2)}{\alpha_1^{\,2}}$$

$$f_6 = \frac{(a_1 a_2)}{4\alpha_1^2} \qquad f_7 = \frac{(a_1^2 + a_2^2)}{8\alpha_1^2}$$

$$f_8 = -f_9 h_1 - f_1 \frac{h_1^2}{2} - f_2 f_3 h_1^2 - f_2 f_4 Sinh[\alpha_1 h_1] - f_2 f_5 Cosh[\alpha_1 h_1] - f_2 f_6 Cosh[2\alpha_1 h_1] - f_2 f_7 Sinh[2\alpha_1 h_1]$$

$$f_9 = \left(\frac{1}{(h_1 - h_2)}\right)\Big[-1 - f_1\left[\frac{h_1^2}{2} - \frac{h_2^2}{2}\right] - f_2 f_3 (h_1^2 - h_2^2) - f_2 f_4 (Sinh[\alpha_1 h_1] - Sinh[\alpha_1 h_2]) - f_2 f_5 (Cosh[\alpha_1 h_1] - Cosh[\alpha_1 h_2]) - f_2 f_6 (Cosh[2\alpha_1 h_1] - Cosh[2\alpha_1 h_2]) - f_2 f_7 (Sinh[2\alpha_1 h_1] - Sinh[2\alpha_1 h_2])\Big]$$

The coefficients of the heat transfer Zh_1 and Zh_2 at the walls $y = h_1$ and $y = h_2$ respectively, are given by

$$Zh_1 = \theta_y h_{1x} \tag{3.15}$$

$$Zh_2 = \theta_y h_{2x} \tag{3.16}$$

The solutions of the coefficient of heat transfer at $y = h_1$ and $y = h_2$ are given by

$$Zh_1 = \theta_y h_{1x} = \left[a_4 - \beta \Pr y - M^2 B_r\left[\frac{y}{2}(2A^2 - a_1^2 + a_2^2) + \frac{2Aa_1 Cosh[\alpha_1 y]}{\alpha_1} + \frac{2Aa_2 Sinh[\alpha_1 y]}{\alpha_1}\right] - M^2 B_r\left[\frac{a_1 a_2 Cosh[2\alpha_1 y]}{2\alpha_1} + \frac{(a_1^2 + a_2^2) Sinh[2\alpha_1 y]}{4\alpha_1}\right]\right] * [-2\pi\varepsilon Cos[2\pi(x - t) + \phi] - k_1] \tag{3.17}$$

$$Zh_2 = \theta_y h_{2x} = \left[a_4 - \beta \Pr y - M^2 B_r\left[\frac{y}{2}(2A^2 - a_1^2 + a_2^2) + \frac{2Aa_1 Cosh[\alpha_1 y]}{\alpha_1} + \frac{2Aa_2 Sinh[\alpha_1 y]}{\alpha_1}\right] - M^2 B_r\left[\frac{a_1 a_2 Cosh[2\alpha_1 y]}{2\alpha_1} + \frac{(a_1^2 + a_2^2) Sinh[2\alpha_1 y]}{4\alpha_1}\right]\right] * [2\pi\varepsilon\, Cos[2\pi(-t + x)] + k_1] \tag{3.18}$$

Volumetric Flow Rate

The volumetric flow rate in the wave frame is defined by

$$q = \int_{h_1}^{h_2} u\, dy = \int_{h_1}^{h_2} (a_1 Sinh[\alpha_1 y] + a_2 Cosh[\alpha_1 y] + A) dy$$

$$\left(\frac{a_1}{\alpha_1}\right)(Cosh[\alpha_1 h_2] - Cosh[\alpha_1 h_1]) + \left(\frac{a_2}{\alpha_1}\right)(Sinh[\alpha_1 h_2] - Sinh[\alpha_1 h_1]) + A(h_2 - h_1) \tag{3.19}$$

The pressure gradient obtained from equation (3.19) can be expressed as

$$\frac{dp}{dx} = -\left(M^2 + \frac{1}{Da}\right)(a_3) \tag{3.20}$$

Where

$$a_3 = \left(\frac{q - \left(\frac{(Cosh[\alpha_1 h_2] - Cosh[\alpha_1 h_1])}{\alpha} \right)(a_1) - \left(\frac{(Sinh[\alpha_1 h_2] - Sinh[\alpha_1 h_1])}{\alpha} \right)(a_2)}{\left(\frac{(Cosh[\alpha_1 h_2] - Cosh[\alpha_1 h_1])}{\alpha} \right)(a_1) + \left(\frac{(Sinh[\alpha_1 h_2] - Sinh[\alpha_1 h_1])}{\alpha} \right)(a_2) + (h_2 - h_1)} \right)$$

The instantaneous flux Q (x, t) in the laboratory frame is

$$Q = \int_{h_2}^{h_1} (u+1)\,dy = q - h \tag{3.21}$$

The average volume flow rate over one wave period (T = λ/c) of the peristaltic wave is defined as

$$\overline{Q} = \frac{1}{T}\int_0^T Q\,dt = q + 1 + d \tag{3.22}$$

From the equations (3.20) and (3.22), the pressure gradient can be expressed as

$$\frac{dp}{dx} = -\left(M^2 + \frac{1}{Da} \right) \left(\frac{(\overline{Q} - 1 - d) - \left(\frac{(Cosh[\alpha_1 h_2] - Cosh[\alpha_1 h_1])}{\alpha} \right)(a_1) - \left(\frac{(Sinh[\alpha_1 h_2] - Sinh[\alpha_1 h_1])}{\alpha} \right)(a_2)}{\left(\frac{(Cosh[\alpha_1 h_2] - Cosh[\alpha_1 h_1])}{\alpha} \right)(a_1) + \left(\frac{(Sinh[\alpha_1 h_2] - Sinh[\alpha_1 h_1])}{\alpha} \right)(a_2) + (h_2 - h_1)} \right) \tag{3.23}$$

4. Numerical Results and Discussion

The main object of this investigation has been to study Joule heating and mass transfer on MHD

peristaltic Jeffery fluid with porous medium through a vertical asymmetric tapered channel. The analytical expressions for velocity distribution, pressure gradient, and temperature and heat transfer coefficient have been derived in the previous section. The numerical and computational results are discussed through the graphical illustration. **Mathematica** software is used to find out numerical results. Figure 2 represents the variation of axial velocity with y for different values of Jeffery fluid (λ_1) with Da = 0.5, M= 1, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, x = 0.6, t = 0.4, dp/dx = 0.5. We observe from this figure that the axial velocity increases with increase in Jeffery fluid λ_1 ($\lambda_1 = 0.5, 1, 1.5$).Effect of Hartmann number M on axial velocity (u) is depicted in figure (3) with Da = 0.5, $\lambda_1 = 0.5$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, x = 0.6, t = 0.4, dp/dx = 0.5. It has been inferred that the axial velocity increases with increasing the values of Hartmann number M (M = 0.5, 1, 1.5) Figure 4 illustrates the variation in axial velocity for different values of porosity parameter Da with M = 1, $\lambda_1 = 0.5$, $\phi = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, x = 0.6, t = 0.4, dp/dx = 0.5. It can be seen that the axial velocity diminished with increase in porosity parameter Da (Da = 0.1, 0.3, 0.5). Figure 5 is drawn to study the effect of non-uniform parameter (k_1) on axial velocity distribution (u) with M = 1, $\lambda_1 = 0.5$, Da = 0.5, $\phi = \pi/6$, $\varepsilon = 0.2$, x = 0.6, t = 0.4, dp/dx = 0.5. We notice from this figure that the increase in non-uniform parameter (k_1) increases the velocity distribution.

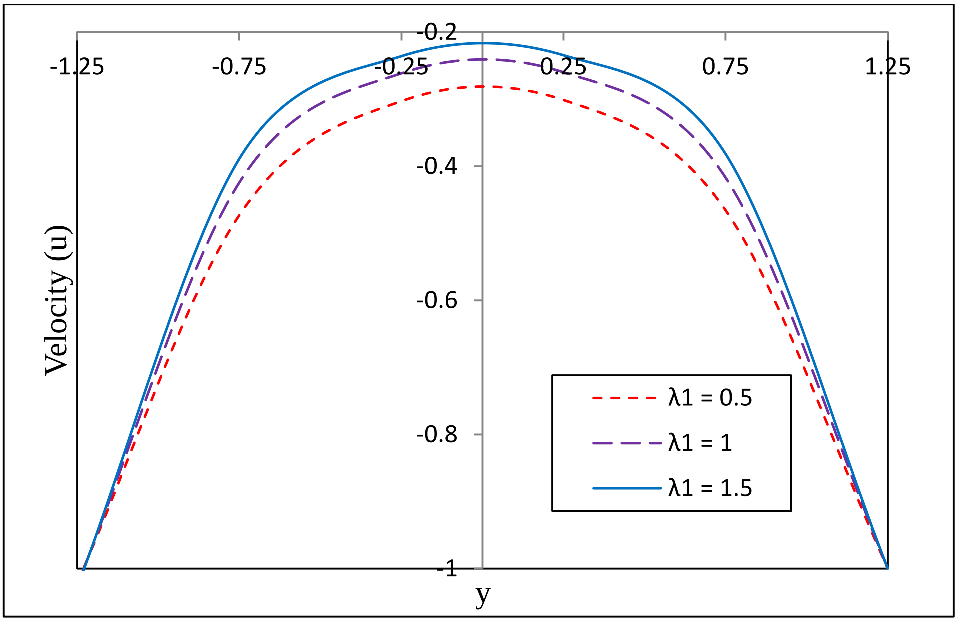

Figure 2. Velocity distribution for different Values of $\lambda_1$with Da = 0.5, M= 1, ϕ = $\pi/6$, ε = 0.2, k_1= 0.1, x = 0.6, t = 0.4, dp/dx = 0.5

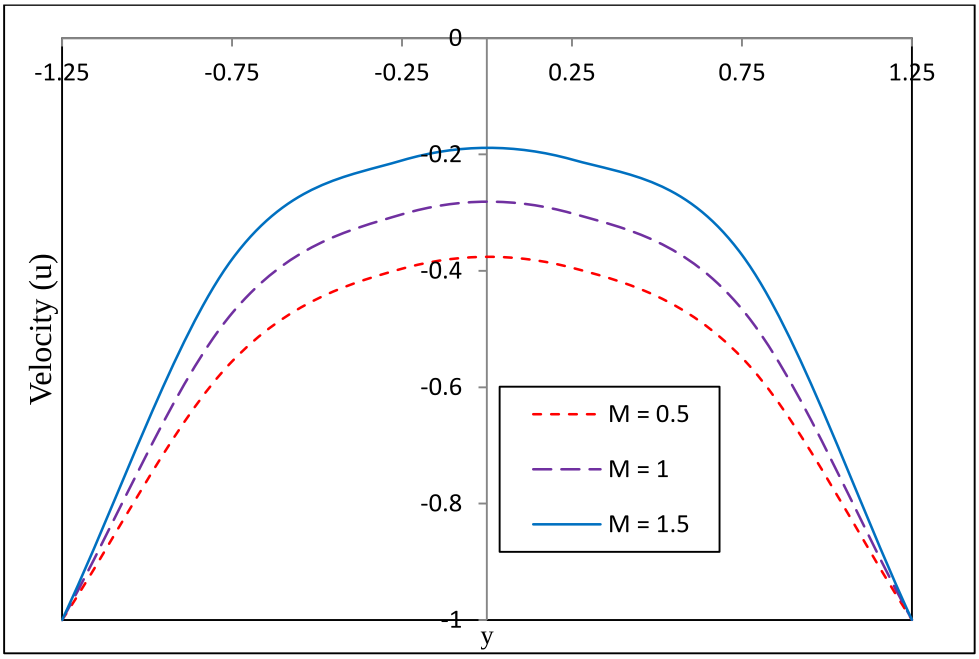

Figure 3. Velocity distribution for different Values M of with Da = 0.5, λ_1= 0.5, ϕ = $\pi/6$, ε = 0.2, k_1= 0.1, x = 0.6, t = 0.4, dp/dx = 0.5

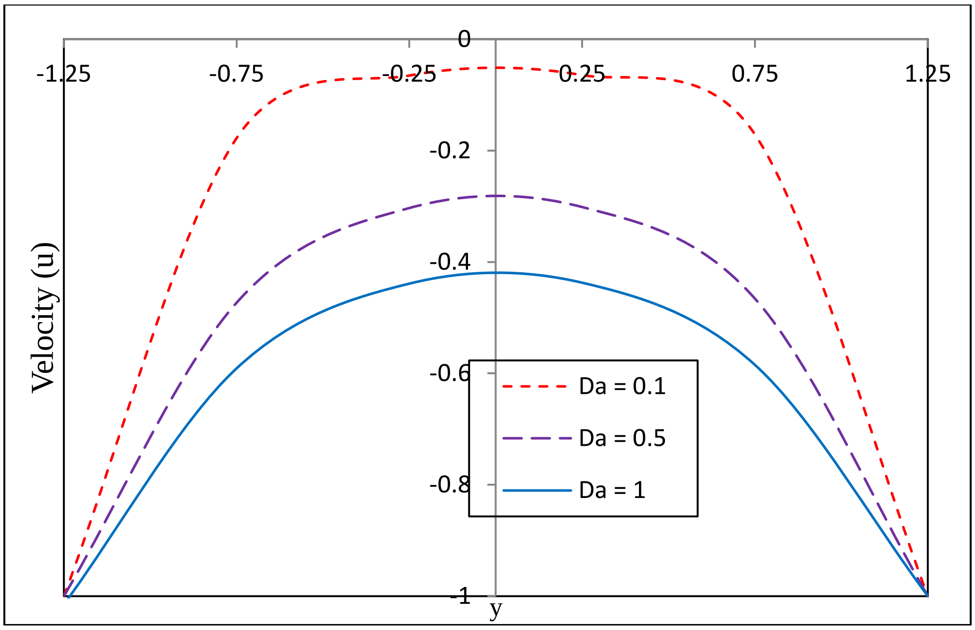

Figure 4. Velocity distribution for different Values Da of with M = 1, λ_1= 0.5, $\phi = \pi/6$, ε = 0.2, k_1= 0.1, x = 0.6, t = 0.4, dp/dx = 0.5

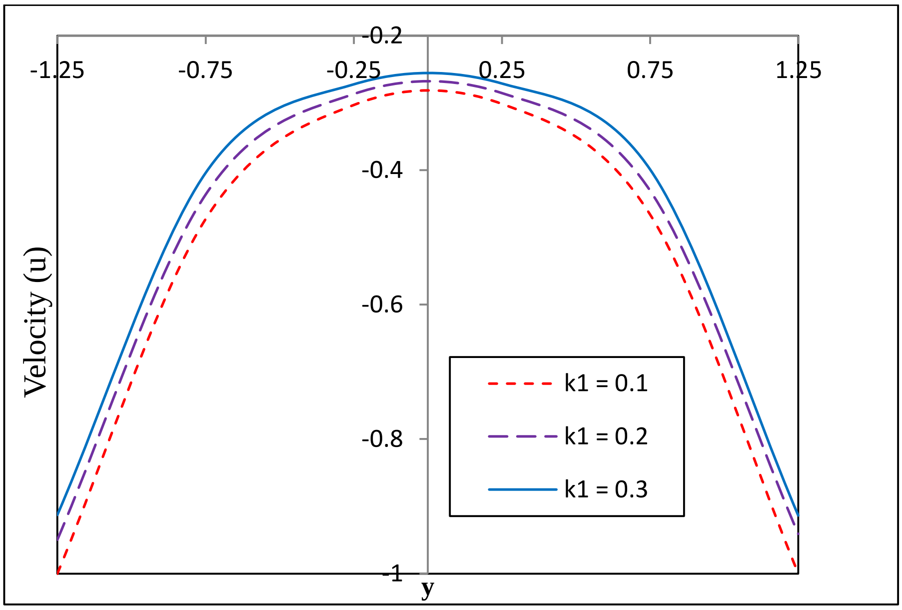

Figure 5. Velocity distribution for different Values k_1 of with M = 1, λ_1= 0.5, Da = 0.5, $\phi = \pi/6$, ε = 0.2, x = 0.6, t = 0.4, dp/dx = 0.5

Figure 6 is plotted to study the effect of Magnetic parameter M on the axial pressure gradient (dp/dx) with Da = 0.5, λ_1= 0.5, $\phi = \pi/6$, $\varepsilon = 0.2$, k_1= 0.1, $t = \pi/4$, $\bar{Q} = 0.2$, d = 2. We notice from this graph that the axial pressure gradient increases with increase in magnetic field M (M = 0.5, 1, 1.5).

Figure 7 depicts to examine the effect of Porosity parameter on pressure gradient (dp/dx) with M = 1, λ_1= 0.5, $\phi = \pi/6$, $\varepsilon = 0.2$, k_1= 0.1, $t = \pi/4$, $\bar{Q} = 0.2$, d = 2. This figure shows that the value of pressure gradient decreases when porosity parameter enhances. Figure 8 analyzes the influence of Jeffery fluid λ_1 on axial pressure gradient (dp/dx) with M = 1, Da= 0.5, $\phi = \pi/6$, $\varepsilon = 0.2$, k_1= 0.1, $t = \pi/4$, $\bar{Q} = 0.2$, d = 2. It is notice that the pressure gradient diminished when Jeffery fluid enhances. Figure9 displays the effect of volumetric flow rate $\bar{Q}$ on pressure gradient (dp/dx) M = 1, Da= 0.5, $\phi = \pi/6$, $\lambda_1 = 0.5$, ε = 0.2, k_1= 0.1, t = $\pi/4$, d = 2.It shows from this figure that an increase in the value of volumetric flow rate results in the pressure gradient diminished.

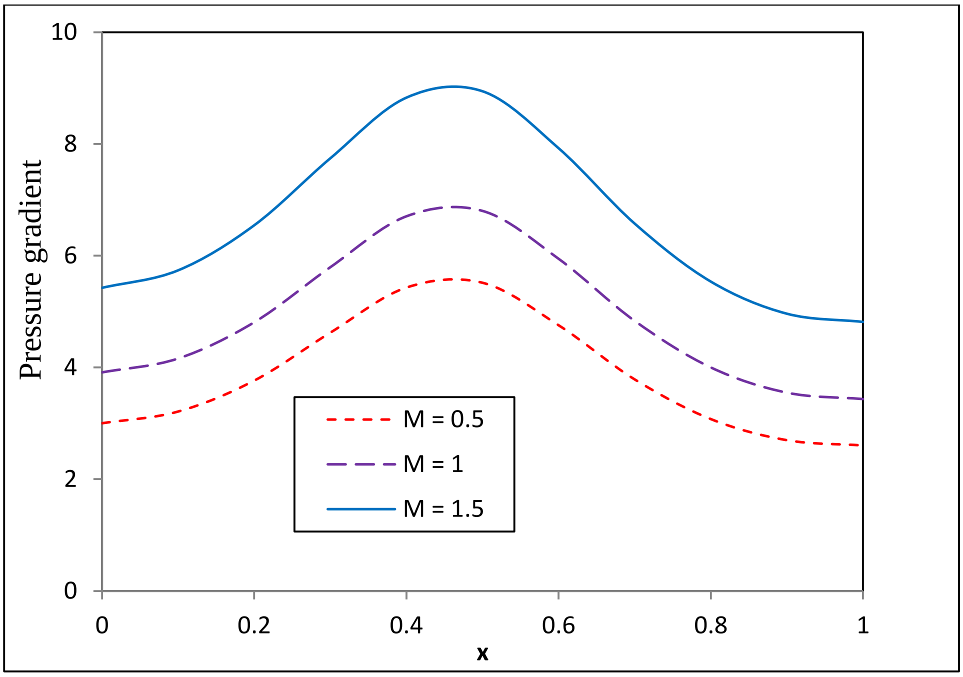

Figure 6. Pressure Gradient for different Values M with Da = 0.5, λ_1= 0.5, ϕ = $\pi/6$, ε = 0.2, k_1= 0.1, t = $\pi/4$, $\bar{Q}$ = 0.2, d = 2

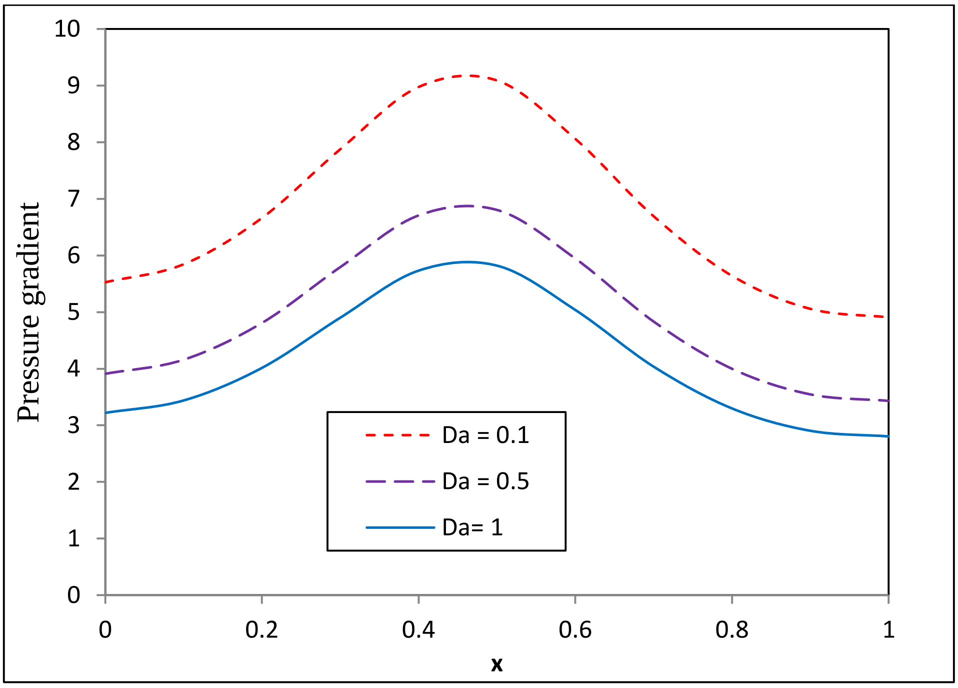

Figure 7. Pressure gradient for different values Da with M = 1, λ_1= 0.5, ϕ = $\pi/6$, ε = 0.2, k_1= 0.1, t = $\pi/4$, $\bar{Q}$ = 0.2, d = 2

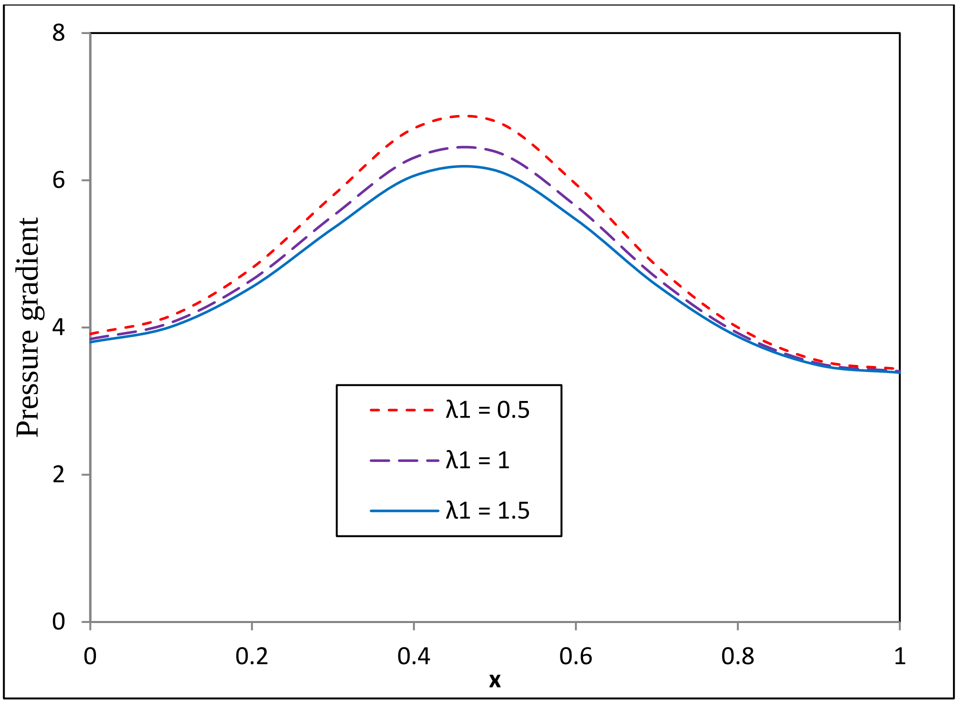

Figure 8. Pressure gradient for different values λ_1 with M = 1, Da= 0.5, $\phi = \pi/6$, ε = 0.2, k_1= 0.1, t = $\pi/4$, $\bar{Q}$ = 0.2, d = 2

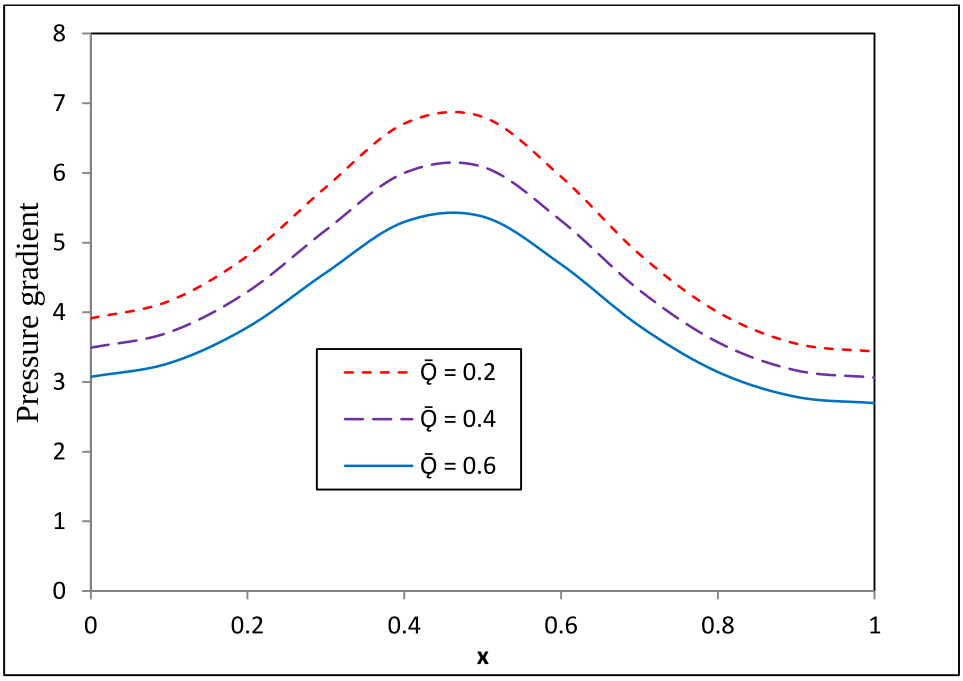

Figure 9. Pressure Gradient for different Values $\bar{Q}$ with M = 1, Da= 0.5, $\phi = \pi/6$, λ_1 = 0.5, ε = 0.2, k_1= 0.1, t = $\pi/4$, d = 2

Figure 10 depicts the variation in the temperature of the fluid for different values of heat generation parameter β with Da = 0.5, M = 1, Pr =2, Br = 1, λ_1 = 0.5, $\phi = \pi/6$, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5.It shows that the temperature distribution enhances with increase in heat generation parameter β (β = 0.1, 0.3, 0.5). Figure 11 analyzes the effect Prandtl number on temperature distribution (θ) with Da = 0.5, M = 1, β =0.1, Br = 1, λ_1 = 0.5, $\phi = \pi/6$, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5.We notice from this figure that the temperature of the fluid enhances with an increase in Prandtl number Pr (Pr = 2, 4, 6). Figure 12 displays the effect of Jeffery fluid on temperature distribution (θ) with Da = 0.5, M = 1, β =0.1, Br = 1, λ_1 = 0.5, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5. It is interested to notice from this figure that the temperature of the fluid diminished when Jeffery fluid enhances (λ_1= 0.5, 1, 1.5). Influence of Brinkman number on temperature distribution (θ) is shown in Figure 13 with Da = 0.5, M = 1, β =0.1, λ_1 = 0.5, Pr = 2, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5.It can be seen that the temperature distribution gradually enhances with increase in Brinkman number (Br = 0.1, 0.5, 1).The temperature distribution (θ) for different values of porosity parameter (Da) is plotted in Figure 14. We observe from this figure that the temperature of the fluid enhances with an increase in porosity parameter (Da 0.1, 0.5, 1) with fixed other parameters. Figure 15 depicts to examine the effect of magnetic field on temperature distribution (θ) with Da = 0.5, β =0.1, λ_1 = 0.5, Br = 0.1, Pr = 2, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5. Indeed, the temperature of the fluid enhances with an increasing the values of magnetic field (M = 0.5, 1, 1.5).

Therefore, we conclude from these figures (Figures 10-15) that the temperature of the enhances with an increase in β, Br, Pr, Da and M whereas the temperature of the fluid diminished with an increase in Jeffery fluid and We notice that the temperature profile is found almost parabolic in nature.

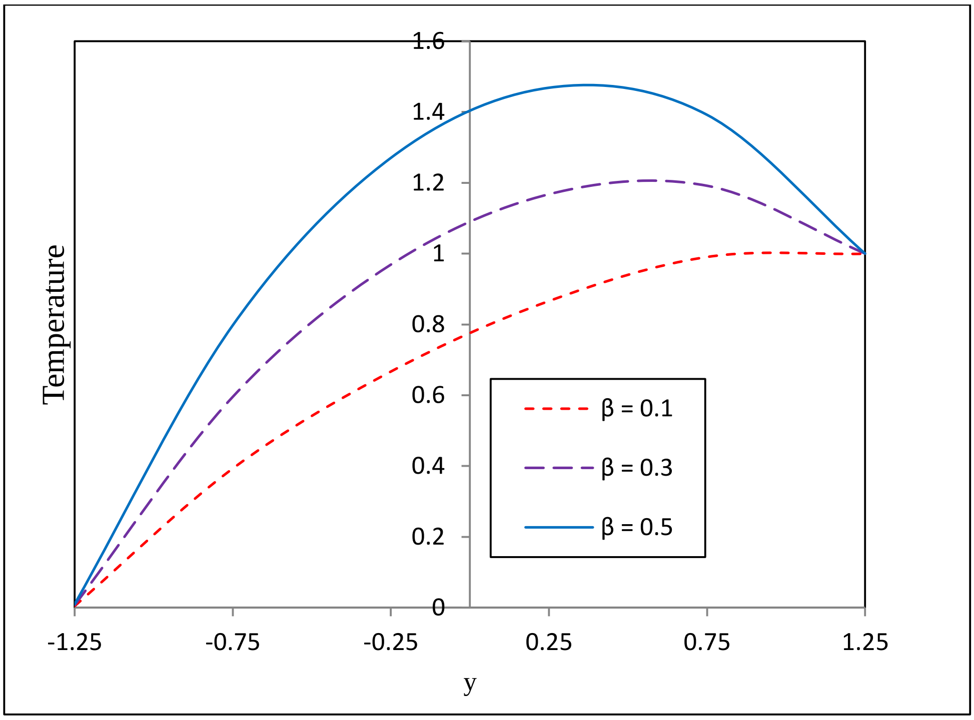

Figure 10. Temperature distribution (θ) for different Values of β with Da = 0.5, M = 1, Pr =2, Br = 1, λ_1 = 0.5, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5

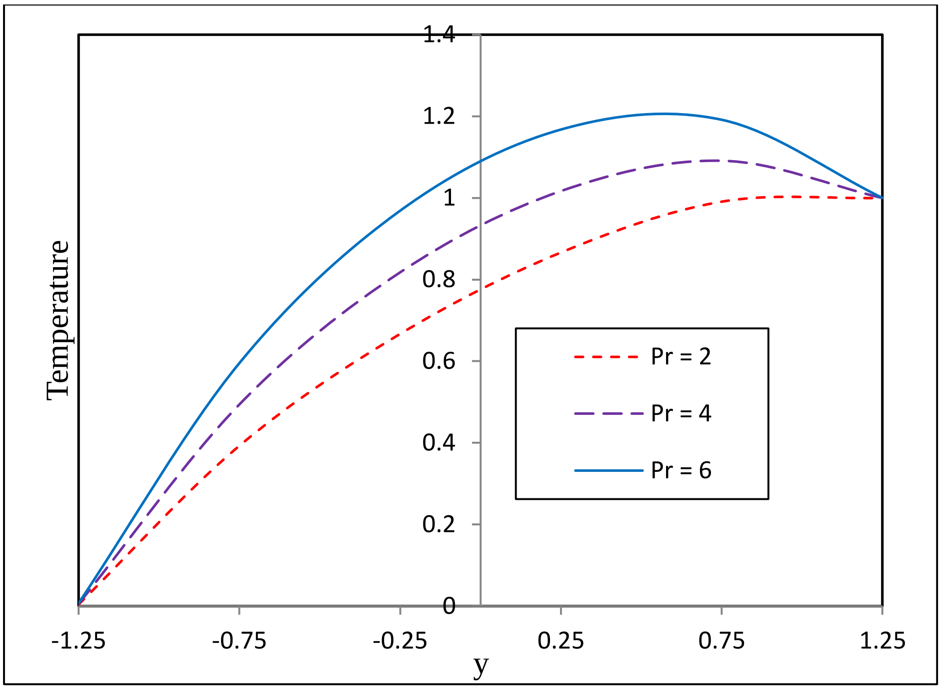

Figure 11. Temperature distribution (θ) for different Values of Pr with Da = 0.5, M = 1, β =0.1, Br = 1, λ_1 = 0.5, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5

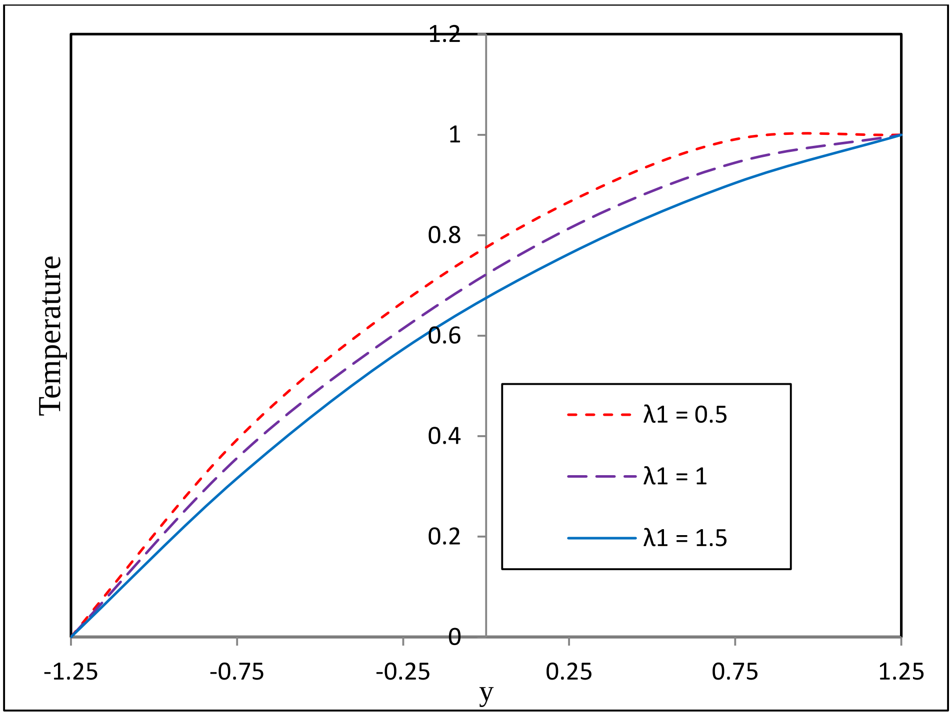

Figure 12. Temperature Distribution (θ) for different Values of λ_1 with Da = 0.5, M = 1, β =0.1, Br = 1, Pr = 2, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5

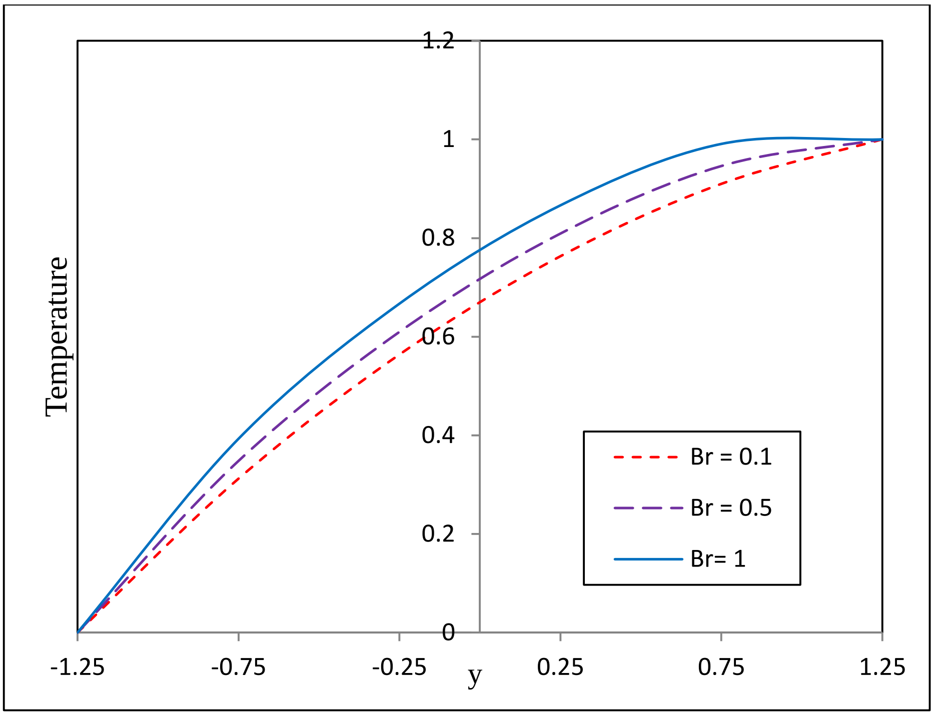

Figure 13. Temperature Distribution (θ) for different Values of Br with Da = 0.5, M = 1, β =0.1, λ_1 = 0.5, Pr = 2, ϕ = π/6, ε = 0.2, k_1= 0.1, x = 0.6, t = 0.4, p = 0.5

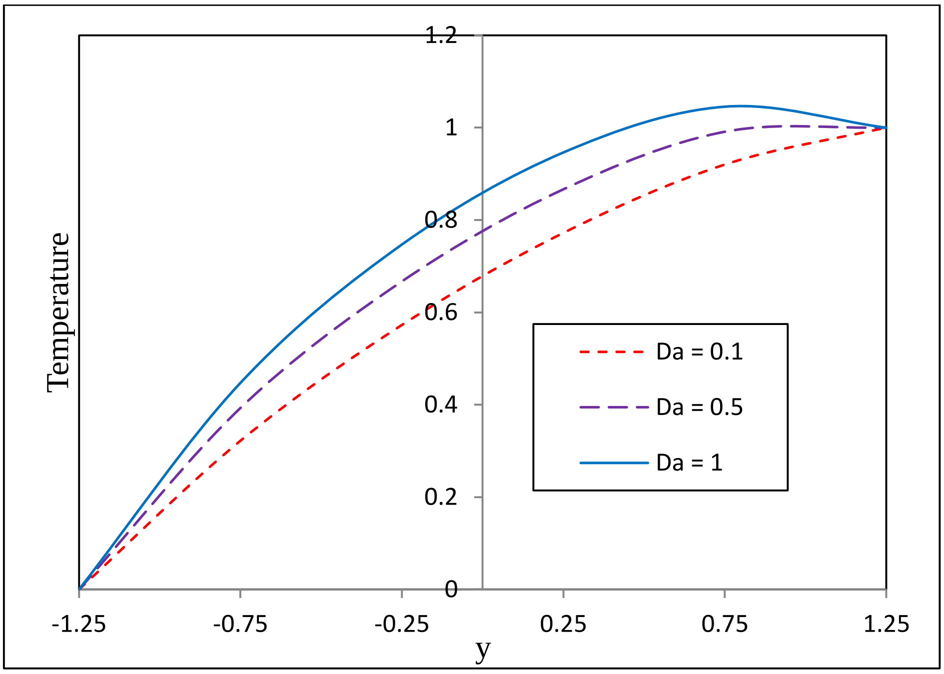

Figure 14. Temperature Distribution (θ) for different Values of Da with M = 1, β =0.1, λ_1 = 0.5, Br = 0.1, Pr = 2, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5

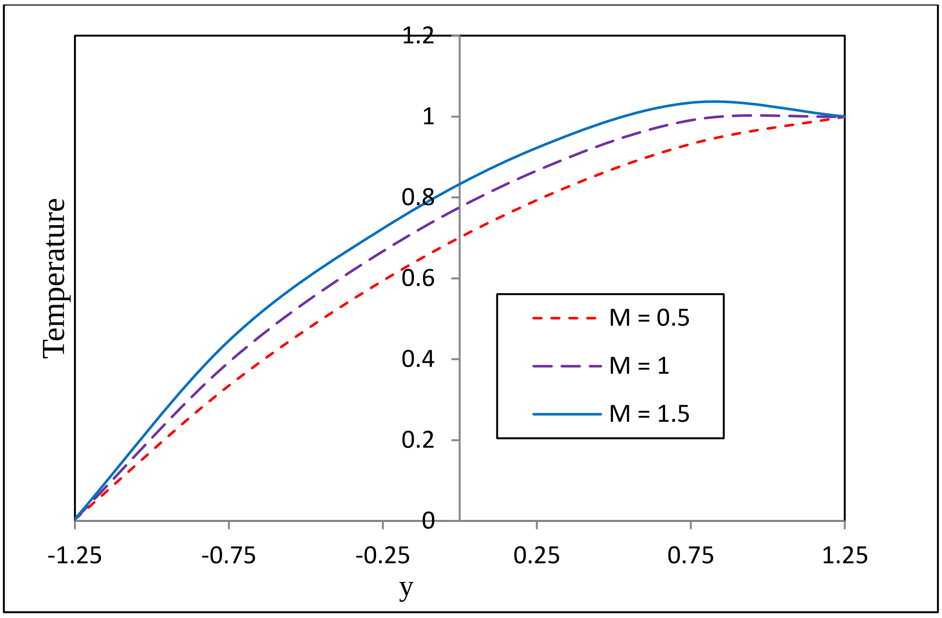

Figure 15. Temperature distribution (θ) for different Values of M with Da = 0.5, β =0.1, λ_1 = 0.5, Br = 0.1, Pr = 2, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5

Figures (16) - (20) are plotted to study the effects of β, Pr, Sc, Br and Sr on the concentration profile.Figure16 shows that the effect of heat source generation parameter on concentration distribution (Φ) with Da = 0.5, Sc = 0.5, Sr = 3, M = 1, Pr =2, Br = 0.1, λ_1 = 0.5, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5.We observe from this figure that the concentration field decreases with increase in heat source generation parameter (β = 0.1, 0.3, 0.5). Influence of Prandtl number (pr) on concentration distribution (Φ) is depicted in Figure17 with fixed Da = 0.5, Sc = 0.5, Sr = 3, M = 1, β = 0.1, Br = 0.1, λ_1 = 0.5, ϕ = π/6, ε = 0.2, k_1= 0.1, x = 0.6, t = 0.4, p = 0.5.It can be seen that the concentration profile diminishes with increase in Prandtl number (Pr = 2, 4, 6). The concentration distribution (Φ) for different values of Schmidt number (Sc) is plotted in Figure18 with fixed Da = 0.5, Pr = 2, Sr = 3, M = 1, β = 0.1, Br = 0.1, λ_1 = 0.5, ϕ = π/6, ε = 0.2, k_1= 0.1, x = 0.6, t = 0.4, p = 0.5.We notice from this figure that an increase in Schmidt number (Sc = 0.1, 0.3, 0.5) results in concentration field diminished. Figure19 depicts the variation in concentration profile (Φ) for different values of brinkman number (Br) with Da = 0.5, Pr = 2, Sc = 0.5, Sr = 3, M = 1, β = 0.1, λ_1 = 0.5, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5. This figure shows that an increase in brinkman number results in concentration profile diminished.Figure20 depicts to examine the effect of Soret number (Sr) on concentration distribution (Φ) with Da = 0.5, Pr = 2, Sc = 0.5, Br = 0.1, M = 1, β = 0.1, λ_1 = 0.5, ϕ = π/6, ε = 0.2, k_1= 0.1, x = 0.6, t = 0.4, p = 0.5. It shows that the concentration field decreases when the Soret number increases (Sr = 1, 3, 5).

Therefore, we conclude that the concentration distribution decreases when increase in β, Pr, Sc, Br and Sr in an entire asymmetric vertical tapered channel.

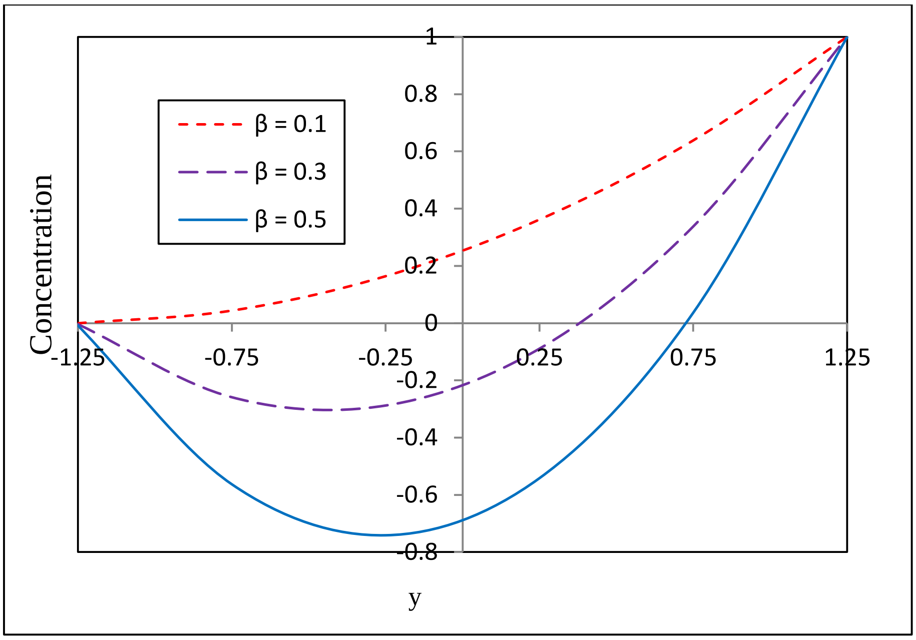

Figure 16. Concentration distribution for different values of β with Da = 0.5, Sc = 0.5, Sr = 3, M = 1, Pr =2, Br = 0.1, λ_1 = 0.5, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5

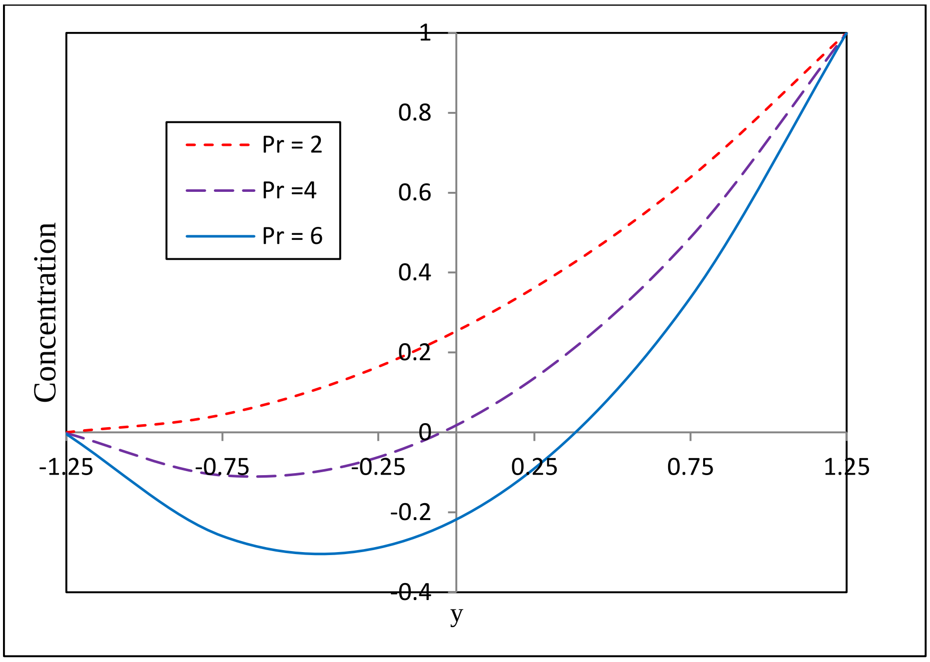

Figure 17. Concentration distribution for different values of Pr with Da = 0.5, Sc = 0.5, Sr = 3, M = 1, β = 0.1, Br = 0.1, λ_1 = 0.5, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5

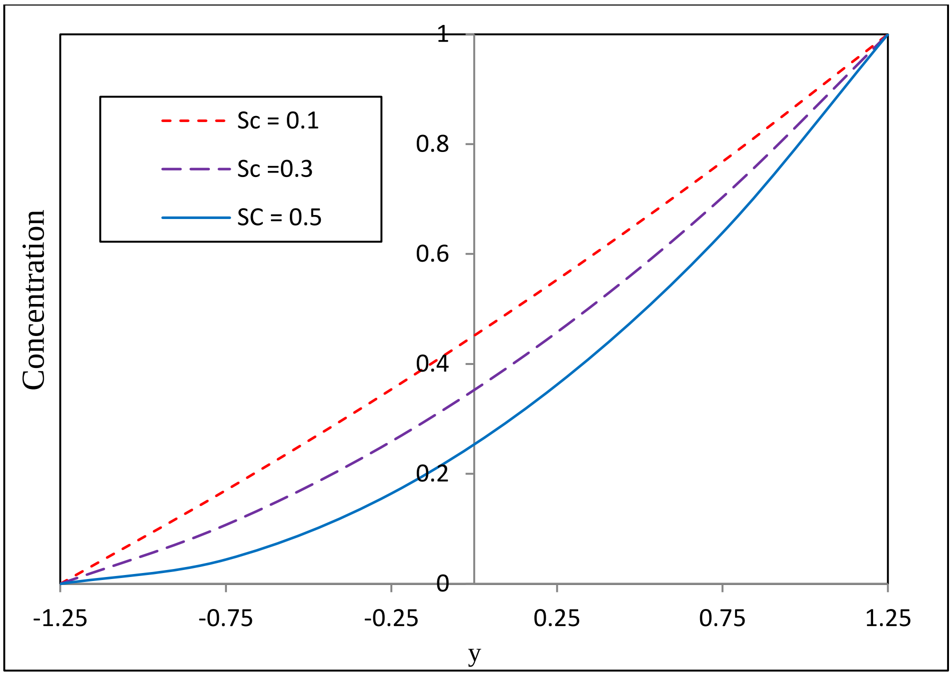

Figure 18. Concentration distribution for different values of Sc with Da = 0.5, Pr = 2, Sr = 3, M = 1, β = 0.1, Br = 0.1, λ_1 = 0.5, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5

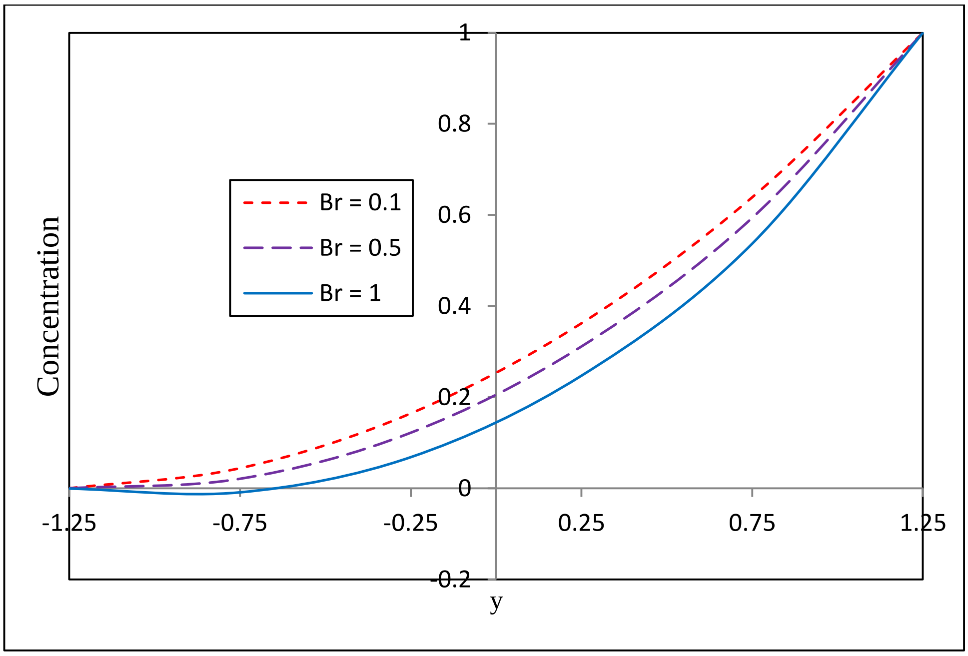

Figure 19. Concentration distribution for different values Of Br with Da = 0.5, Pr = 2, Sc = 0.5, Sr = 3, M = 1, β = 0.1, λ_1 = 0.5, ϕ = π/6, ε = 0.2, k_1= 0.1, x= 0.6, t = 0.4, p = 0.5

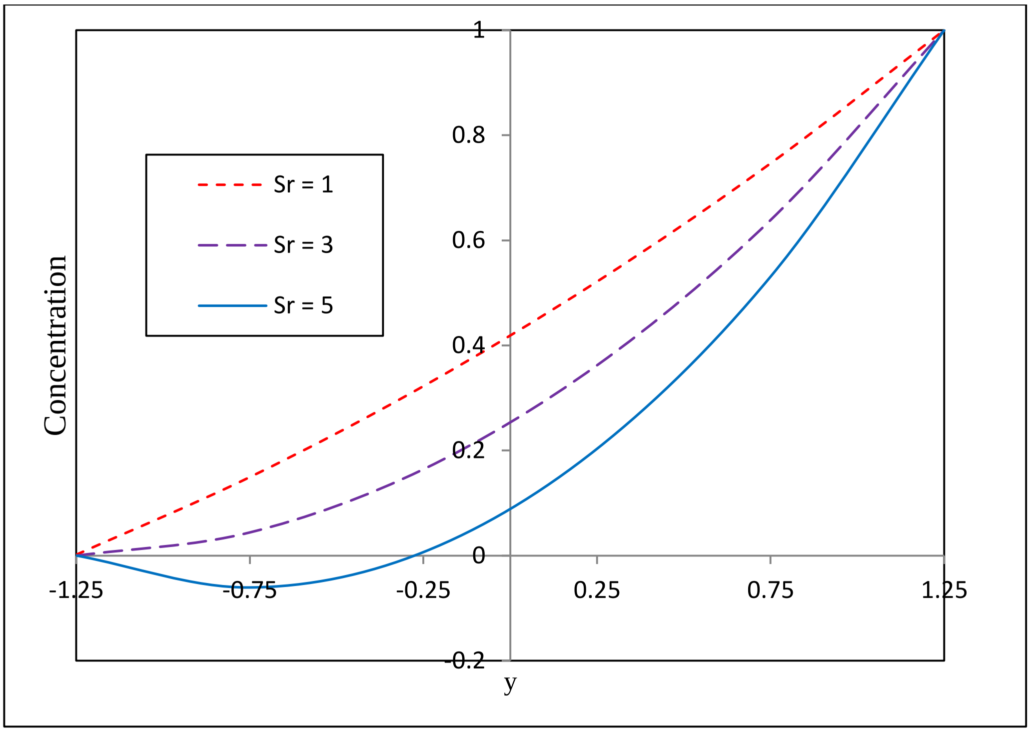

Figure 20. Concentration distribution (Mass transfer distribution) for different Values of Br with Da = 0.5, Pr = 2, Sc = 0.5, Br = 0.1, M = 1, β = 0.1, λ_1 = 0.5, ϕ = π/6, ε = 0.2, k_1= 0.1, x = 0.6, t = 0.4, p = 0.5

5. Conclusions

In this paper, we have proposed a theoretical study on simultaneous effects of joule heating and mass transfer on hydromagnetic peristaltic Jeffery fluid model filled with porous medium in a tapered vertical channel. We have concluded the following key observations:

(1) Axial velocity increases with an increase in Jeffery fluid λ_1, Magnetic field (M) and non-uniform parameter (k_1).

(2) The axial velocity diminished with an increase in Porosity parameter (Da).

(3) Pressure gradient enhances with an increase in Hartmann number M

(4) Pressure gradient diminished with an increase in porosity parameter Da, Jeffery fluid $\lambda_1$and volumetric flow rate $\bar{Q}$.

(5) The temperature of the fluid enhances with an increase in β, Br, Pr, Da and M.

(6) The temperature of the fluid diminished with an increase in Jeffery fluid $\lambda_{1.}$

(7) The concentration distribution decreases when increase in β, Pr, Sc, Br and Sr in an entire asymmetric vertical tapered channel.

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